Lecture 37 – Failure theories

Reading assignment: Ch.15 lecturebook

PURDUE UNIVERSITY | Mechanical Engineering

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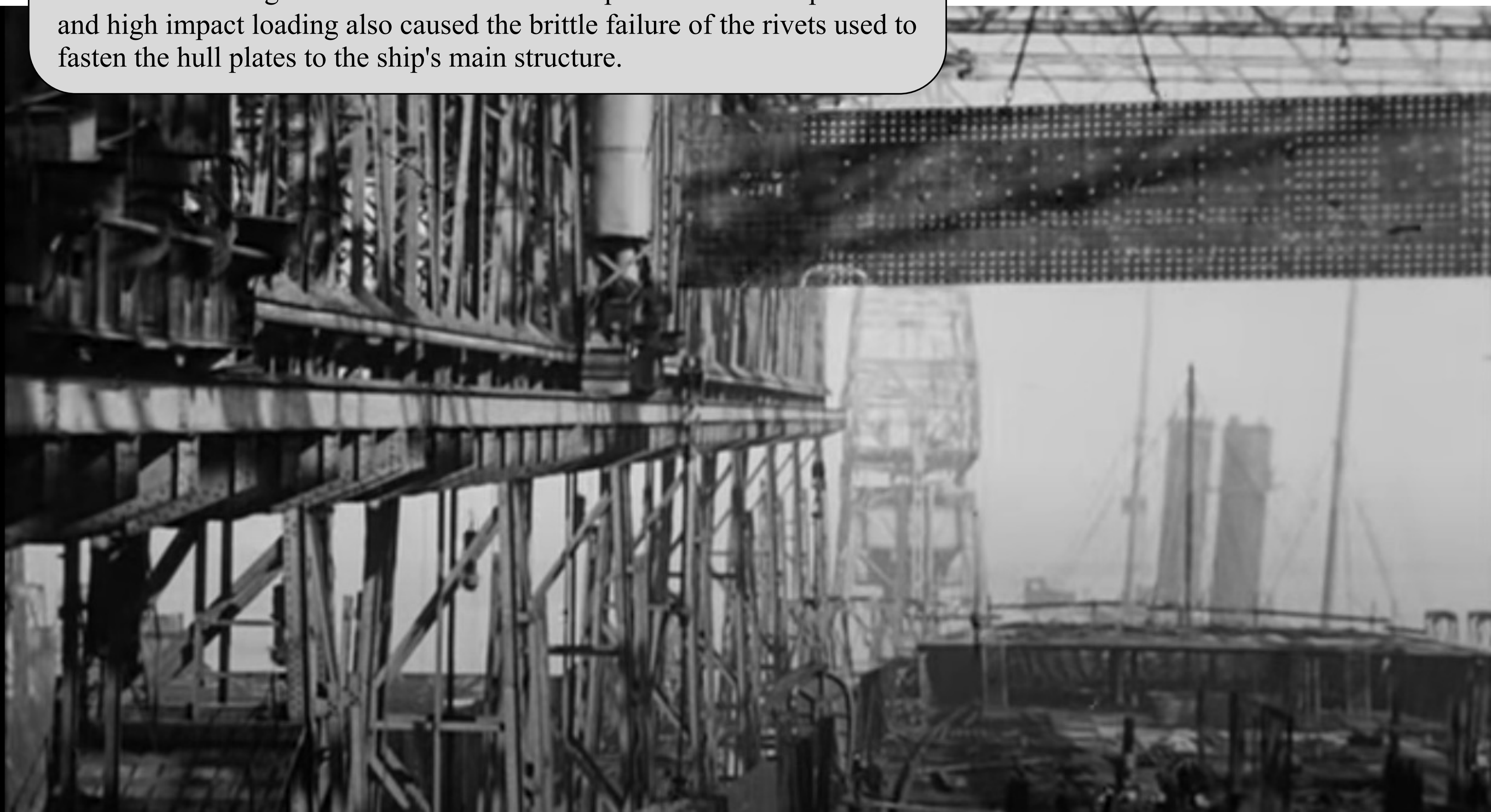
Last modified: 11/30/22 10:54:10 AM

No so unsinkable – Learning from our mistakes

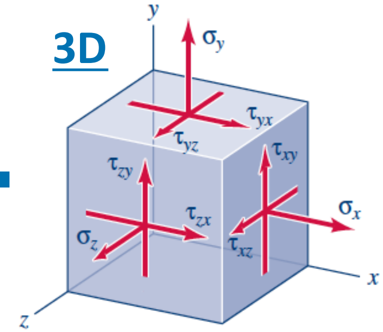
The failure of the hull steel resulted from brittle fractures caused by the high sulphur content of the steel, the low temperature water on the night of the disaster, and the high impact loading of the collision with the iceberg. When the Titanic hit the iceberg, the hull plates split open and continued cracking as the water flooded the ship. Low water temperatures and high impact loading also caused the brittle failure of the rivets used to fasten the hull plates to the ship's main structure.

