

Sixth Lecture

Example 2.1

$h[n]$

$$\begin{array}{c} 1 \\ \hline 0 \ 1 \ 2 \end{array}$$

$x[n]$

$$\begin{array}{c} 0.5 \\ | \\ 1 \\ \hline 0 \ 1 \end{array}$$

$$y[n] = \sum_{k=-\infty}^{\infty} x[k] h[n-k]$$

$$= 0.5 h[n] + 2 h[n-1]$$

Example 2.2 (Same but different
way of thinking about the
solution)

Example 2.3

$$x[n] = \alpha^n u[n]$$

$$h[n] = u[n]$$

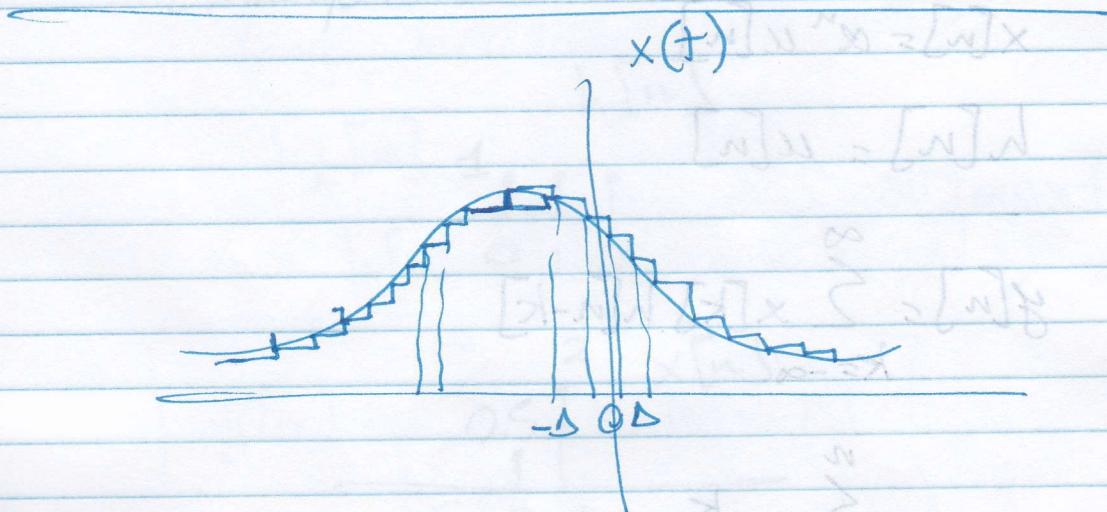
$$y[n] = \sum_{k=-\infty}^{\infty} x[k] h[n-k]$$

$$= \sum_{k=0}^n \alpha^k$$

$$= \left(\frac{1 - \alpha^{n+1}}{1 - \alpha} \right) u[n]$$

Continuous-time LTI Systems:

The Convolution Integral



$$s_{\Delta}(t) = \begin{cases} \frac{1}{\Delta}, & 0 \leq t \leq \Delta \\ 0, & \text{otherwise} \end{cases}$$

$$\hat{x}_{\Delta}(t) = \sum_{k=-\infty}^{\infty} x(k\Delta) s_{\Delta}(t-k\Delta) \Delta$$

$$x(t) = \lim_{\Delta \rightarrow 0} \hat{x}_{\Delta}(t)$$

$$x(t) = \int_{-\infty}^{\infty} x(z) \delta(t-z) dz$$

~~-inf~~

Scaled Impulse

at $t=z$ with area $x(t)$
and zero elsewhere

$$\int_{-\infty}^{\infty} x(z) \delta(t-z) dz$$

$$= \int_{-\infty}^{\infty} x(t) \delta(t-z) dz$$

$$= x(t) \int_{-\infty}^{\infty} \delta(t-z) dz = x(t)$$

For

~~As $\Delta \rightarrow 0$,~~

As $\Delta \rightarrow 0$, duration of $\delta_{\Delta}(t-k\Delta)$
is insignificant

$$(t)_d * (t)_x = (t)_y$$

As far as the system is concerned,
 the response to this pulse is, the
 essentially
 as the response to a unit impulse
 at the same point in time.

