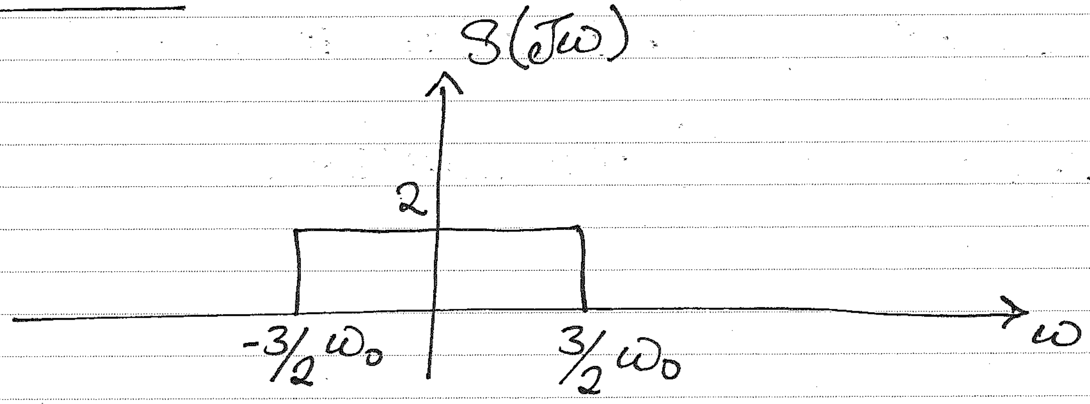
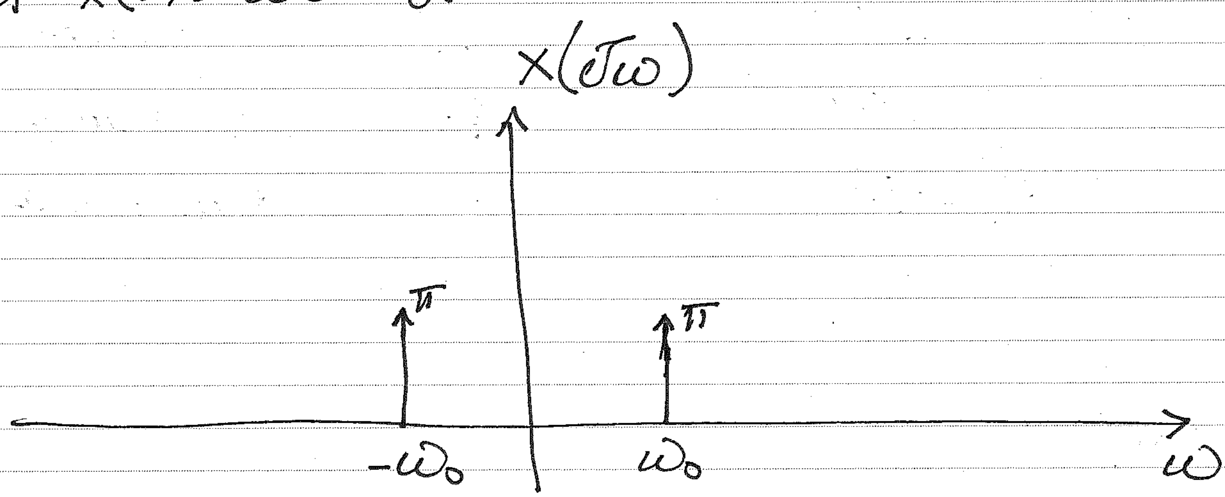
Lecture 33 - Monday Nov. 30

Exam 2 Solutions

Problem 4



Let $x(t) = \cos \omega_0 t$



$$R(j\omega) = \frac{1}{2\pi} S(j\omega) * X(j\omega)$$

Convolution

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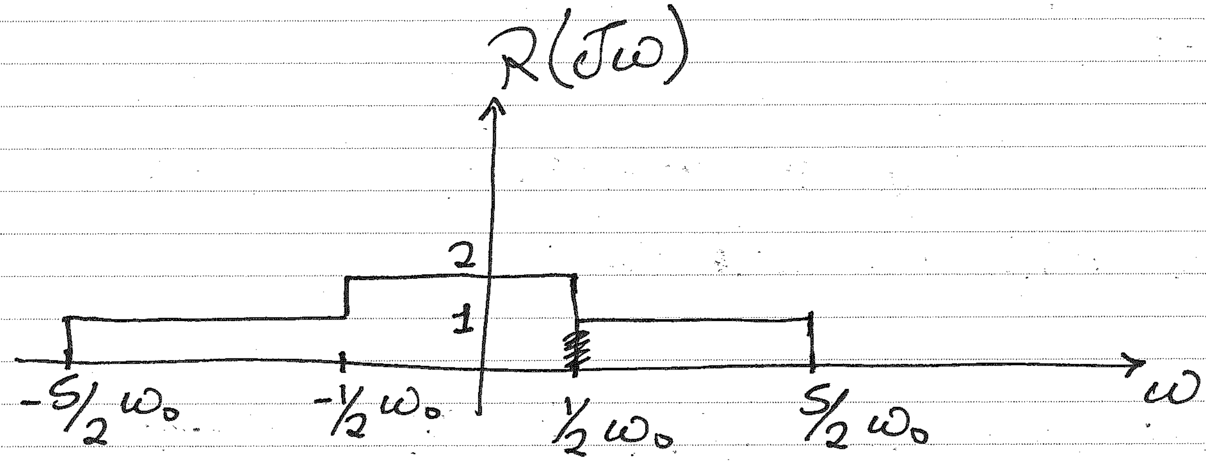

