

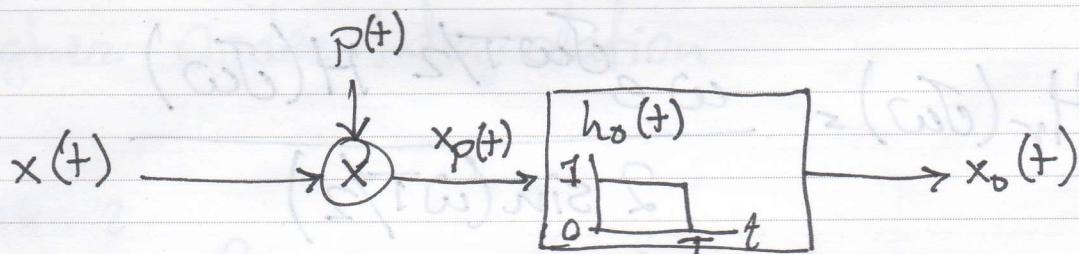
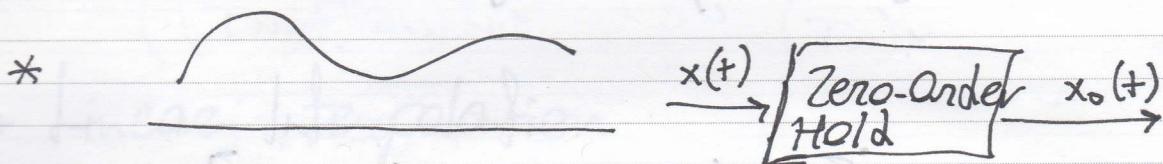
JAN FEB MAR APR MAY JUN JUL AUG SEP OCT NOV DEC
 01 02 03 04 05 06 07 08 09 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31

Lecture - Wednesday Nov. 18

Sampling with a Zero-Order Hold

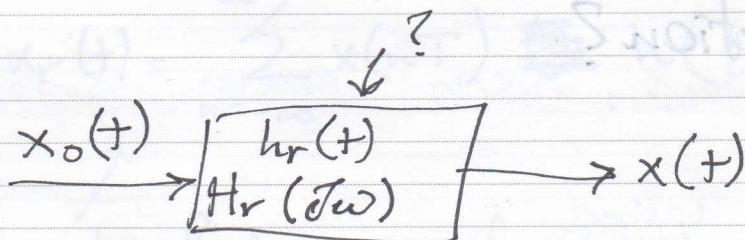
* Generating & Transmitting Impulse

approximation is difficult

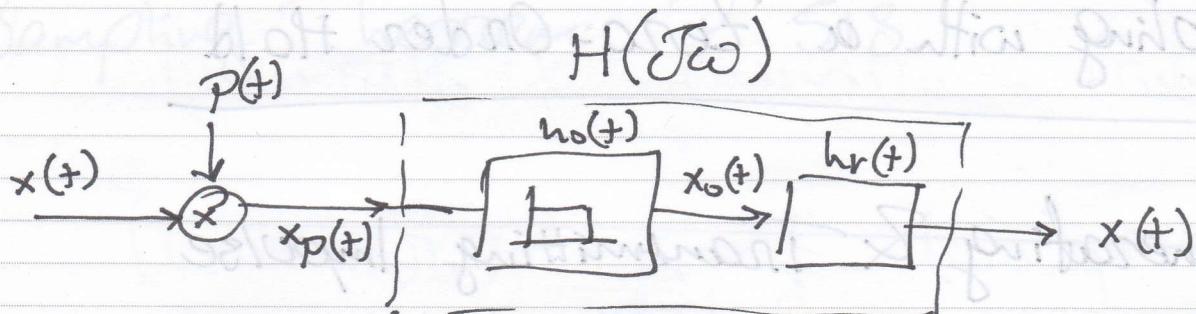


Illustrate why $x_o(t) = x_p(t) \neq h_o(t)$

To reconstruct $x(t)$ from $x_o(t)$



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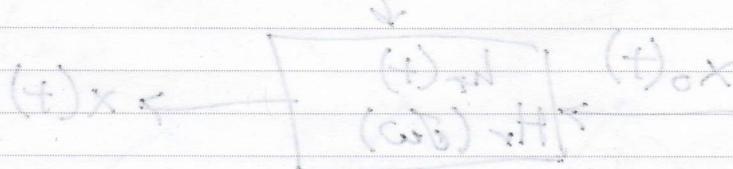
Ideal low-pass filter

$$H_o(j\omega) = e^{-j\omega T/2} \left[\frac{2 \sin(\omega T/2)}{\omega} \right]$$

$$\Rightarrow H_r(j\omega) = \frac{\omega e^{j\omega T/2} H(j\omega)}{2 \sin(\omega T/2)}$$

Output of Zero-Order Hold is an ~~approximation~~
approximation to the original signal

Smoother Interpolation?



Reconstruction of a signal from its samples using Interpolation

- * Fitting of a continuous signal to a set of sample values.