$$\hat{x}(t) =$$

$$x(t) = \lim_{\Delta \rightarrow 0} \sum_{k=-\infty}^{\infty} x(k\Delta) \delta_{\Delta}(t-k\Delta)$$

$$y(t) = \lim_{\Delta \rightarrow 0} \sum_{k=-\infty}^{\infty} x(k\Delta) h(t-k\Delta)$$

$$y(t) = \int_{-\infty}^{\infty} x(z) h(t-z) dz$$

Convolution Integral

$$y(t) = x(t) * h(t)$$

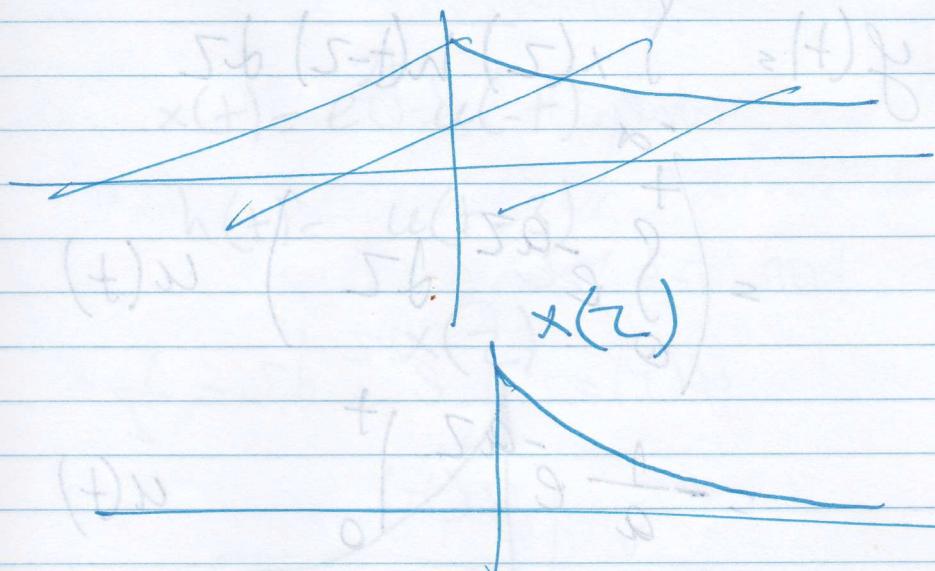
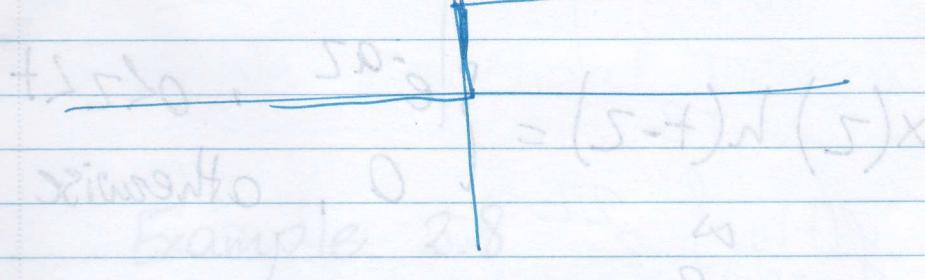
Example 2.6

