The Region of Convergence for Laplace Transform

Property 1: The ROC of $X(s)$ consists of strips parallel to the $j\omega$-axis

The ROC of $X(s)$ consists of the values of $s = \sigma + j\omega$ for which the Fourier Transform of $x(t)\,e^{-\sigma t}$ converges.

$$\int_{-\infty}^{\infty} |x(t)|\, e^{-\sigma t}\, dt < \infty$$

Depends only on $Re\{s\}$

Property 2: For rational Laplace Transforms, the ROC does not contain any poles.

Property 3: If $x(t)$ is of finite duration and is absolutely integrable, then the ROC is the entire s-plane

the multiplicative exponential weight is never unbounded

Let $x(t) \neq 0$ only for $T_1 \leq t \leq T_2$

For $Re\{s\} = 0$

$$\int_{T_1}^{T_2} |x(t)| \, e^{-at} \, dt = \int_{T_1}^{T_2} |x(t)| \, dt < \infty$$

For $Re\{s\} < 0$

$$\int_{T_1}^{T_2} |x(t)| \, e^{-at} \, dt < e^{-aT_1} \int_{T_1}^{T_2} |x(t)| \, dt < \infty$$

For $Re\{s\} > 0$

$$\int_{T_1}^{T_2} |x(t)| \, e^{-at} \, dt < e^{-aT_2} \int_{T_1}^{T_2} |x(t)| \, dt < \infty$$

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Example 9.6

Let $x(t) = \begin{cases} e^{-at}, & 0 < t < T \\ \cancel{oth}\ 0 & \text{otherwise} \end{cases}$

$$X(s) = \int_0^T e^{-at} e^{-st}\, dt = \underline{\frac{1}{s+a}\left[1 - e^{-(s+a)T}\right]}$$

at $s = -a$

$$\lim_{s \to -a} X(s) = \lim_{s \to a} \left[\frac{\frac{d}{ds}\left(1 - e^{-(s+a)T}\right)}{\frac{d}{ds}(s+a)} \right]$$

$$= \lim_{s \to -a} T e^{-aT} e^{-sT}$$

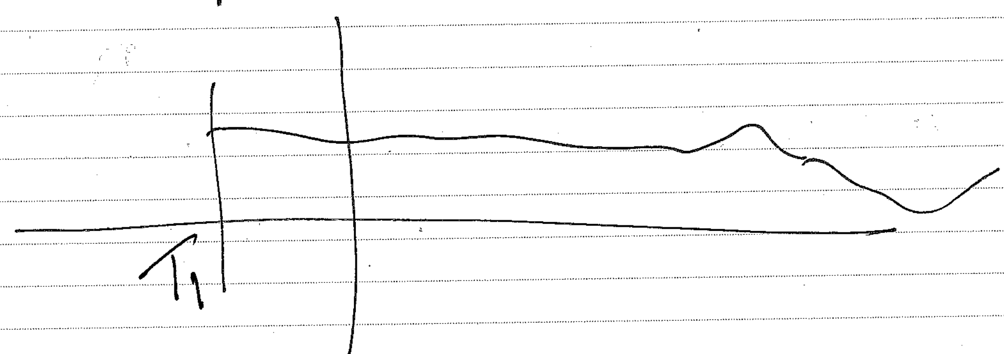
so that

$$X(-a) = T$$

We said that the ROC does not contain poles

Property 4: If $x(t)$ is right sided, and if the line $Re\{s\} = a_0$ is in the ROC, then all values of s for which $Re\{s\} > a_0$ will also be in the ROC.

