Cooperation in Large Cellular Networks: Insights and Fundamental Limits

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Cloud-Based Communication

Global Knowledge / Control available at Central nodes
Cloud-Based Coordinated Multi-Point (CoMP)

Enabling centralized approaches:

1. Cell association decisions
2. Transmission schedules

Enabling CoMP Gains for Cell Edge Users
Locally Connected Interference Networks

\[ W_1 \xrightarrow{Tx_1} Rx_1 \xrightarrow{\hat{W}_1} \]
\[ W_2 \xrightarrow{Tx_2} Rx_2 \xrightarrow{\hat{W}_2} \]
\[ W_3 \xrightarrow{Tx_3} Rx_3 \xrightarrow{\hat{W}_3} \]
\[ W_4 \xrightarrow{Tx_4} Rx_4 \xrightarrow{\hat{W}_4} \]

Generic Time Varying Channel

\( T_{x \ i} \) connected to \( R_{x \ \{i, i+1, \ldots, i+L\}} \)
Locally Connected Interference Networks

BS: Base Station
MT: Mobile Terminal
Cloud-Based Cell Associations

Each Mobile Terminal can be associated with $N$ Base Stations
Degrees of Freedom (DoF)

\[ \text{DoF} = \lim_{\text{SNR} \to \infty} \frac{\text{sum capacity}}{\log \text{SNR}} \]

- **Objective:** Determine Per User DoF as a function of \( N \).

\[ \text{PUDoF}(N) = \lim_{K \to \infty} \frac{\text{DoF}(K, N)}{K} \]

**What is the optimal cell association?**

- **Downlink:** \( \text{PUDoF}_D(N) \)
- **Uplink:** \( \text{PUDoF}_U(N) \)
- **Average:** \( \text{PUDoF}_{UD}(N) \)
Justifying Choices: Network Topology

Local Connectivity:
- Reflects path loss
- Simplifies problem, only consider local cooperation

Large Networks:
- Understand scalability
- Derive insights

Solutions generalize to cellular network models
Justifying Choices: Network Topology

Cellular Network Model
Solutions for $L = 2$ are applicable
Modelling a limited capacity backhaul:

- Each mobile terminal can be associated with $N$ base stations
- Associations reflect the allocation of messages to transmitters in the downlink
- In uplink, associations allow base stations to decode the mobile terminal’s message

Digital Backhaul in both Uplink and Downlink

Solutions can generalize to more practical constraints
Justifying Choices: Degrees of Freedom

Advantages:

1. Simplicity
2. Captures the interference effect (without noise)
3. Highlights the combinatorial part of the problem

Drawbacks:

1. Insensitive to Gaussian noise
2. Insensitive to varying channel strengths
Results: Downlink

Using One-Shot Zero-Forcing:

$$\text{PUDoF}_{ZF}^D(N) = \frac{2N}{2N + L}$$

$$\geq \frac{1}{2}, \forall N \geq \frac{L}{2}$$

Optimal for $L = 1$

Assigning $W_i$ to $\text{Tx} \{i, i + 1, \ldots, i + N - 1\} \Rightarrow \frac{N}{N + L}$
Results: Downlink

Average Backhaul Load $B$ (Associations per message):

$$\text{PUDoF}^{ZF}_D(B, L = 1) = \text{PUDoF}_D(B, L = 1) = \frac{4B - 1}{4B}$$

Compare to $\frac{2N}{2N+1}$

Can achieve $\frac{1}{2}$ using zero-forcing and $B = 1$ for $L \leq 6$

Achieved using convex combination of $N = 2B$ and $N = 2B - 1$
Uplink: Achieving Full DoF

Associating each MT with two BSs connected to it

Message Passing Decoding: Interference-free Degrees of Freedom
Results: Uplink

$$PUDoF^ZF_U(N) \geq \begin{cases} 
1 & L + 1 \leq N \\
\frac{N+1}{L+2} & \frac{L}{2} \leq N \leq L \\
\frac{2N}{2N+L} & 1 \leq N \leq \frac{L}{2} - 1 
\end{cases}$$

$$\geq \frac{1}{2}, \forall N \geq \frac{L}{2}$$

Higher than Downlink

Is Cooperation useful for $$N < \frac{L}{2}$$?