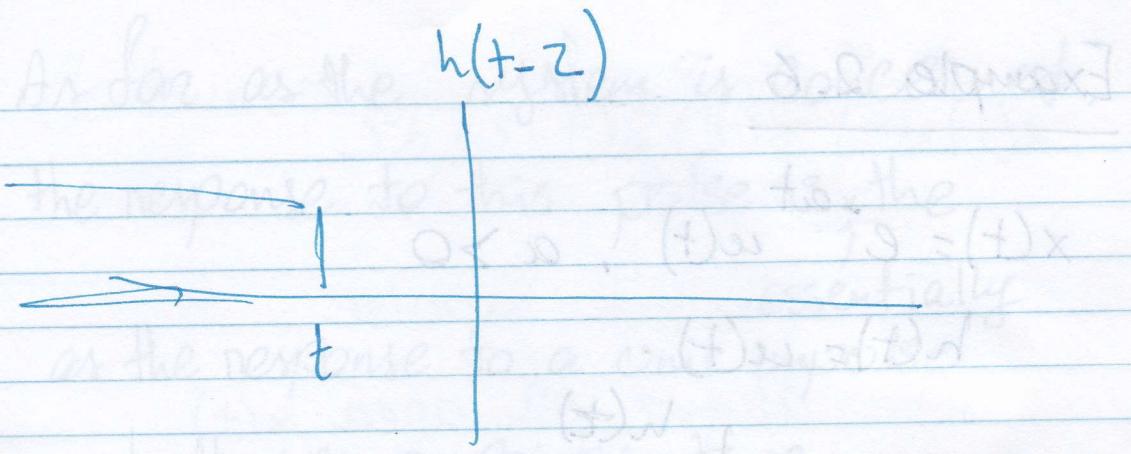
$$(s-t)u$$

$$x(t) = e^{-at} u(t), \alpha > 0$$

$$u(t) = u(t)$$

$$u(t)$$





$$x(z) h(t-z) = \begin{cases} e^{-az}, & 0 \leq z \leq t \\ 0, & \text{otherwise} \end{cases}$$

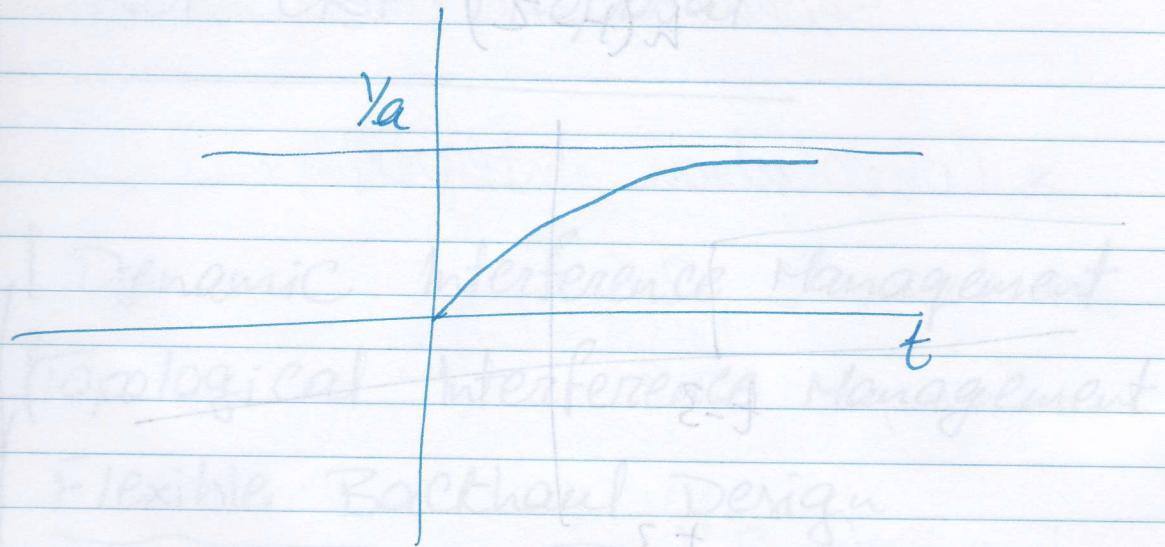
$$y(t) = \int_{-\infty}^t x(z) h(t-z) dz$$