$x(t) = 0$ prior to some finite T_1



$$\int_{-\infty}^{\infty} |x(t)| e^{-a_0 t} dt < \infty$$

$$\int_{T_1}^{\infty} |x(t)| e^{-a_0 t} dt < \infty$$

If $a_1 > a_0$

$$\int_{T_1}^{\infty} |x(t)| e^{-a_1 t} dt = \int_{T_1}^{\infty} |x(t)| e^{-a_0 t} e^{-(a_1 - a_0)t} dt$$

$$< e^{-\underbrace{(a_1 - a_0)}_{}T_1} \int_{T_1}^{\infty} |x(t)| e^{-a_0 t} dt$$

ROC in this case is called a right-half plane

Property 5: If $x(t)$ is left sided, and if the line $Re\{s\} = a_0$ is in the ROC, then all values of s for which $Re\{s\} < a_0$ will also be in the ROC

the ROC in this case is called a ~~left-sided~~ ~~signal~~ left-half plane

Property 6: If $x(t)$ is two-sided, and if the line $Re\{s\} = a_0$ is in the ROC, then the ROC will consist of a strip in the s-plane that includes the line $Re\{s\} = a_0$

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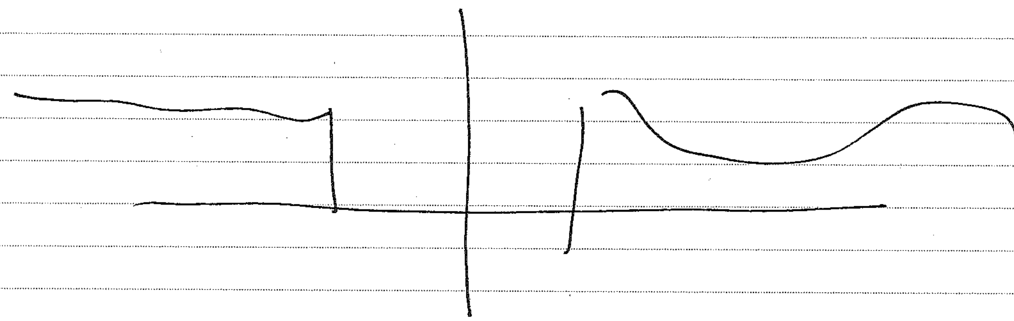
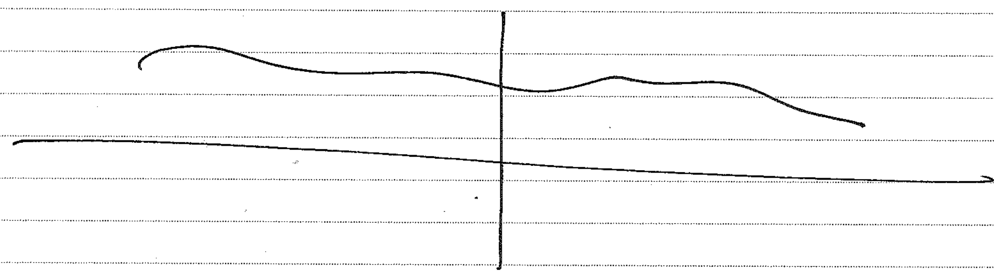




