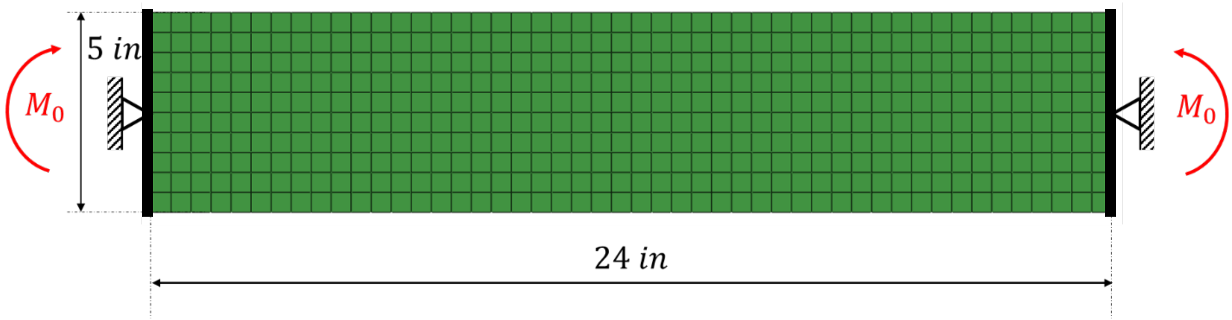
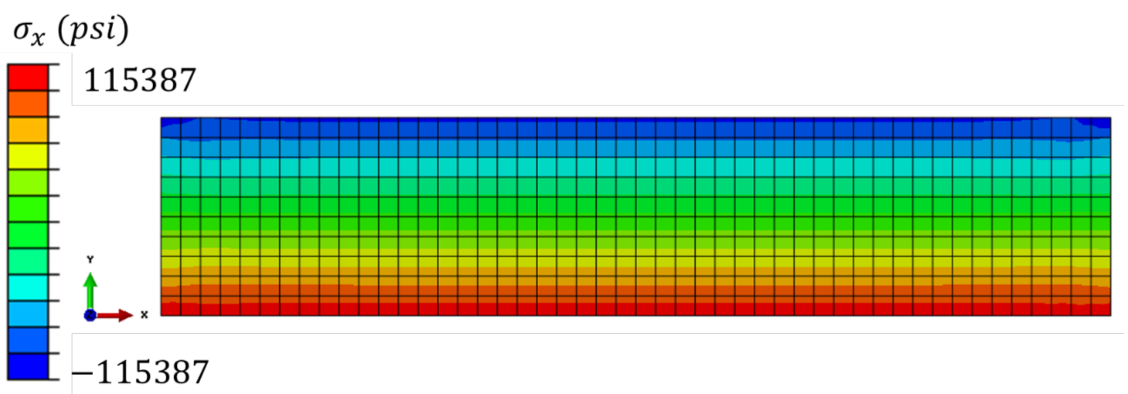
Example A

Use ABAQUS or ANSYS to calculate the plane stress distribution in a thin beam under pure bending loading conditions. The beam width, length and height are $b = 1$ in, $L = 24$ in and $h = 5$ in, respectively. The two end faces of the beam rotate rigidly under a bending moment equal to $M_0 = 38876$ *ft-kip*. These two end faces are pineed on the neutral plane, as shown in the figure. Compare the stress distribution on a cross section perpendicular to the x-axis with that of a thin beam.

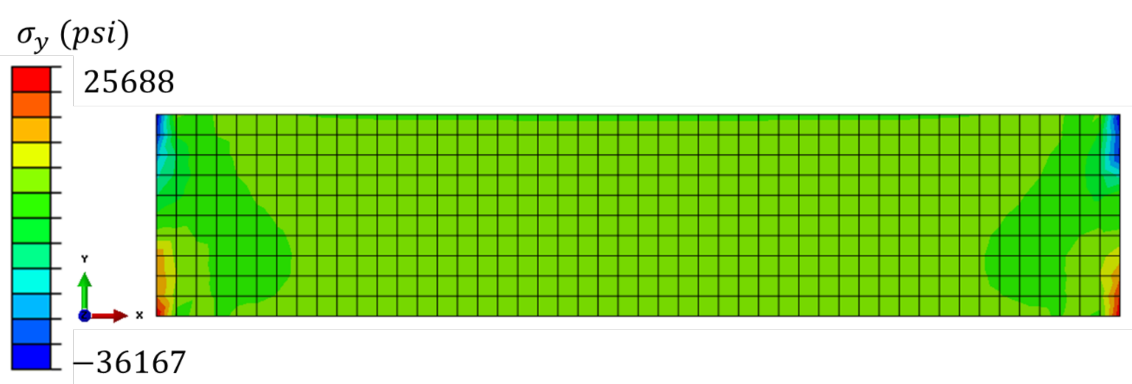


<u>Stress Distribution</u>:

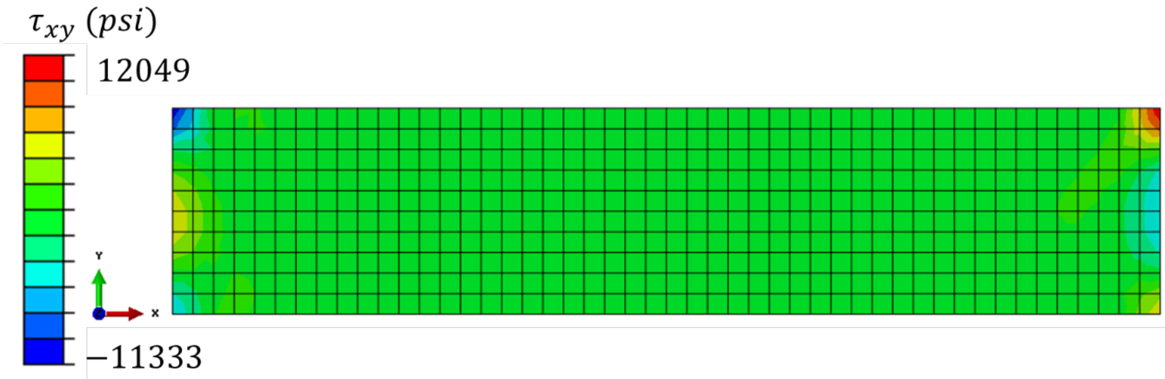
a) Flexural normal stress σ_x



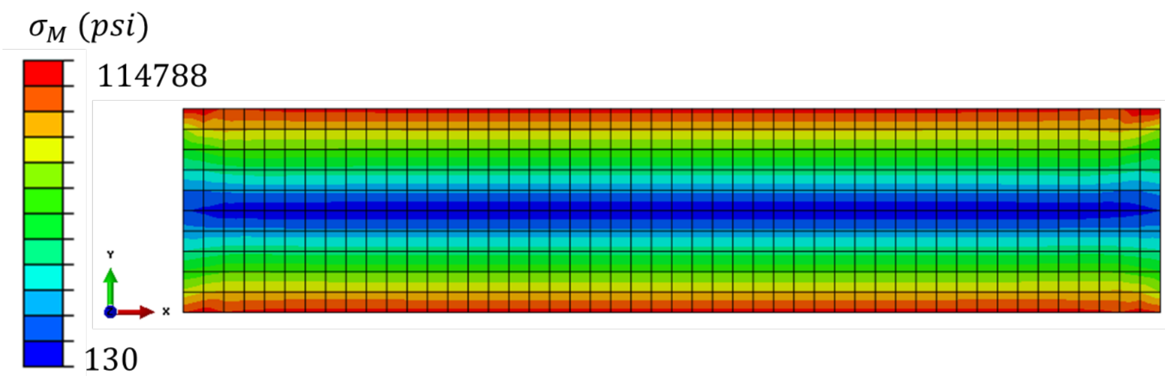
b) Transverse normal stress σ_y



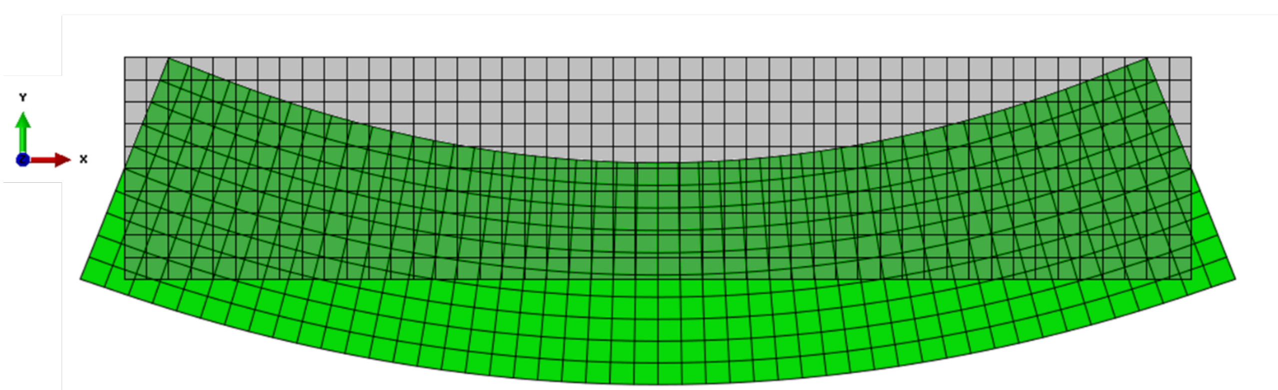
c) Shear stress τ_{xy}



d) von Mises equivalent stress σ_M



Deformed configuration: Deformation scale: x20, $u_y^{max} = -0.105\ in$, measured at the neutral surface.



Analytical Solution

Second moment of inertia of the cross section

$$I = \frac{bh^3}{12} = \frac{(1)(5)^3}{12} = 10.417\ in^4$$

Bending moment: $M(x) = M_0 = 38876 \times 12 = 466512\ in\ lb$

Maximum magnitude of flexural stress:

$$\sigma_x = \left|\frac{My_{max}}{I}\right| = \left|\frac{M}{I}\left(\frac{h}{2}\right)\right| = \frac{466512}{10.417} \times 2.5 = 111959.297\ psi$$

Boundary conditions: $\theta(L/2) = 0\ (by\ symmetry),\ v(0) = v(L) = 0$

$$\theta(x) = \theta(0) + \frac{1}{EI}\int_0^x M(x)dx = \theta(0) + \frac{1}{EI}M_0 x$$

Enforcing boundary condition on slope:

$$\theta(L/2) = 0 = \theta(0) + \frac{M_0 L}{2EI} \Rightarrow \theta(0) = -\frac{M_0 L}{2EI}$$

$$\Rightarrow \theta(x) = \frac{M_0}{EI}\left(x - \frac{L}{2}\right)$$

By symmetry, maximum deflection occurs at the middle $x = L/2$

$$\delta_{max} = v(L/2) = v(0) + \int_0^{L/2} \theta(x)dx = \int_0^{L/2} \frac{M_0}{EI}\left(x - \frac{L}{2}\right)dx = -\frac{M_0 L^2}{8EI}$$

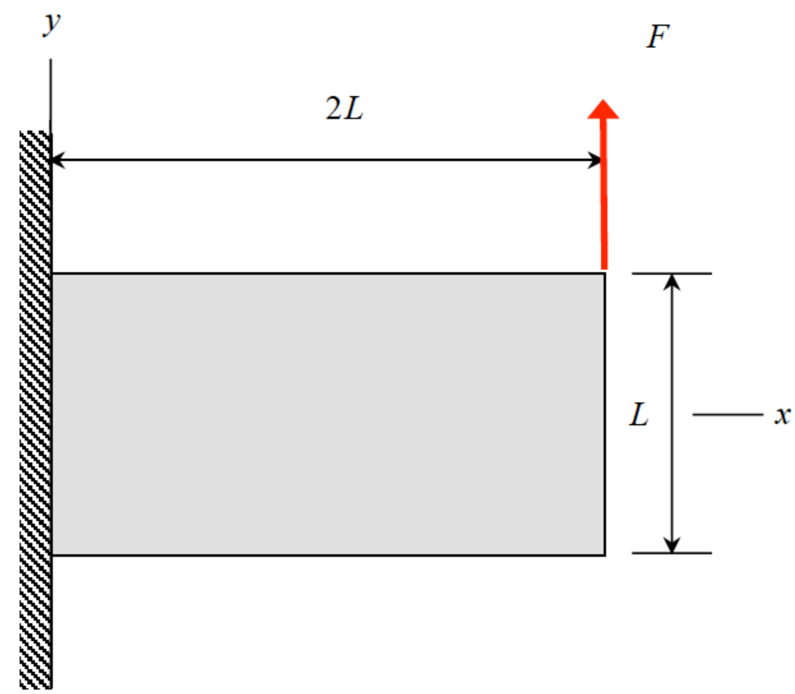
$$\Rightarrow \delta_{max} = -\frac{(466512)(24^2)}{8(30 \times 10^6)(10.417)} = -0.107\ in$$

Notes:

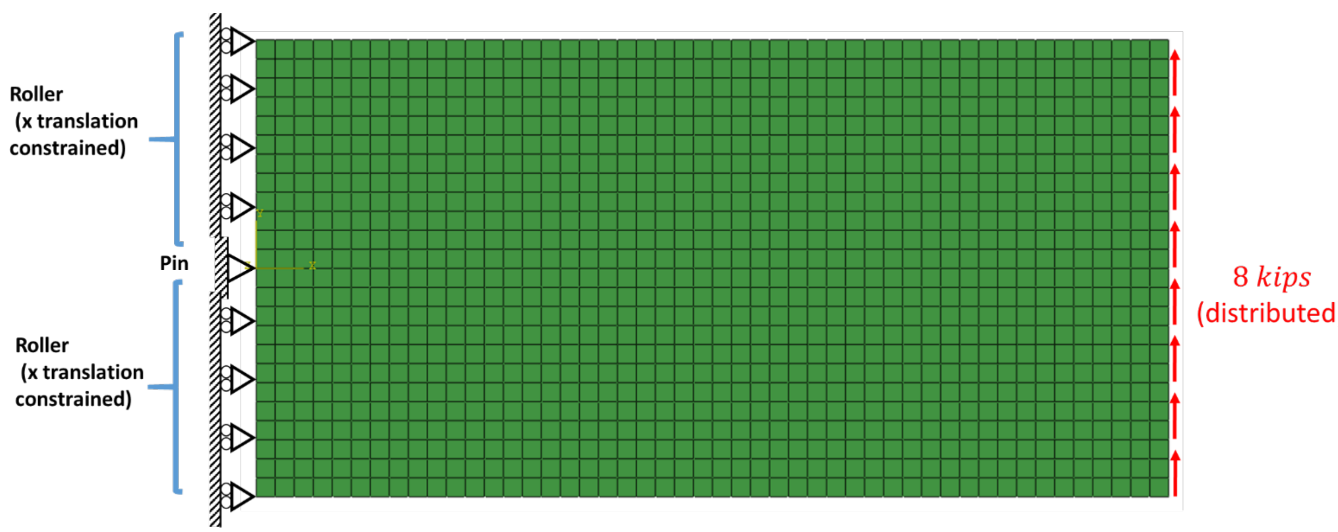
1) The pure bending loading conditions are achieved by enforcing a rigid rotation on all the nodes located on the two end faces of the beam.
2) Since the rigid rotation neglects the deformation in the vertical direction due to Poisson's effect, σ_y and τ_{xy} stresses are developed at the two ends of the beam.
3) These boundary effects are localized, are an order of magnitude smaller than the axial stresses, and do not affect the solution in the rest of the structure.
4) The analytical one-dimensional approximate solution should be very close to the numerical two-dimensional solution. Indeed, the normal stress and the maximum deflection obtained from the finite element simulation are close to the analytical approximation.
5) It is worth noting that the von Mises equivalent stress does not indicate whether the state of stress is dominated by compressive or tensile stresses. So, for brittle material where failure may be asymmetric, von Mises stress may not be sufficient for a stress analysis.

Example B

Use ABAQUS or ANSYS to calculate the plane stress distribution in the structural component below. Compare the shear and normal stress distribution on a cross section perpendicular to the x-axis with that of a thin beam. Use $F = 8$ kips , $E = 30 \times 10^6$ psi , $\nu = 0.3$, $L = 12$ in and $b = 1$ in.