Problem 5

We did a very similar problem in class with $x[n] = \frac{\sin(\pi n/2)}{\pi n}$

The solution to this problem is the same except that $T_1 = \frac{7\pi}{8}$ instead of $\frac{\pi}{2}$

$$X(j\omega) = \begin{cases} 1 & , |\omega| \leq \frac{7\pi}{8} \\ 0 & , \frac{7\pi}{8} < |\omega| \leq \pi \end{cases}$$

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Problem 6

a) $\omega_s = 1000\pi$ is sufficient to sample $x[n]$ with no aliasing

$\Rightarrow X(j\omega) = 0$ for $|\omega| \geq \frac{1000\pi}{2} = 5000\pi$

b) $x(t)$ is obtained at the output of an ideal low pass filter with cutoff frequency $\omega_c = 1000\pi$

$\Rightarrow X(j\omega) = 0$ for $|\omega| \geq 1000\pi$

$\Rightarrow \omega_s \geq 2000\pi$ guarantees the sampling of $x(t)$ with no aliasing

a) $T = 0.5 \times 10^{-3} \Rightarrow \omega_s = \frac{2\pi}{T} = 4000\pi$ ✓

b) $T = 2 \times 10^{-3} \Rightarrow \omega_s = \frac{2\pi}{T} = 1000\pi$ ×

c) $T = 10^{-4} \Rightarrow \omega_s = 20000\pi$ ✓

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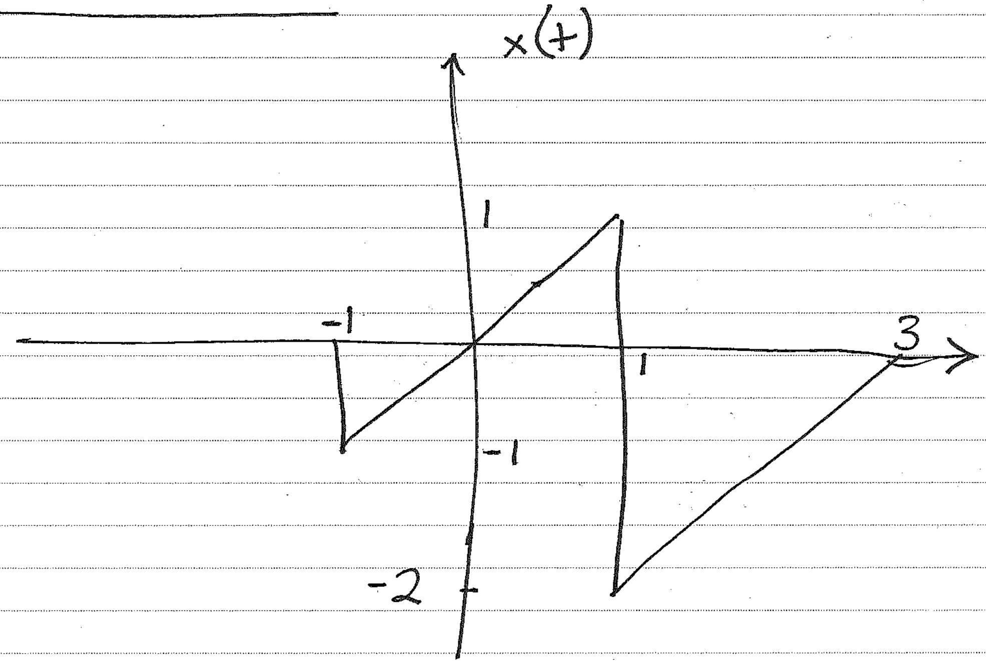
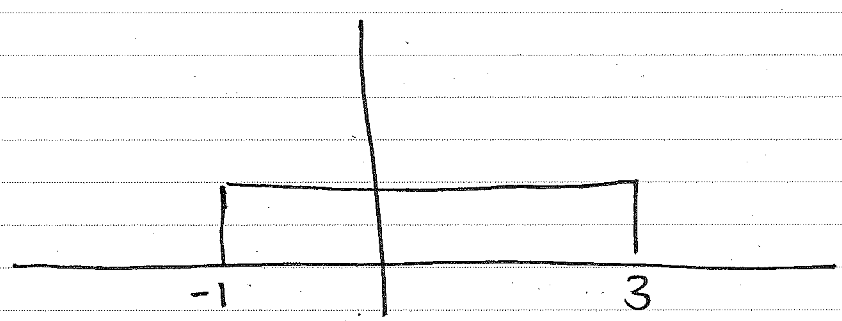
Problem 2

