

Lecture Nine

Linear Constant-Coefficient Difference Equations

for continuous-time, N^{th} order linear constant coefficient differential equation

$$\sum_{k=0}^{N} a_k \frac{d^k y(t)}{dt^k} = \sum_{k=0}^{M} b_k \frac{d^k x(t)}{dt^k}$$

Discrete-time difference equations

$$\sum_{k=0}^{N} a_k y[n-k] = \sum_{k=0}^{M} b_k x[n-k] \quad (2)$$

With the condition of initial rest,
the system described by (2) is
LTI and causal

(2) can be rearranged

$$y[n] = \frac{1}{a_0} \left[\sum_{k=0}^{M-1} b_k x[n-k] - \sum_{k=1}^N a_k y[n-k] \right]$$

Recursive equation

Example

$$y[n] - \frac{1}{2} y[n-1] = x[n]$$

$$y[n] = x[n] + \frac{1}{2} y[n-1]$$

Assume initial rest and

$$x[n] = k \delta[n]$$

$$y[n] = \left(\frac{1}{2}\right)^n k$$

$$\text{for } k \neq 1, \quad y[n] = \left(\frac{1}{2}\right)^n$$

$$h[n] = \left(\frac{1}{2}\right)^n u[n]$$

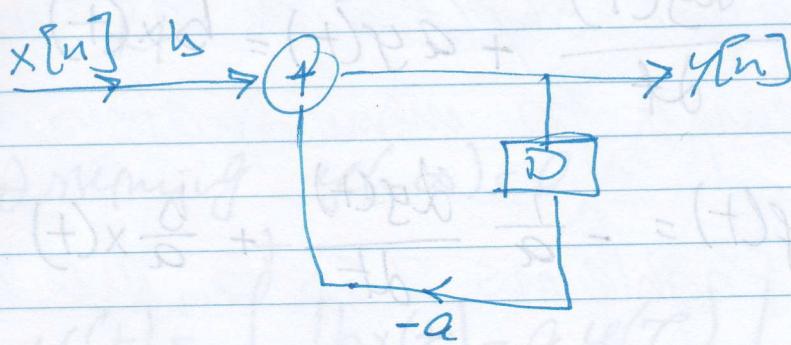
IIR System

Block Diagram Representation of
First-Order Systems Described by
Differential and Difference Equations

Consider the causal discrete-time system described by the first-order difference equation

$$y[n] + a y[n-1] = b x[n]$$

$$y[n] = -a y[n-1] + b x[n]$$



Used for

useful for early analog computer simulations of systems described by differential equations