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1M. Singhal, A. El Gamal, “Joint Uplink-Downlink Cell Associations for Interference Networks with Local Connectivity,” submitted to ISIT ’17
Average Uplink-Downlink DoF

Downlink Associations

\[ N = 3 \]

Uplink Associations

\[ \text{PUDoF} = \frac{1 + \frac{4}{5}}{2} = \frac{9}{10} \]
Average Uplink-Downlink DoF

\[
PUDoF_{UD}(N, L = 1) = \frac{4N-3}{4N-2}
\]
Average Uplink-Downlink DoF

For \( L \geq 2 \):

\[
P_{UD}^{ZF}(N) \geq \begin{cases} 
\frac{1}{2} \left( 1 + \left( \frac{\left\lceil \frac{L}{2} \right\rceil + \delta + N - (L+1)}{N} \right) \right) & \text{if } L + 1 \leq N \\
\frac{2N}{2N+L} & \text{if } 1 \leq N \leq L 
\end{cases}
\]

where \( \delta = (L + 1) \mod 2 \).

For \( L + 1 \leq N \), scheme is different from both downlink and uplink.
Further Questions

1. General network topologies

2. When to simplify into optimizing for uplink / downlink only

3. Constrain average number of cell associations
Deep Fading Block Erasures\textsuperscript{2}

Communication takes place over blocks of time slots.

- Link block erasure probability $p$ (long-term fluctuations).
- Non-erased links are generic (short-term fluctuations).

Maximize average performance

\textsuperscript{2} A. El Gamal, V. Veeravalli, “Dynamic Interference Management,” Asilomar ’13
Dynamic Linear Interference Network

$\text{Tx } i \text{ can only be connected to receivers } \{i, i + 1\}$

Each of the dashed links can be erased with probability $p$
Average Degrees of Freedom (DoF)

\[
\text{DoF} = \lim_{\text{SNR} \to \infty} \frac{\text{sum capacity}}{\log \text{SNR}}
\]

\[
PUDoF(N) = \lim_{K \to \infty} \frac{\text{DoF}(K, N)}{K}
\]

- For dynamic topology: PUDoF is a function of \( p \) and \( N \)

\[
PUDoF(p, N) = \mathbb{E}_p [PUDoF(N)]
\]
Cell Association \((N = 1)\)

**Theorem**

*For the Cell Association problem in dynamic Wyner’s linear model,*

\[
PUDoF(p, N = 1) = \max \left\{ PUDoF^{(1)}(p), PUDoF^{(2)}(p), PUDoF^{(3)}(p) \right\}
\]

\(PUDoF^{(1)}(p)\): Optimal at high values of \(p\)

\(PUDoF^{(2)}(p)\): Optimal at low values of \(p\)

\(PUDoF^{(3)}(p)\): Optimal at middle values of \(p\)

Achievable through TDMA
Cell Association ($N = 1$): High Erasure Probability

Maximize probability of message delivery
Cell Association ($N = 1$): Low Erasure Probability

Avoiding Interference
Cell Association ($N = 1$): Low Erasure Probability

Avoiding Interference
Cell Association ($N = 1$)

Optimal at middle values of $p$
Cell Association \((N = 2)^3\)

1. Identified optimal zero-forcing associations

2. As \(p\) goes from 1 to 0, role of cooperation shifts to interference management

3. As \(p\) goes from 0 to 1, role of cooperation shifts to coverage extension

Knowledge of \(p\) is necessary

Needed level of accuracy?

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\(^3\)Y. Karacora, T. Seyfi, A. El Gamal, “The Role of Transmitter Cooperation in Linear Interference Networks with Block Erasures,” submitted to ISIT ’17
Conclusions

Cloud-Based Wireless Networks:

- Enabling centralized approaches
- new questions and conclusions
- Value of flexible cell association
- Significant CoMP gains
- Learning network topology?
- Benefit with no CSIT / Ad-hoc networks?
We are writing a book!