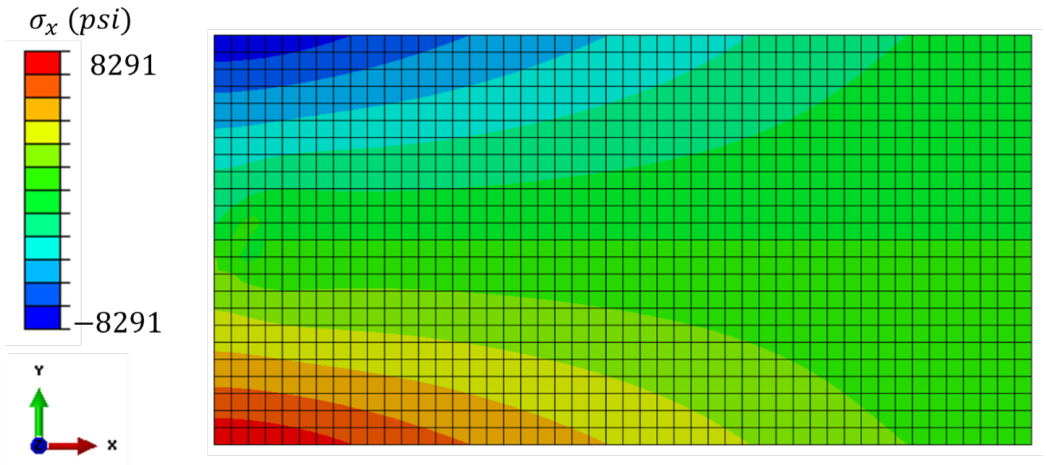


Finite Element Mesh and Boundary Conditions

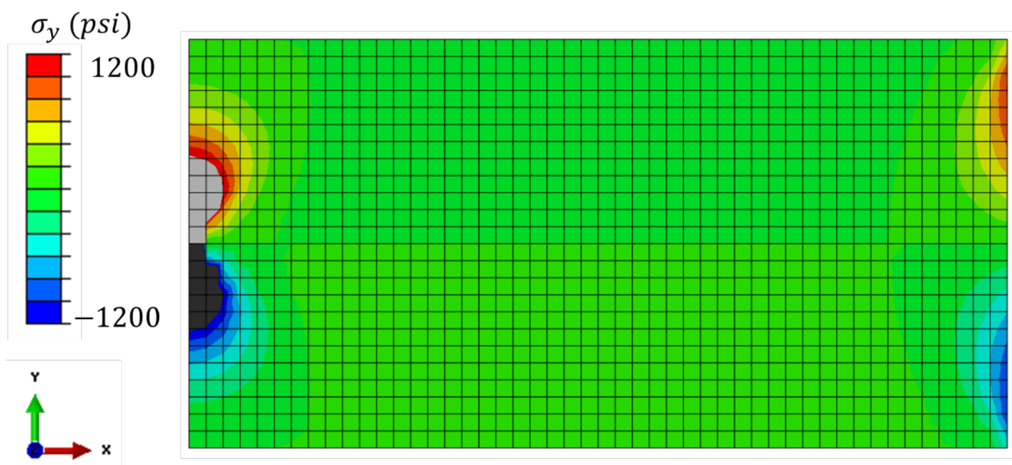


Stress Distribution

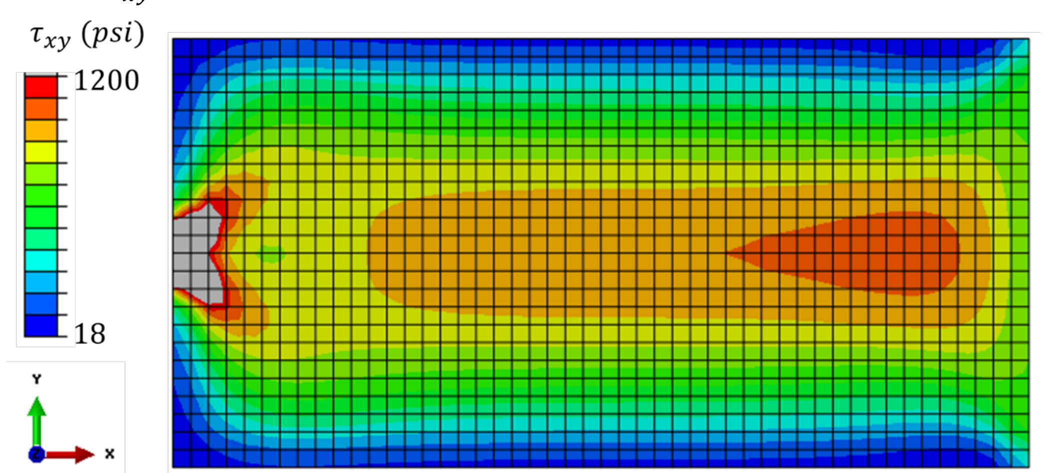
a) Flexural normal stress σ_x



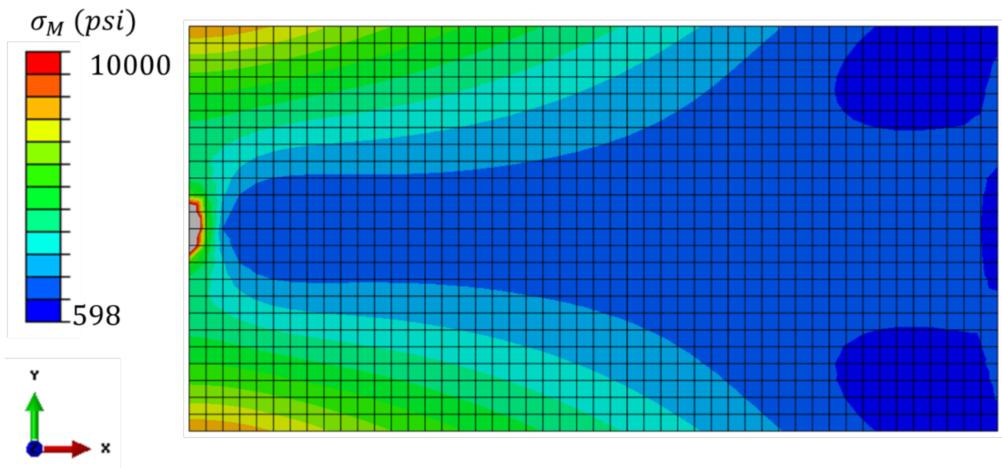
b) Transverse normal stress σ_y



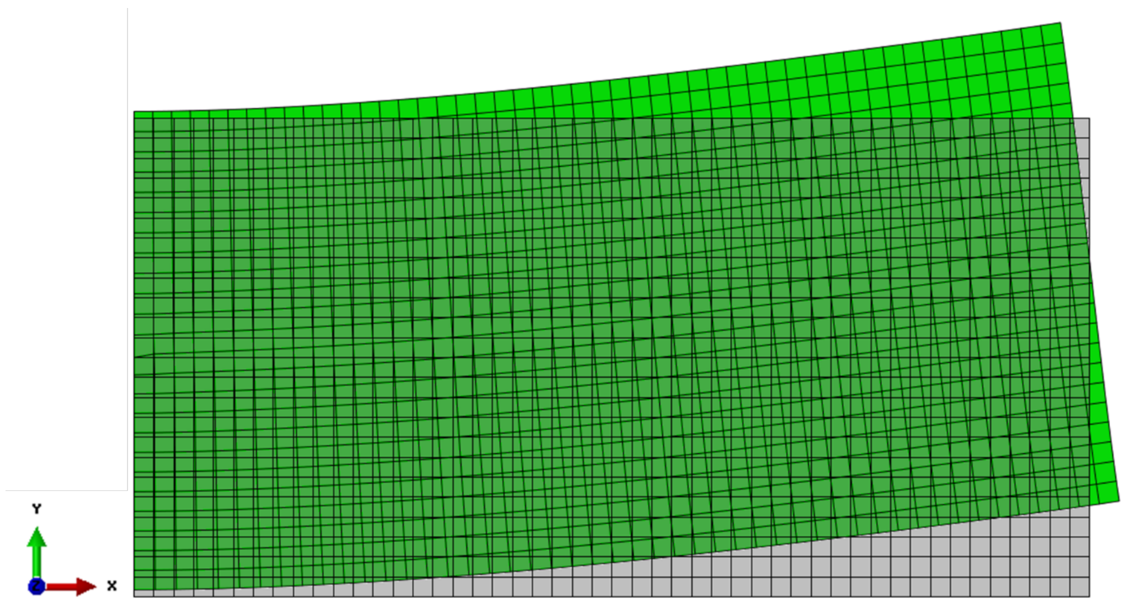
c) Shear stress τ_{xy}



d) von Mises equivalent stress σ_M

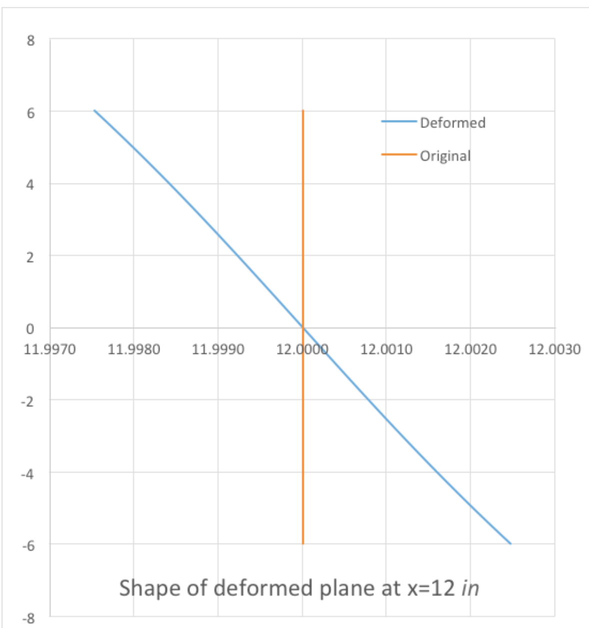


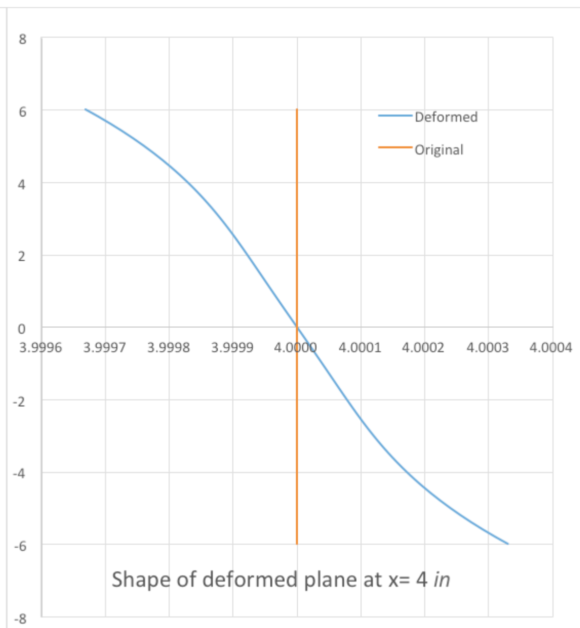
Deformed configuration: Deformation scale: x220, $u_y^{max} = 0.0107\ in$, measured at the neutral surface.



Notes:

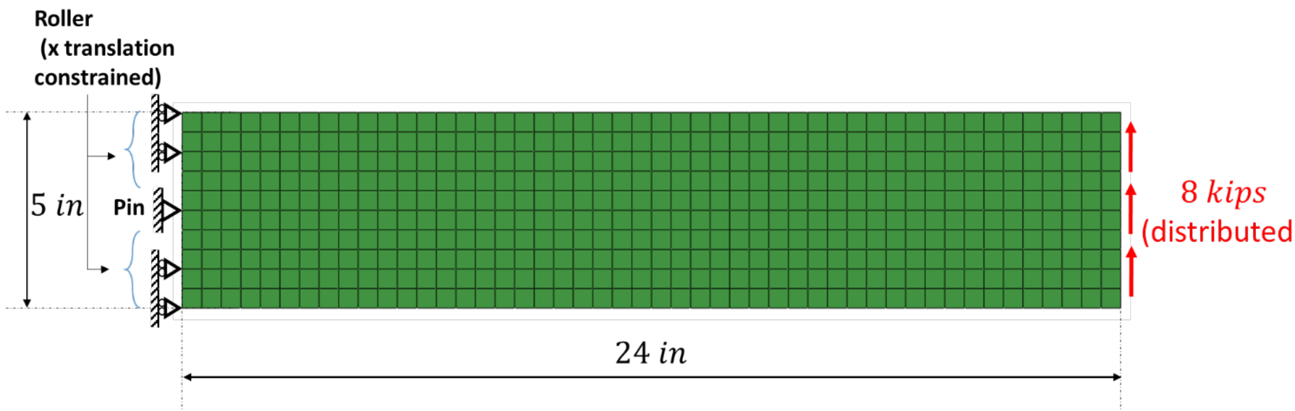
1) The concentrated load acting on the right end face of the beam is modeled with a distributed load to minimize boundary effects. In practice, the load should be modeled to best resemble the true loading conditions (a concentrated load may be accurate in some situations, a distributed load may be accurate in other).