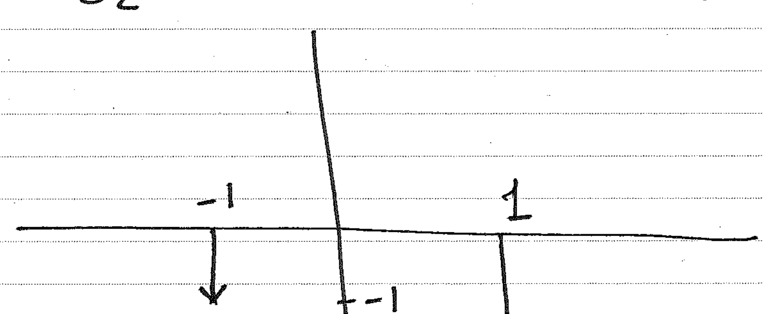


Let $g(t) = g_1(t) + g_2(t)$

where $g_1(t) = u(t+1) - u(t-3)$



and $g_2(t) = -\delta(t+1) - 3\delta(t-1)$



then $g(t) = \frac{dx(t)}{dt}$

$g_1(t) = y(t-1)$

where $y(t) = \begin{cases} 1, & |t| \leq 2 \\ 0, & |t| > 2 \end{cases}$

$$Y(j\omega) = \frac{2 \sin 2\omega}{\omega}$$

$$G_1(j\omega) = e^{-j\omega} Y(j\omega) = e^{-j\omega} \frac{2\sin(2\omega)}{\omega}$$

$$G_2(j\omega) = -e^{j\omega} - 3e^{-j\omega}$$

$$G(j\omega) = e^{-j\omega} \frac{2\sin(2\omega)}{\omega} - e^{j\omega} - 3e^{-j\omega}$$

$$X(j\omega) = \frac{1}{j\omega} G(j\omega) + \pi G(0)\delta(\omega)$$

$$G(0) = -4$$

$$X(j\omega) = \frac{1}{j\omega}\left[e^{-j\omega} \frac{2\sin(2\omega)}{\omega} - e^{j\omega} - 3e^{-j\omega}\right] - 4\pi\delta(\omega)$$

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Problem 3

(i) $$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n] e^{-j\omega n}$$

$$X^*(e^{-j\omega}) = \sum_{n=-\infty}^{\infty} x^*[n] e^{-j\omega n}$$

$$\Rightarrow x^*[n] \xleftrightarrow{FT} X^*(e^{-j\omega})$$

$$Re\{x[n]\} = \frac{x[n]+x^*[n]}{2} \xleftrightarrow{FT} \frac{X(e^{j\omega})+X^*(e^{-j\omega})}{2}$$

(ii) $$X(e^{-j\omega}) = \sum_{n=-\infty}^{\infty} \underline{x[-n] e^{-j\omega n}}$$

same as $\sum x[n] e^{j\omega n}$

$$x[-n] \xleftrightarrow{FT} X(e^{-j\omega})$$

$$x^*[n] \xleftrightarrow{FT} X^*(e^{-j\omega})$$

$$\Rightarrow x^*[-n] \xleftrightarrow{FT} X^*(e^{j\omega})$$

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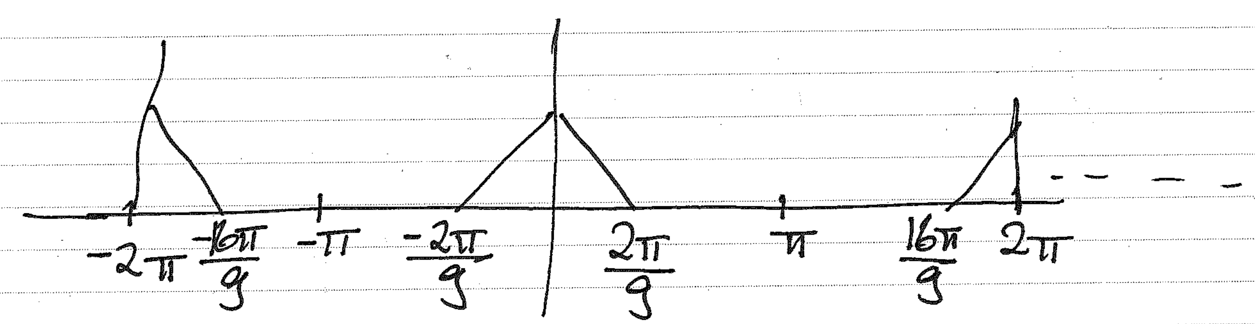
iii) $\mathcal{E}v\{x[n]\} = \dfrac{x[n]+x[-n]}{2} \xleftrightarrow{FT} \dfrac{X(e^{j\omega})+X(e^{-j\omega})}{2}$

Example 7.5

The maximum possible downsampling is achieved once the non-zero portion of one period of the discrete-time spectrum has expanded to fill the entire band from $-\pi$ to ~~#~~ π



Lowest sampling rate with no aliasing : $\frac{2\pi}{4}$