$$= \left(\int_0^t e^{-az} dz \right) u(t)$$

$$= -\frac{1}{a} e^{-az} \Big|_0^t u(t)$$

$$= \frac{1}{a} (1 - e^{-at}) u(t)$$

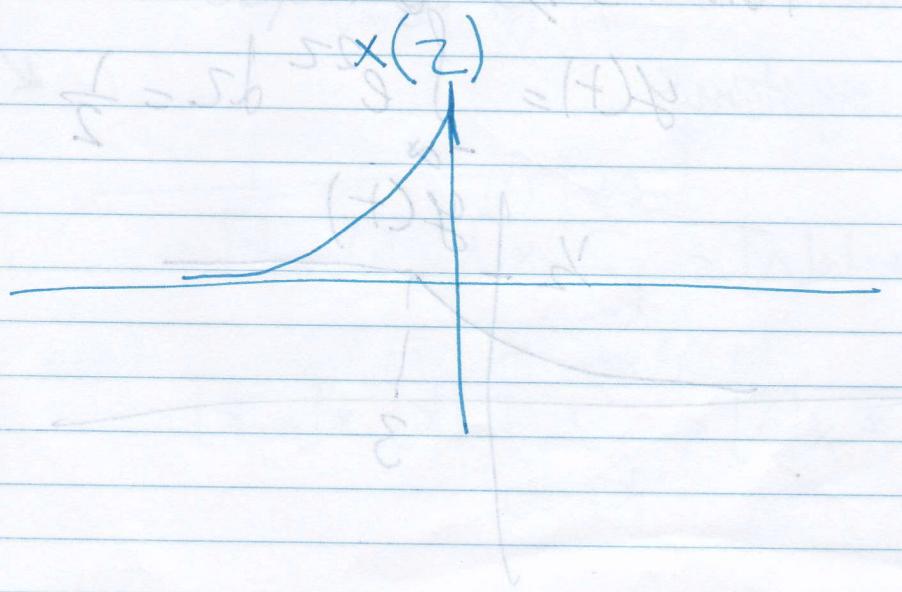
IEE-CRF Proposal



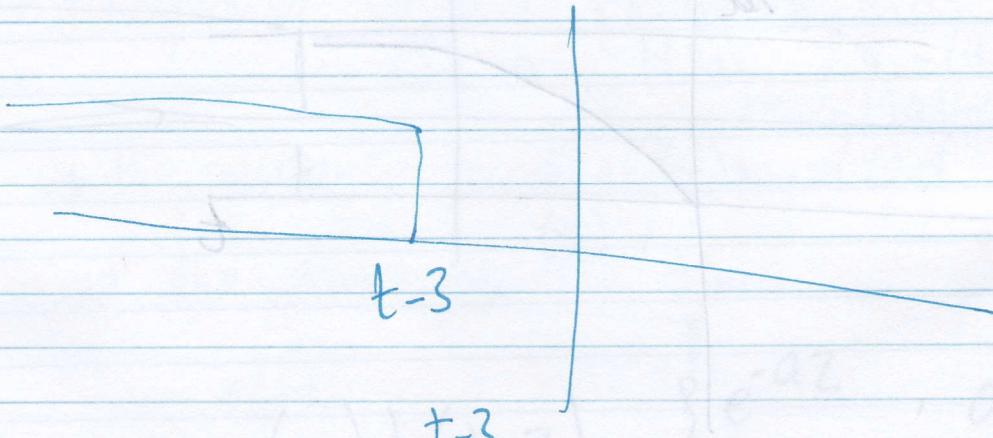
Example 2.8

$$x(t) = e^{2t} u(-t)$$

$$h(t) = u(t - 3)$$



$h(t-z)$



$$y(t) = \int_{-\infty}^{t-3} e^{2z} dz = \frac{1}{2} e^{2(t-3)}$$

when $t-3 \leq 0$

For $t-3 > 0$

$$y(t) = \int_{-\infty}^0 e^{2z} dz = \frac{1}{2}$$