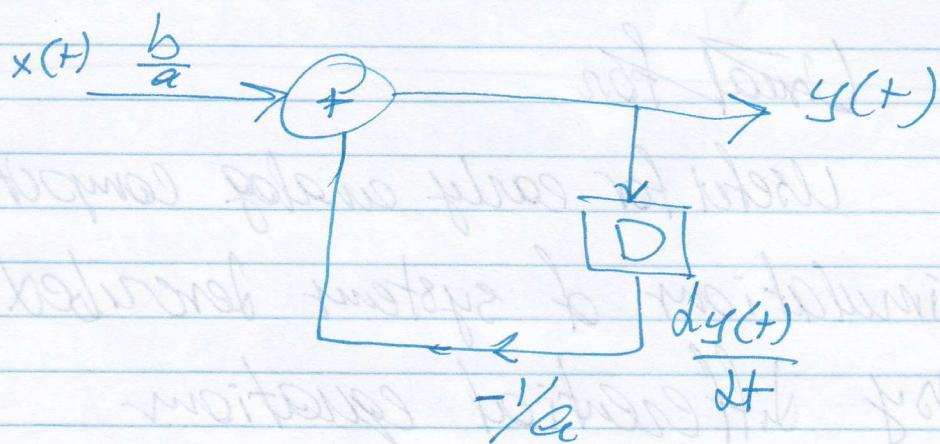
D is a memory element

initial value determines initial condition of system

Consider next

$$\frac{dy(t)}{dt} + ay(t) = bx(t)$$

$$y(t) = -\frac{1}{a} \frac{dy(t)}{dt} + \frac{b}{a}x(t)$$



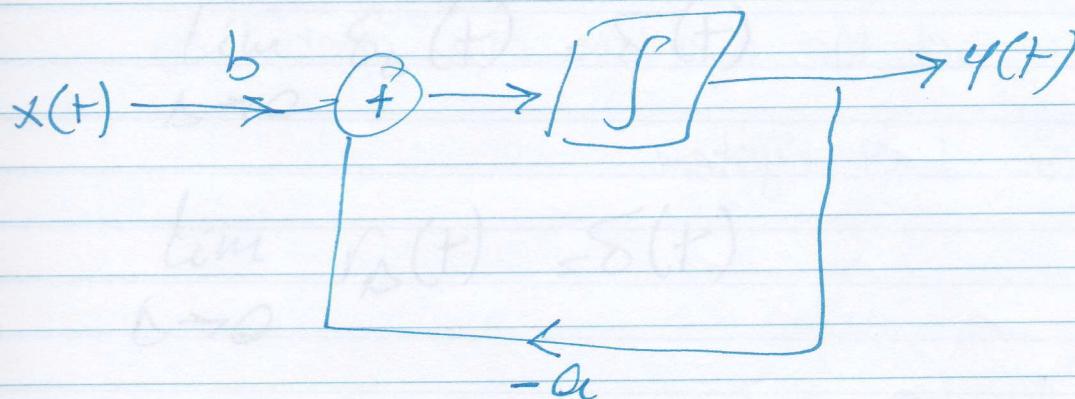
Differentiators are both difficult to implement and extremely sensitive to errors and noise.

Alternative Implementation

$$\frac{dy(t)}{dt} = b \times(t) - a y(t)$$

Assuming $y(-\alpha) = 0$

$$y(t) = \int_{-\alpha}^t [b \times(z) - a y(z)] dz$$



Integrators are implemented

using OP AMPS

Analog Computational Systems

* Tell them a bit about history
of digital computation as an
attempt to mimic nature *

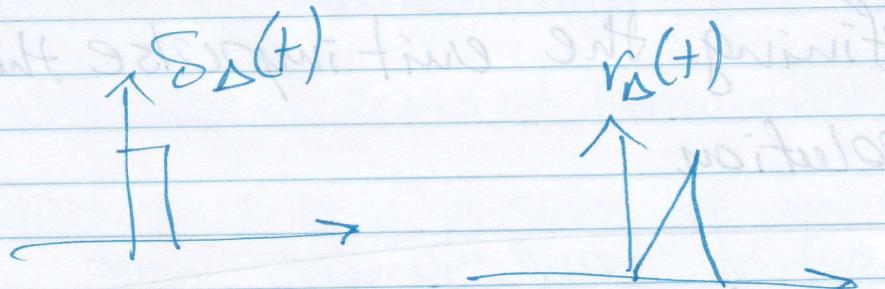
* Problems 2.58 and 2.60 *

Singularity Functions

The unit impulse as an Idealized Short Pulse

$$\epsilon(t) = \delta(t) * \delta(t)$$

$$r_\Delta(t) = \delta_\Delta(t) * \delta_\Delta(t)$$



$$\lim_{\Delta \rightarrow 0} \delta_\Delta(t) = \delta(t)$$

$$\lim_{\Delta \rightarrow 0} r_\Delta(t) = \delta(t)$$

Signals that behave like an impulse

As long the pulse is short enough, the response of an LTI system to all these signals is essentially identical

* Defining the unit impulse through Convolution

The signals $\delta_D(t)$, $r_D(t)$, $r_D(t) * \delta_D(t)$,
 $, r_D(t) * r_D(t)$

will act like impulses when applied to

an LTI system.

referred to as the Fourier series

The primary importance of the unit impulse is not what is at each value of t ,
but rather what it does under convolution

Energy signals

not S. All vib. are forth
along w/ vibration; average IT

the field of motion with the rod is an

series of vibrations of harmonics (pri-

related frequencies) of periodic

and along fixed to the int

Complex frequencies

2 (vibration & equal) 980

Vibration of the string

out is not sin, regard with in

relation to vibration

classical vibration, diff. from