

## Third Lecture

Periodicity properties of discrete-time complex exponentials

How is it different from continuous-time?

Signal with frequency  $\omega_0 + 2\pi$  is the same as that at frequency  $\omega_0$

$$e^{j(\omega_0 + 2\pi)n} = e^{j2\pi n} e^{j\omega_0 n}$$

Described by a frequency interval of length  $2\pi$

As we increase  $\omega_0$  from 0 to  $\pi$ , we get signals that oscillate more rapidly

As we increase  $\omega_0$  from  $\pi$  to  $2\pi$ ,  
we decrease the rate of oscillation

high are located near  $\omega_0 = \pm\pi$   
frequencies

$$e^{j\omega_0 n} = (e^{j\pi})^n = (-1)^n$$

Changing sign at each point  
in time

- For a discrete-time sequence to have period  $N$

$$e^{j\omega_0(n+N)} = e^{j\omega_0 n}$$
$$e^{j\omega_0 N} = 1 \quad (1)$$

For (1) to hold,  $\omega_0 N$  must be a multiple  
of  $2\pi$

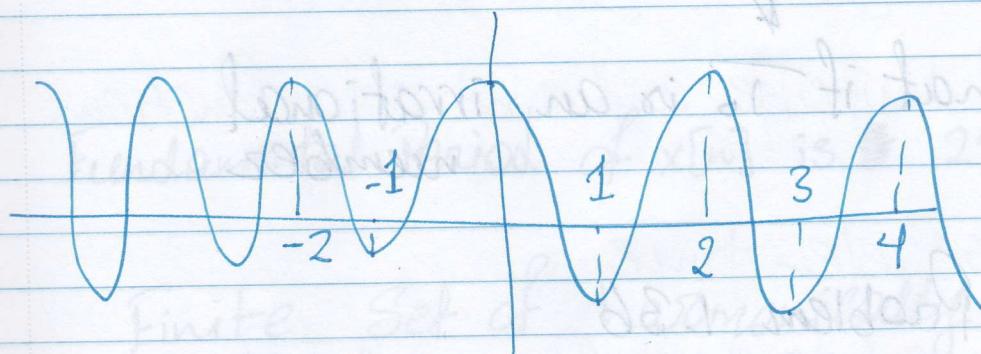
$$3m \text{ s.t. } \omega_0 N = 2\pi m,$$

$$\frac{\omega_0}{2\pi} = \frac{m}{N}$$

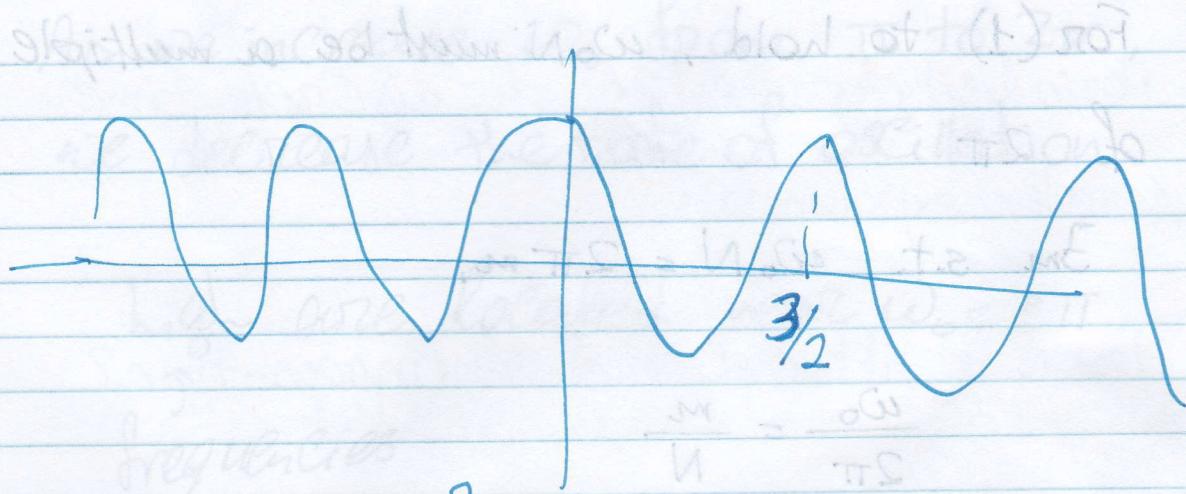
The signal  $e^{j\omega_0 n}$  is periodic iff