Video

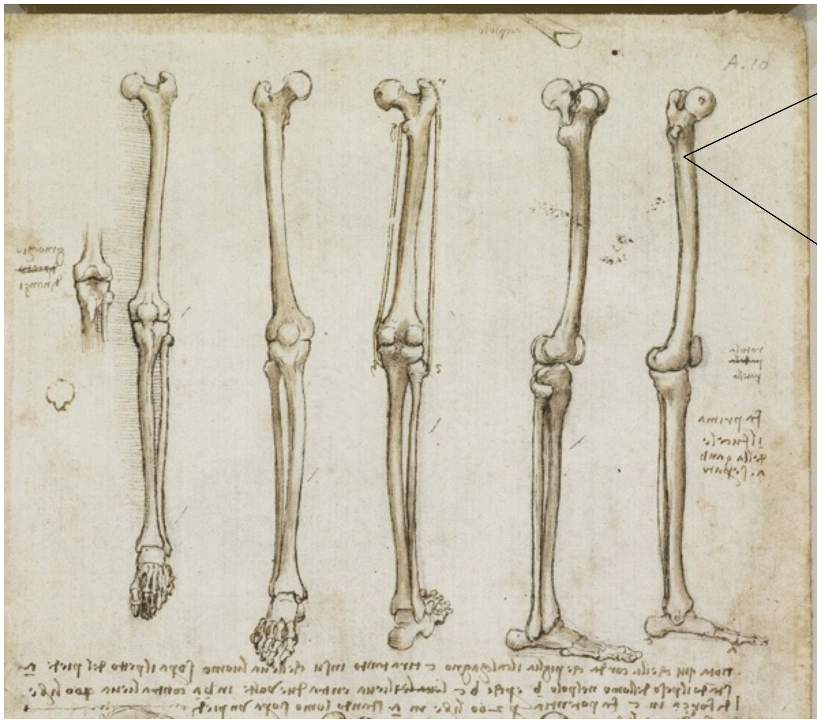


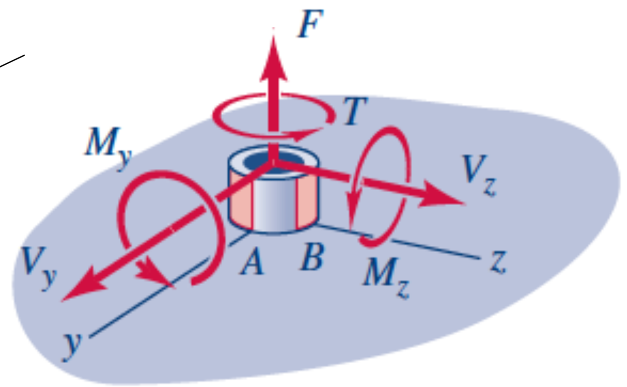
Failure theories

3D



Stress due to combined loads





- Determine the internal resultants at a given point.
- Determine the centroid of the cross section (principal axes pass through the centroid).
- Calculate and combine individual stresses.
- Determine principal stresses and maximum shear stress.
- **Failure due to excessive stress?!**

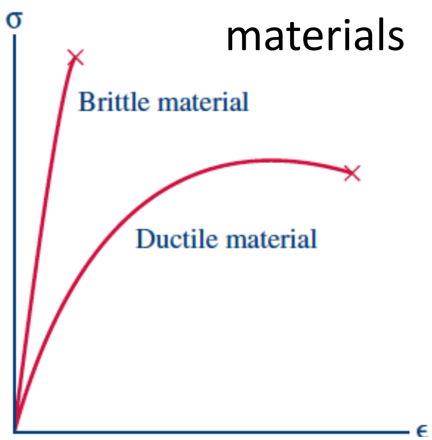
Failure theories

Material properties from stress-strain diagrams (Lecture 3)

- Tensile uniaxial test – two distinct failure mechanisms

Brittle

Brittle vs. ductile materials



Ductile
(yielding before fracture)

- What is the criterion if the solid is under combined loads and the state of stress is triaxial?