2) Since the finite element formulation used is based on an interpolation of displacements, it is not possible to model clamped boundary conditions exactly. Specifically, it is not possible to enforce a slope of zero on the deflection curve of the beam at the clamped end. Therefore, boundary effects are inevitable in the solution. Notice that the developed σ_y and τ_{xy} stresses are of the same order of magnitude as the axial stresses, but they remain localized.
3) Shear effects are evident in the solution. The cross section placed at the mid-half of the beam does not remain plane. A detail of the deformed cross section is show below for two different cases:
   (i) a shear force of 8 kips, a beam of 24 in by 12 in, and a mesh of 120x60 elements;
   (ii) a shear force of 8 kips, a shorter beam of 8 in by 12 in, and a mesh of 40x60 elements.





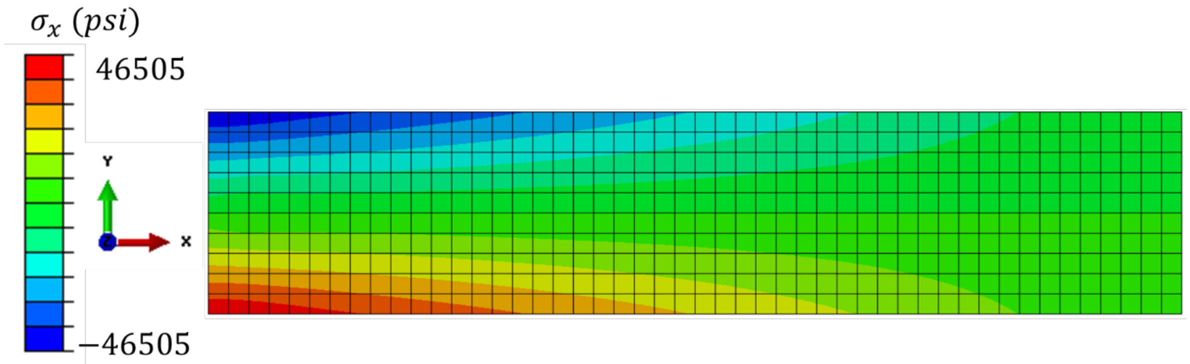
Example C

Use ABAQUS or ANSYS to calculate the plane stress distribution in the structural component below. Compare the shear and normal stress distribution on a cross section perpendicular to the x-axis with that of a thin beam. Use $F = 8$ kips , $E = 30 \times 10^6$ psi , $v = 0.3$, $L = 24$ in, $h = 5$ in. and $b = 1$ in.