$\frac{\omega_0}{2\pi}$  is a rational number

Problem 1.35 \*Homework\*



Discrete-time sampled version has  
same peri fundamental period



$$T_0 = \frac{3}{2}$$

~~NS 3~~

NS 3

What if  $T_0$  is an irrational number

Problem 1.36

Example 1.6

Determine the fundamental period of

$$x[n] = e^{\underbrace{j(2\pi/3)n}_{\text{Fundamental Period}}} + e^{\underbrace{j(3\pi/4)n}_{\text{Fundamental Period}}}$$

$$N = m \left( \frac{2\pi}{\omega_0} \right)$$

$$\text{Try } m=8$$

$$N = \frac{2\pi}{2\pi/3} = 3$$

Fundamental Period of  $x[n]$  is 24

Finite Set of harmonically related  
N distinct complex exponential

$$\phi_{k+N}[n] = e^{j(k+N)(2\pi/N)n}$$

$$= e^{jk(2\pi/N)n} e^{j2\pi n}$$

$$= \phi_k[n]$$

$$\phi_0[n] = 1, \quad \phi_1[n] = e^{j2\pi n/N}$$

$$\phi_2[n] = e^{j4\pi n/N}$$

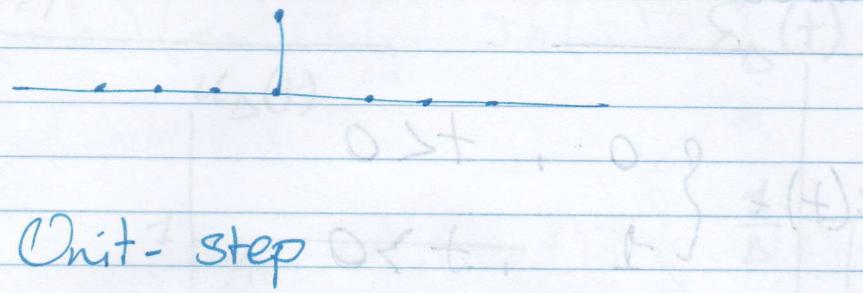
$$\phi_{N-1}[n] = e^{j2\pi(N-1)n/N}$$

$$\phi_N[n] = \phi_0[n] \quad \phi_{-1}[n] = \phi_{N-1}[n]$$

Unit Impulse and Unit Step Functions

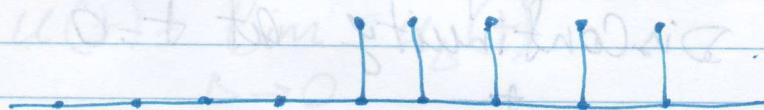
Discrete-time

$$\delta[n] = \begin{cases} 0, & n \neq 0 \\ 1, & n=0 \end{cases}$$



