

## Fourth Lecture

Example 1.8

$$i(t) = \frac{v_s(t) - v_c(t)}{R}$$

$$i(t) = C \frac{dv_c(t)}{dt}$$

$$\frac{dv_c(t)}{dt} + \frac{1}{RC}$$

## Basic System Properties

\* Memoryless System

$$y[n] = (2x[n] - x^2[n])^2$$

$$y(t) = R x(t)$$

↑  
Resistor is a memoryless system

Example system with memory

$$y[n] = \sum_{k=-\infty}^n x[k]$$

Accumulator

$$y[n] = x[n-1]$$

Delay

A Capacitor is a continuous-time system with memory

$$y(t) = \frac{1}{C} \int_{-\infty}^t x(z) dz$$

Our formal definition leads to referring to a system as having memory if the current output is dependent on future values of the input "Causality!?"

## Invertibility and Inverse Systems

Distinct inputs lead to distinct outputs

$$y(t) = 2x(t)$$

$$w(t) = \frac{1}{2}y(t)$$

$$y[n] = \sum_{k=-\infty}^n x[k]$$

$$w[n] = y[n] - y[n-1]$$

$$y[n] = 0$$

$$y(t) = x^2(t)$$

↑  
Non invertible systems

Communication Systems

Causality

Nonanticipative systems

If two inputs to a causal system are identical up to some point in time, the corresponding outputs must also be equal up to this same time.

$$y[n] = x[n] - x[n+1]$$

is not causal

Not an essential constraint  
in applications in which the  
independent variable is not time

Processing recorded data

Example

$$y[n] = x[n]$$

non causal

$$y(t) = x(t) \cos(t+1)$$

is causal

# Stability

Bounded Input  $\rightarrow$  Bounded Output

Applying a constant force to an automobile

Stable system

Accumulator Input: step func.

$$y[n] = \sum_{k=-\infty}^n u[k] = (n+1)u[n]$$

Unstable

Time Invariance

~~If  $y[n]$~~

~~If  $x[n] = x[n-n_0]$~~

~~then  $y[n] = y[n-n_0]$~~

~~If the dependence of the output  $y(t)$  on the time variable  $t$  is only through the input  $x(t)$~~

A system is time invariant if a time shift in the input signal results in an identical time shift in the output signal.

If  $y[n]$  is o/p to  $x[n]$   
then  $y[n-n_0]$  is o/p to  $x[n-n_0]$

$$y(t) = \sin(x(t))$$

time invariant

$$y[n] = n x[n]$$

time varying

$$y(t) = \sin x(2t)$$

time varying

Linearity

Superposition

1. Response to  $x_1(t) + x_2(t)$  is  $y_1(t) + y_2(t)$

2. Response to  $ax_1(t)$  is  $ay_1(t)$

where  $a$  is any complex constant

~~Additive~~

$$x[n] = \sum_k a_k x_k[n]$$

$$y[n] = \sum_k a_k y_k[n]$$

$$0 \cdot x[n] \rightarrow 0 \cdot y[n] = 0$$

$$y(t) = t x(t) \text{ linear}$$

$$y(t) = x^2(t) \text{ Not}$$

$$y[n] = \operatorname{Re}\{x[n]\}$$

Additive

NOT homogeneous

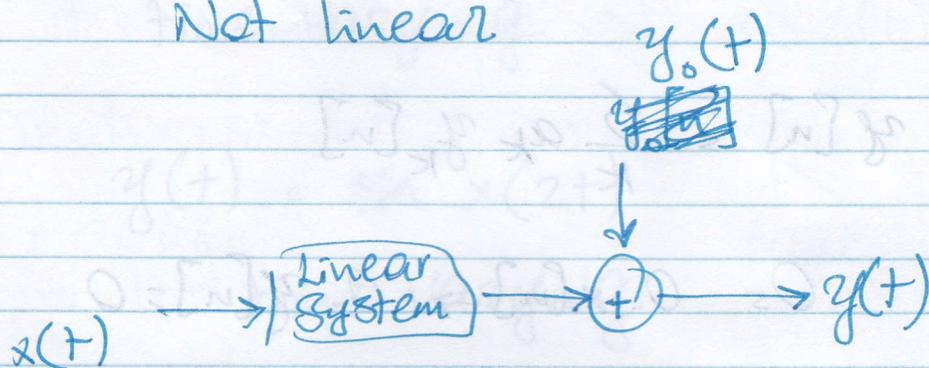
$$x_1[n] = r[n] + j s[n] \quad y_1[n] = r[n]$$

$$x_2[n] = j x_1[n] = -s[n] + j r[n]$$

$$y_2[n] = -s[n] \neq j y_1[n]$$

$$y[n] = 2x[n] + 3 = [n]x$$

Not linear



$y_0(t)$  Zero Input Response

\* Problem 1.47 \*

Incrementally linear systems

Difference between the responses to any two inputs is a linear function of the difference between the two inputs

Systems that respond linearly to changes in the input

$$y_1[n] - y_2[n] = 2x_1[n] + 3 - [2x_2[n] + 3] \\ = 2 \{x_1[n] - x_2[n]\}$$

We only need to know the impulse response to characterize system output

Discrete-time LTI systems:  
The convolution sum