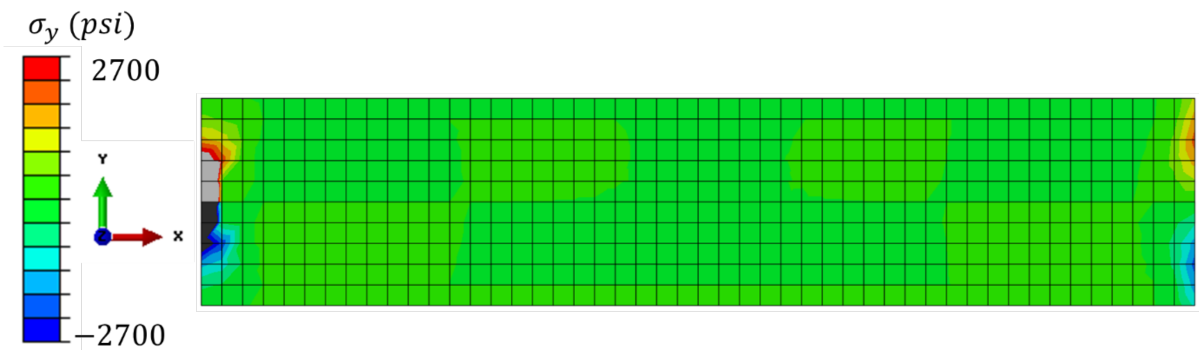


Stress Distribution

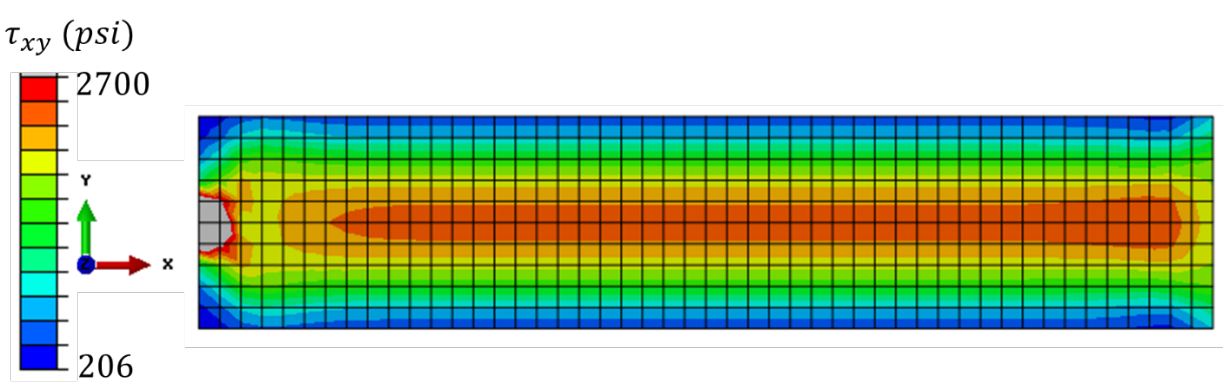
a) Flexural stress σ_x



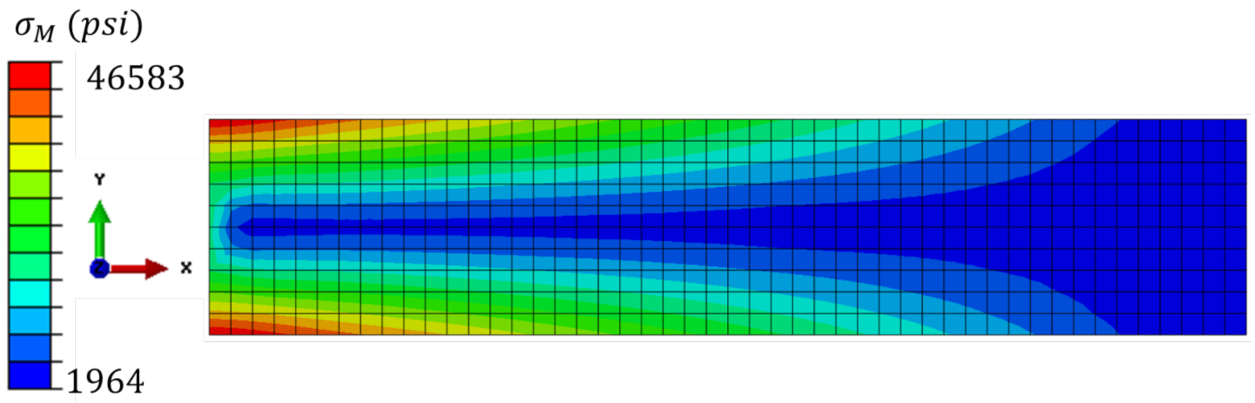
b) Transverse normal stress σ_y



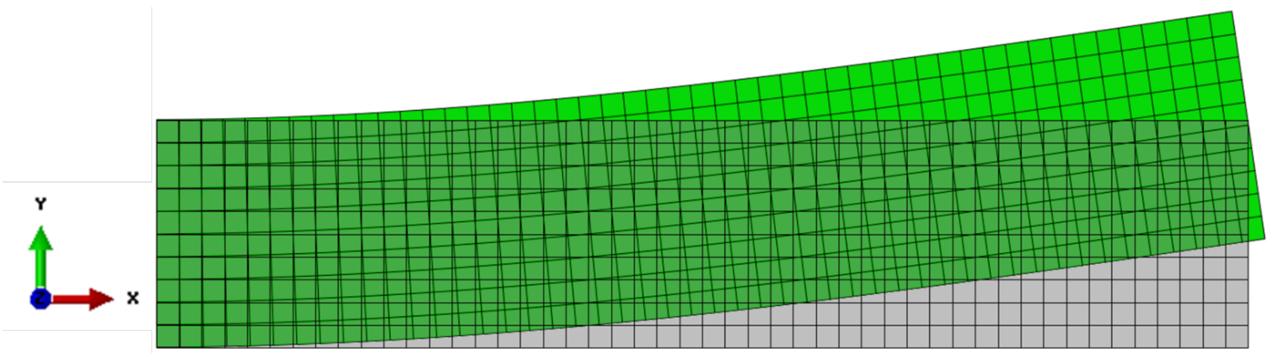
c) Shear stress τ_{xy}



d) von Mises equivalent stress σ_M



Deformed configuration: Deformation scale: x20, $u_y^{max} = 0.122\ in$



Analytical Solution

Second moment of inertia of the cross section

$$I = \frac{bh^3}{12} = \frac{(1)(5)^3}{12} = 10.417\ in^4$$

Maximum bending moment (at the wall):

$$M_{max} = PL = 8000 \times 24 = 192000\ in\ lb$$

Maximum magnitude of flexural stress:

$$\sigma_x = \left|\frac{My_{max}}{I}\right| = \left|\frac{M}{I}\left(\frac{h}{2}\right)\right| = \frac{192000}{10.417} \times 2.5 = 46078.53\ psi$$