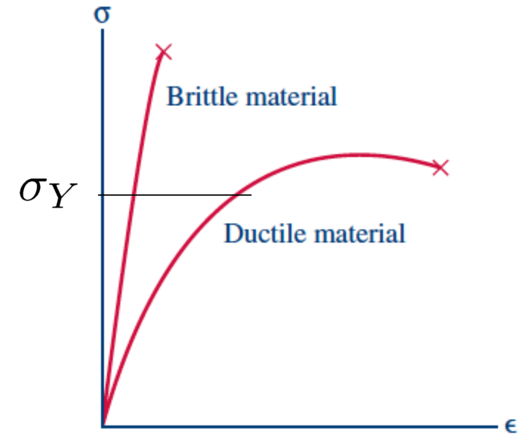
Failure theories - Ductile materials

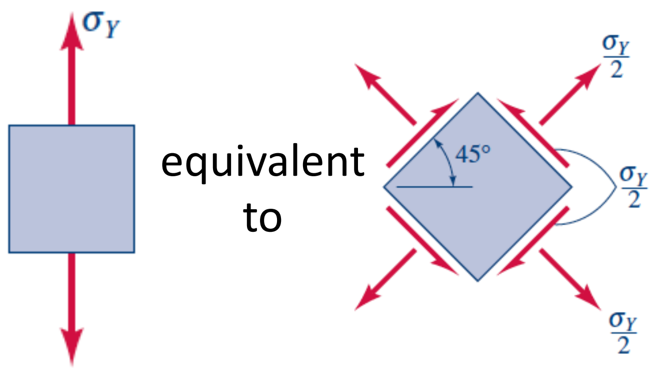
Failure criteria for ductile materials

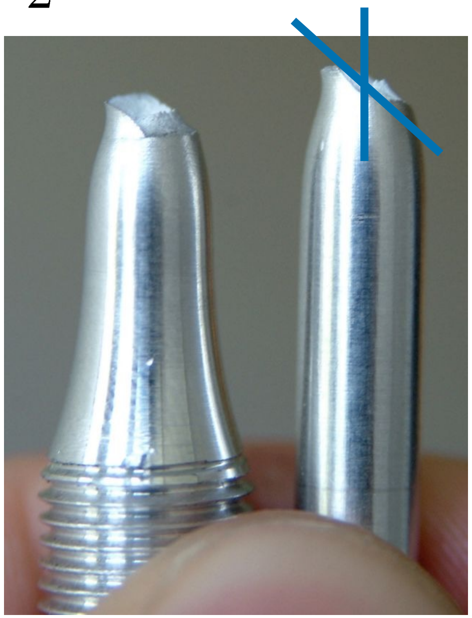
- Maximum-shear-stress theory

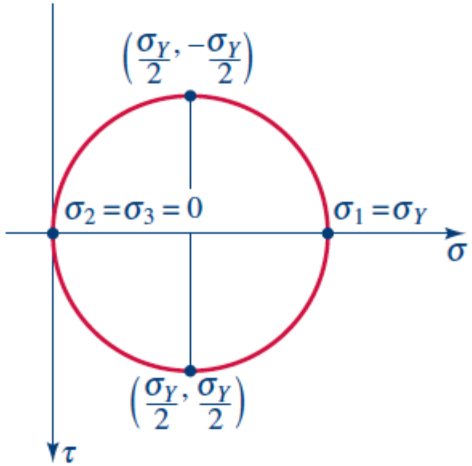
$$\tau_{\max}^{\text{abs}} = \frac{\sigma_Y}{2}$$ failure criterion (where σ_Y is measured)

$$\tau_{\max}^{\text{abs}} = \frac{\sigma_{\max} - \sigma_{\min}}{2} = \frac{\sigma_Y}{2}$$









Failure theories - Ductile materials

Failure criteria for ductile materials

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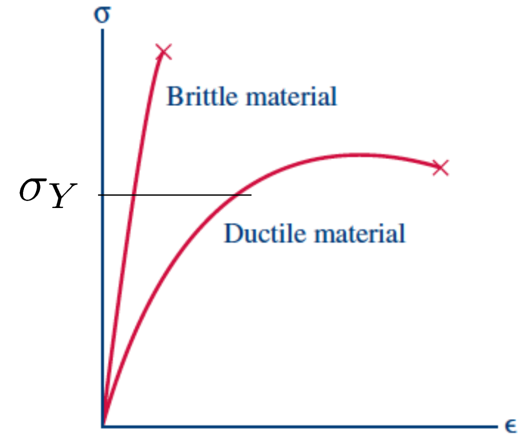
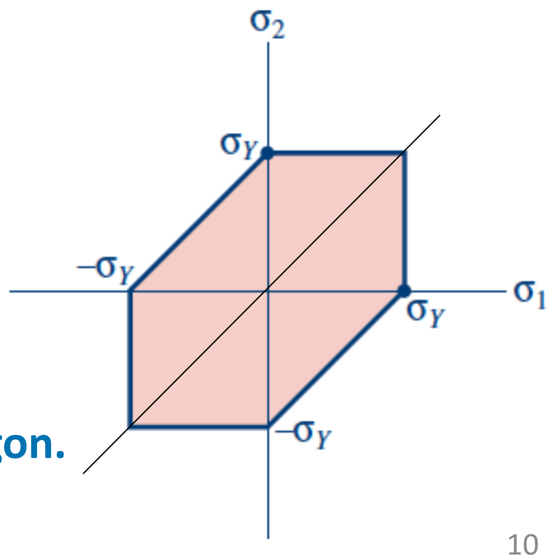