Signal $x(t) = x_R(t) + x_L(t)$

Right Sided — ROC $Re\{s\} > a_R$

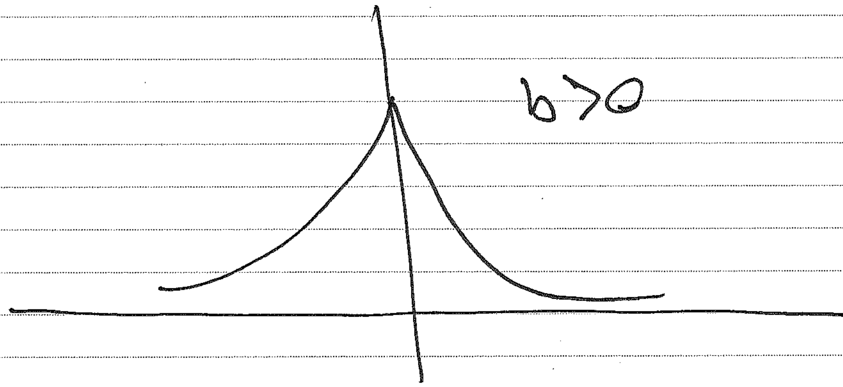
Left Sided — ROC $Re\{s\} < a_L$

If $a_R < a_L$ the ROC $a_R < Re\{s\} < a_L$

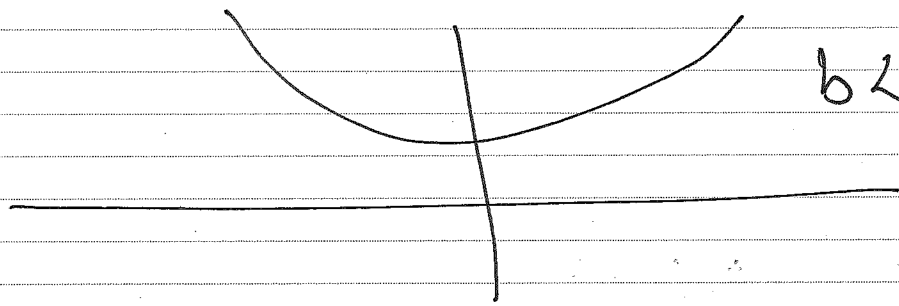
Example 9.7

$$x(t) = e^{-b|t|}$$

$$x(t) = e^{-bt}u(t) + e^{bt}u(-t)$$



$b>0$



$b<0$

$$e^{-bt}u(t) \overset{\mathcal{L}}{\longleftrightarrow} \frac{1}{s+b}, \quad Re\{s\}>-b$$

$$e^{bt}u(-t) \overset{\mathcal{L}}{\longleftrightarrow} \frac{-1}{s-b}, \quad Re\{s\}<b$$

If $b \leq 0$, Laplace Transform does not converge

If $b>0$, the Laplace Transform is

$$e^{-b|t|} \overset{\mathcal{L}}{\longleftrightarrow} \frac{1}{s+b} - \frac{1}{s-b} = \frac{-2b}{s^2-b^2},$$

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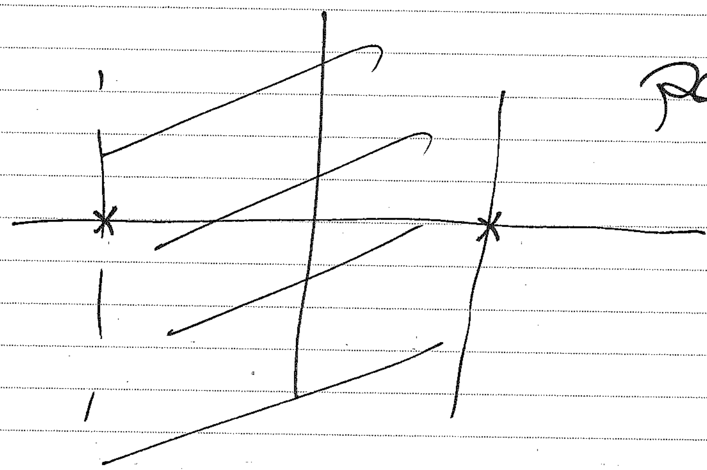

Pole-zero plot

Property 7:

If the Laplace Transform $X(s)$ of $x(t)$ is rational, then its ROC is bounded by poles or extends to infinity. In addition, no poles of $X(s)$ are contained in the ROC.

Validity follows from the facts that a signal with rational Laplace transforms consists of a linear combination of exponentials

~~Propet~~

Property 8

If the Laplace Transform is rational, the if $x(t)$ is right sided, the ROC is the region ~~to~~ ~~in~~ to the right of the rightmost pole. If $x(t)$ is left sided, the ROC is the region to the left of the leftmost pole.

Example 9.8

$$X(s) = \frac{1}{(s+1)(s+2)}$$

Possible ROCs