(Flexural stress is compressive at the top and tensile at the bottom)

From Appendix E.1.3 of text,

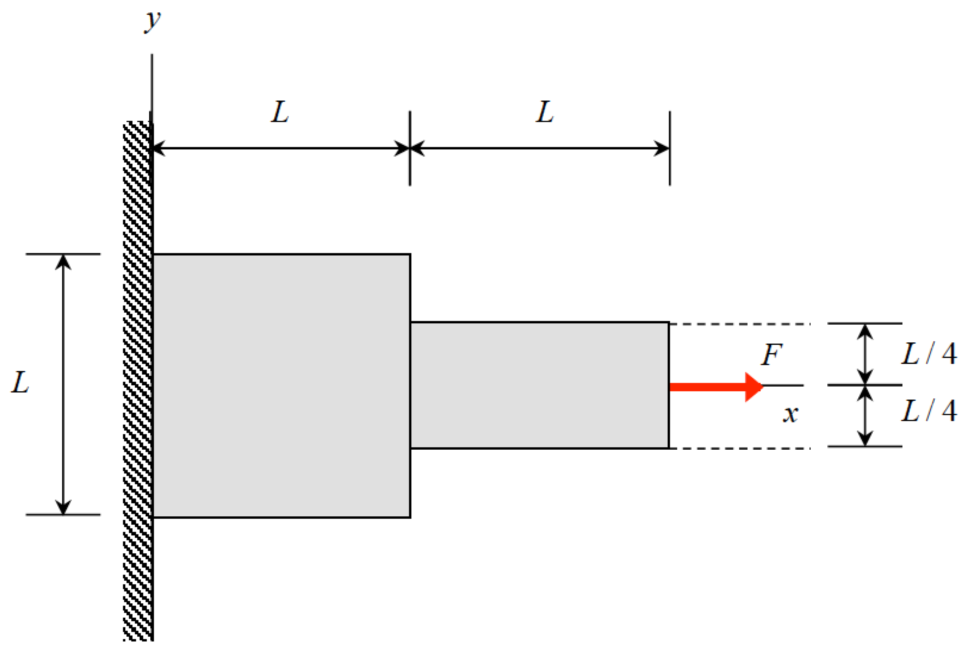
$$\delta_{max} = \frac{PL^3}{3EI} = \frac{(8000)(24^3)}{3(30 \times 10^6)(10.417)} = 0.118\ in$$

Notes:

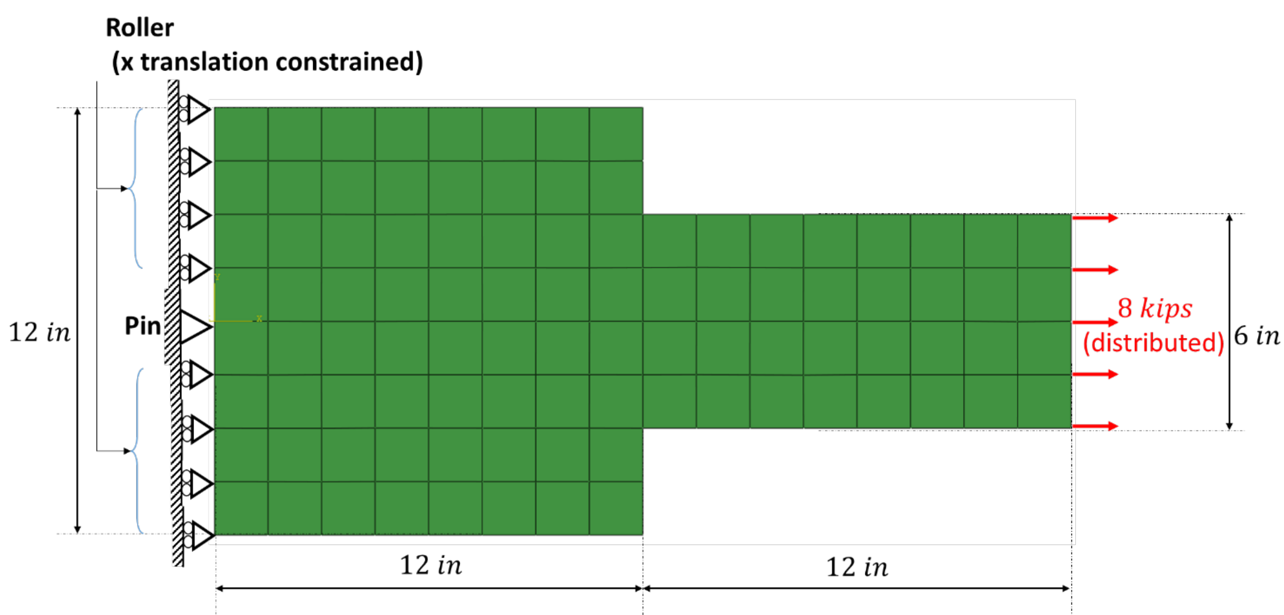
1) As noted above, the finite element formulation used is based on an interpolation of displacements and thus it is not possible to model clamped boundary conditions. Boundary effects are inevitable in the solution. However, notice that the developed σ_y and τ_{xy} stresses, although still of the same order of magnitude as the axial stresses, they are localized to a region that does not scale with the length of the beam.
2) The analytical one-dimensional approximate solution should be very close to the numerical two-dimensional solution. Indeed, the normal and shear stresses and the maximum deflection obtained from the finite element simulation are close to the analytical approximation.