## Unit-step

$$u[n] = \begin{cases} 0, & n < 0 \\ 1, & n \geq 0 \end{cases}$$



$$S[n] = u[n] - u[n-1]$$

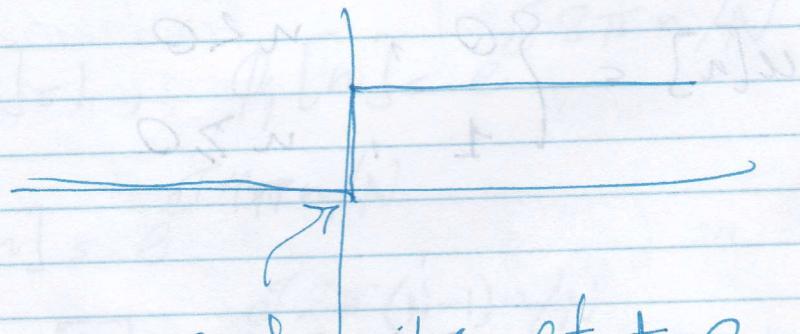
$$u[n] = \sum_{m=-\infty}^n \delta[m]$$

$$= \sum_{k=0}^{\infty} \delta[n-k]$$

$$= \sum_{k=0}^{\infty} \delta[n-k]$$

## Continuous time

$$u(t) = \begin{cases} 0, & t < 0 \\ -1, & t \geq 0 \end{cases}$$



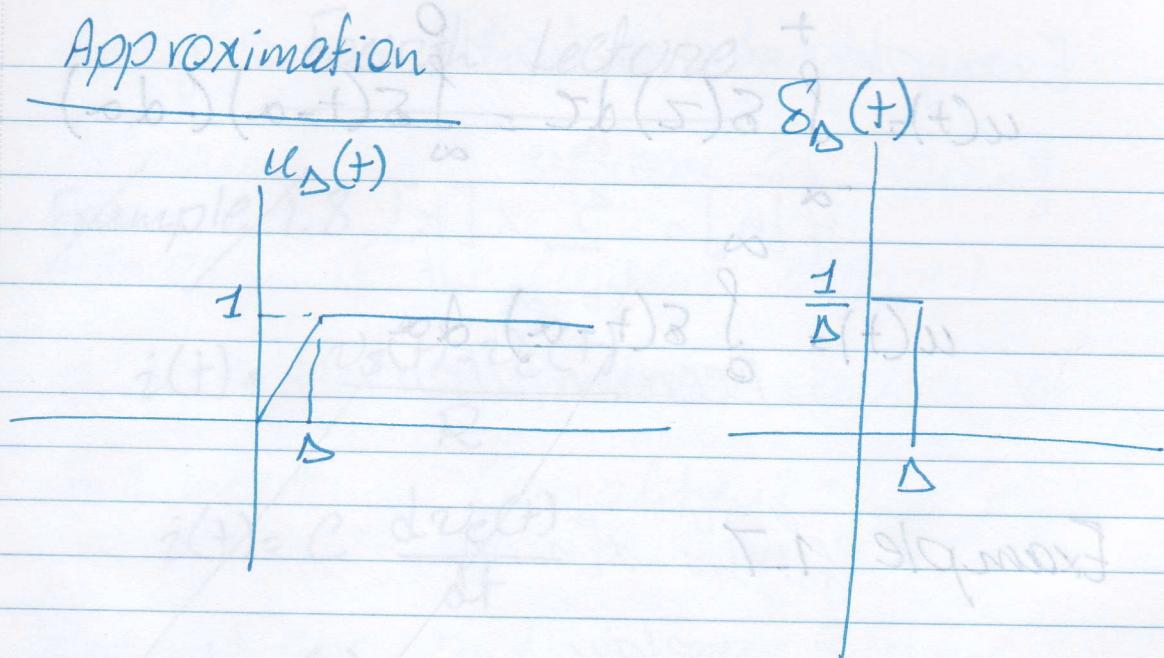
Discontinuity at  $t=0$

$$u(t) = \int_{-\infty}^t \delta(\tau) d\tau$$

$$\boxed{\delta(t) = \frac{du(t)}{dt}}$$

at  $t=0$ !

## Approximation

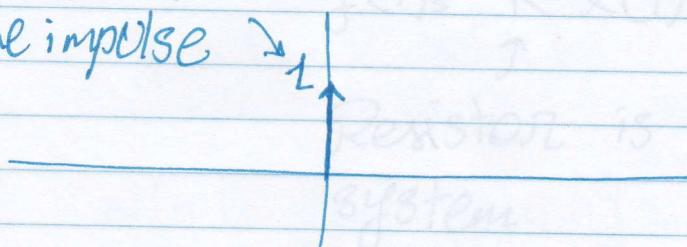


$$u(t) = \lim_{\Delta \rightarrow 0} u_D(t)$$

$$\delta_D(t) \approx \frac{du_D(t)}{dt}$$

$$\delta(t) = \lim_{\Delta \rightarrow 0} \delta_D(t)$$

Area of  
the impulse  $\int_1$



$$u(t) = \int_{-\infty}^{+\infty} \delta(\tau) d\tau = \int_{-\infty}^0 \delta(t-\alpha)(-\alpha) d\alpha$$

$$u(t) = \int_0^\infty \delta(t-\alpha) d\alpha$$

Example 1.7

Reads Section 1.5

$$\frac{(t)_{ab}}{t_b} z(t)_{ab}$$

$$(S(t)_{ab})_{ab} = (t)_{ab}$$

to work  
getting right