- * Linear Interpolation

Connecting Sample points by straight lines

- * Higher Order Interpolation

Consider

Low-pass filtering as Interpolation

(Band-limited Interpolation)

$$x_r(t) = x_p(t) * h(t)$$

$$x_r(t) = \sum_{n=-\infty}^{\infty} x(nT) \cancel{h(t-nT)}$$

How to fit a continuous curve between the sample points

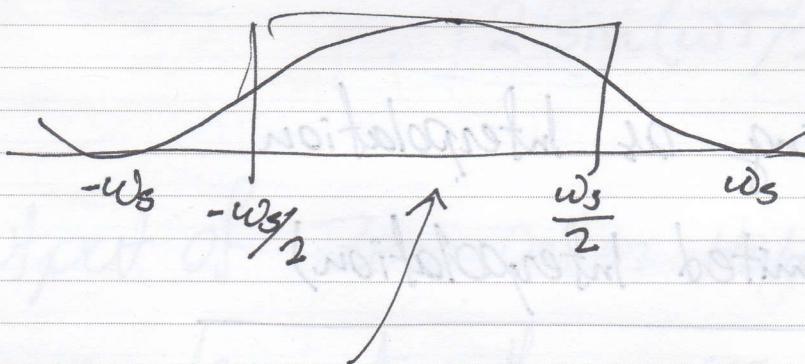
JAN	FEB	MAR	APR	MAY	JUN	JUL	AUG	SEP	OCT	NOV	DEC
01	02	03	04	05	06	07	08	09	10	11	12

for Ideal low pass filter

$$h(t) = \frac{\omega_c T \sin(\omega_c t)}{\pi \omega_c t}$$

$$x_r(t) = \sum_{n=-\infty}^{\infty} x(nT) \frac{\omega_c T}{\pi} \frac{\sin(\omega_c(t-nT))}{\omega_c(t-nT)}$$

- * Zero order hold as interpolation



Can work in image applications

because the human visual system applies natural low-pass filter

JAN	FEB	MAR	APR	MAY	JUN	JUL	AUG	SEP	OCT	NOV	DEC																			
01	02	03	04	05	06	07	08	09	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31

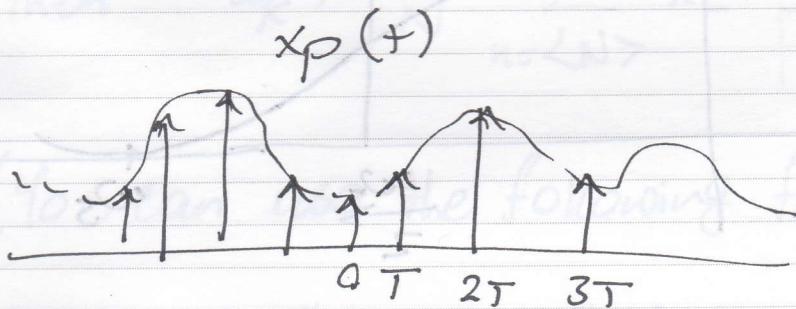
Higher-order holds

Output of Zero-order hold is discontinuous

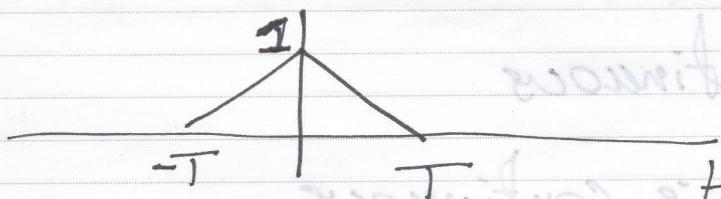
Output of Linear Interpolation is continuous

But derivative is discontinuous

Called First-Order Hold



$$h(t)$$



Explain why!

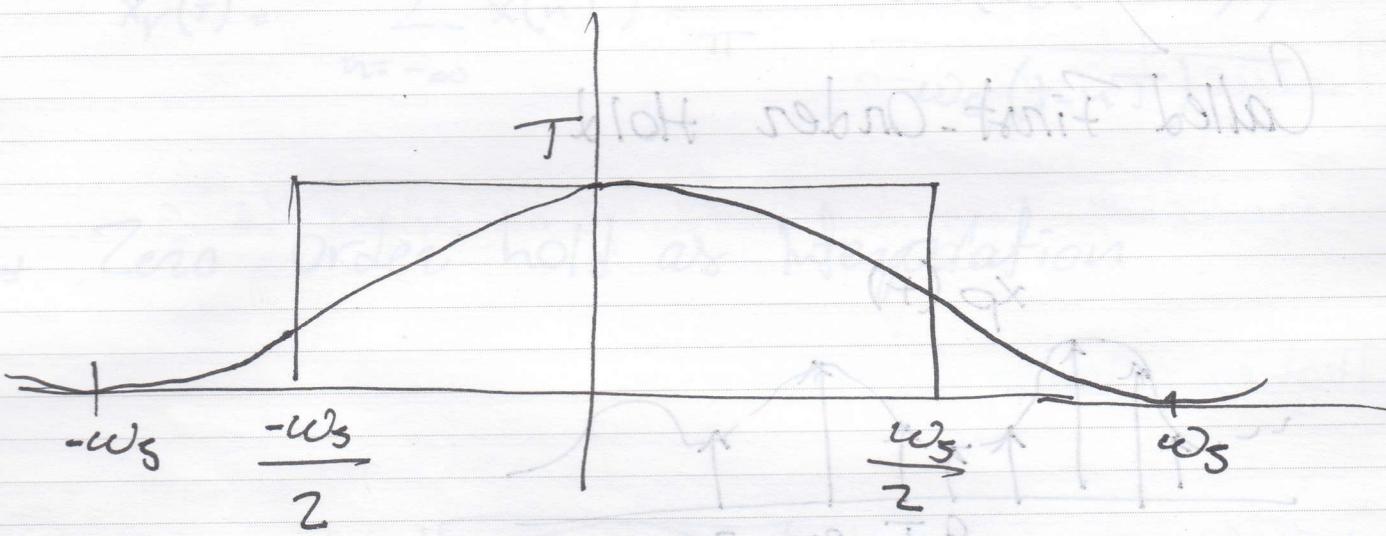
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$$H(j\omega) = \frac{1}{T} \left[\frac{\sin(\omega T/2)}{\omega/2} \right]^2$$

evident overshoot

why?

triangle is two squares convolved



Second Order hold

Output is continuous

first derivative is continuous

second derivative is continuous