Example D

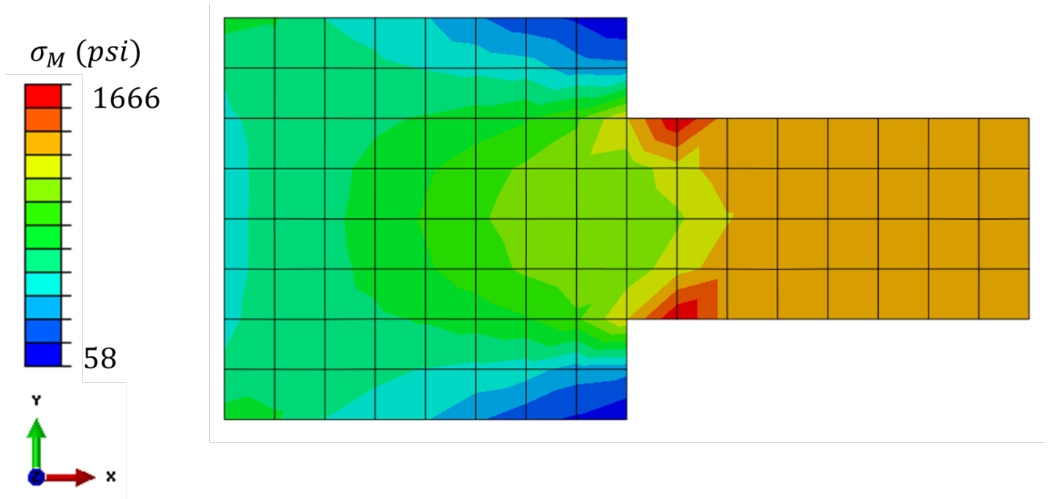
Use ABAQUS or ANSYS to calculate the plane stress distribution in the structural component below. Compare the stress distribution with that found using a thin rod model for the structural component. Use $F = 8$ kips , $E = 30 \times 10^6$ psi , $\nu = 0.3$, L = 12 in and $b = 1$in.



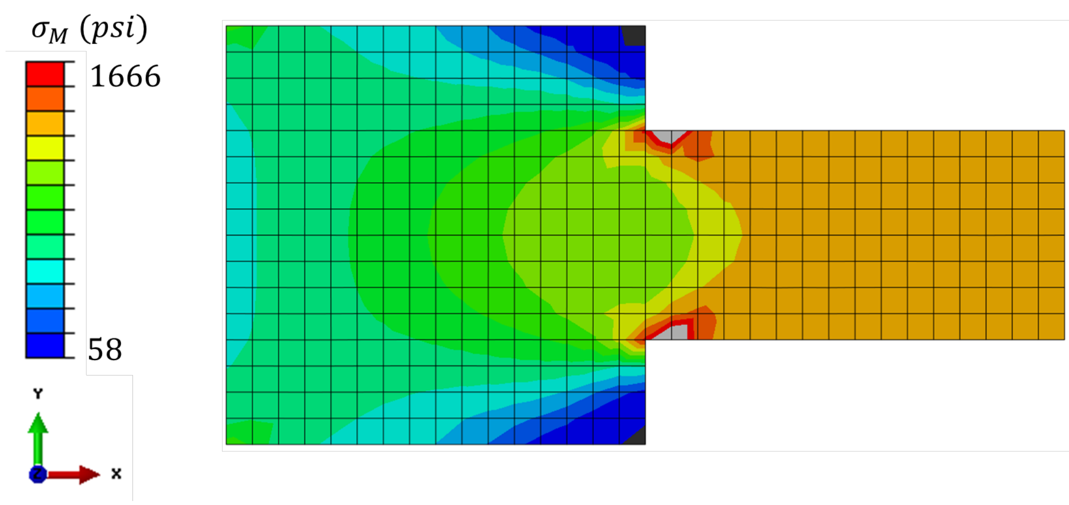
Finite Element Mesh and Boundary Conditions



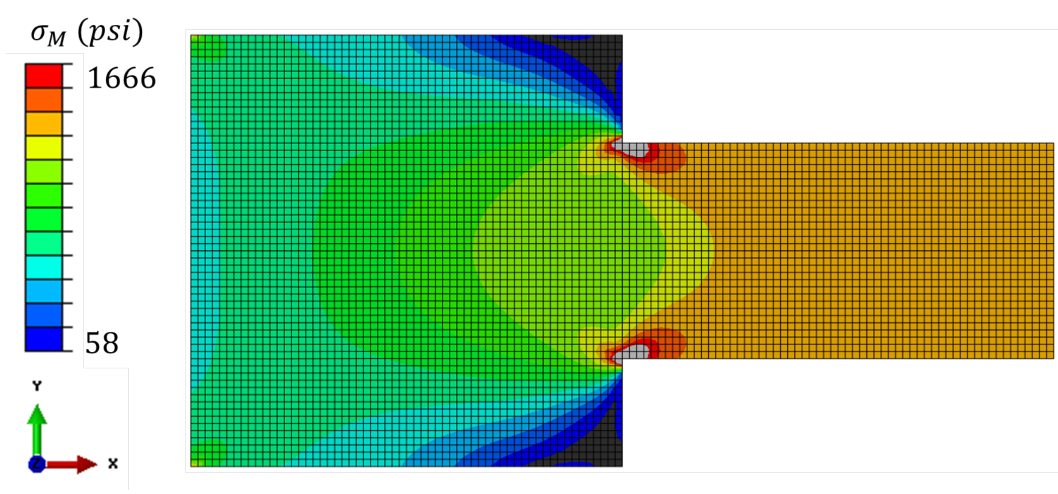
Coarse mesh density (Element size: 1.5 in x 1.5 in)



Medium mesh density (Element size: 0.75 in x 0.75 in)

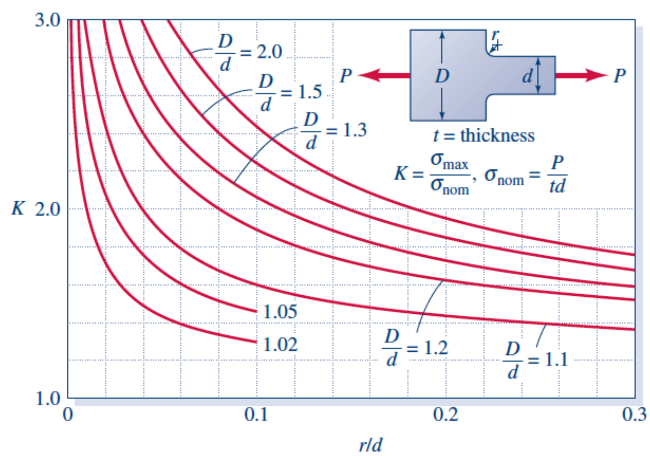


Medium mesh density (Element size: 0.2 in x 0.2 in)



Notes:

1) Stress concentration is indeed part of the physical solution of this problem because the rod exhibits a drastic change in the cross section and a sharp corner (see figure below).
2) Even though the stress concentration is well predicted by the finite element solution, this prediction is only accurate if the mesh is refined. These three cases show the effect of mesh refinement on the solution.
3) In practice, elasto-plastic material behavior or brittle failure should be incorporated in the finite element solution to recover a realistic stress response. Otherwise, the stresses will be unbounded as the mesh size is reduced (for a sharp corner).
4) A uniform solution is recovered far from the transition in the cross sectional area. The analytical approximate solutions studied in ME323 neglect these effects.



Stress concentration factor K for a flat bar with shouldered fillets under tension.