For plane stress, using $\sigma_3 = 0$ in this analysis!)

- $\sigma_1 > 0 \quad \sigma_2 > 0$
- $\sigma_1 < 0 \quad \sigma_2 < 0$
- $\sigma_1 > 0 \quad \sigma_2 < 0$

What are the conditions for each quadrant?



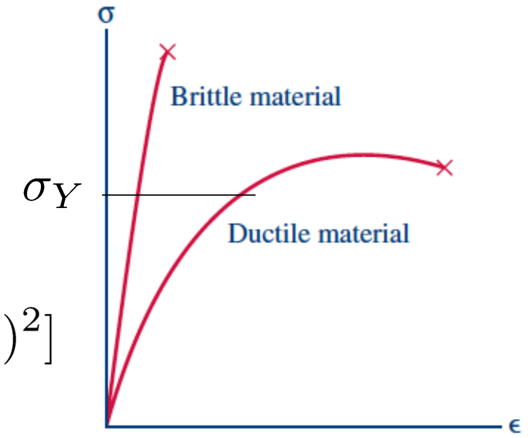
The failure envelope is a hexagon.
+ inside: no failure
+ outside: failure

Failure theories - Ductile materials

Failure criteria for ductile materials

- Maximum-distortion-energy theory

Total elastic energy = Change of volume + Distortion



Distortion energy:

$$\bar{u}_d = \frac{1}{12G}[(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_1-\sigma_3)^2]$$

Distortion energy at yielding point: $(\bar{u}_d)_Y = \dfrac{1}{6G}\sigma_Y^2$

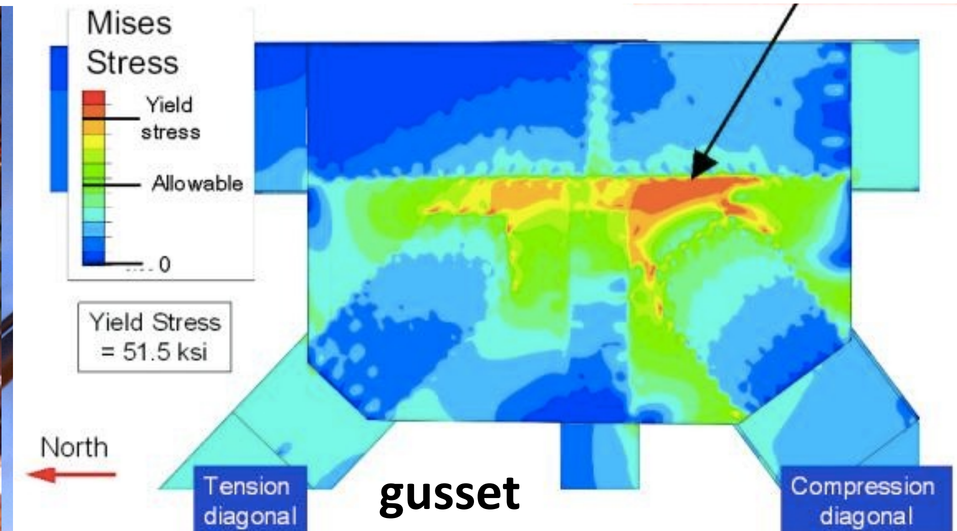
Failure criterion (for a triaxial state of stress)

$$\sigma_M = \frac{1}{\sqrt{2}}[(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_1-\sigma_3)^2]^{1/2} = \sigma_Y$$

$$\sigma_M = \frac{1}{\sqrt{2}}[(\sigma_x-\sigma_y)^2+(\sigma_y-\sigma_z)^2+(\sigma_x-\sigma_z)^2+6(\tau_{xy}^2+\tau_{yz}^2+\tau_{xz}^2)]^{1/2} = \sigma_Y$$

Mises equivalent stress

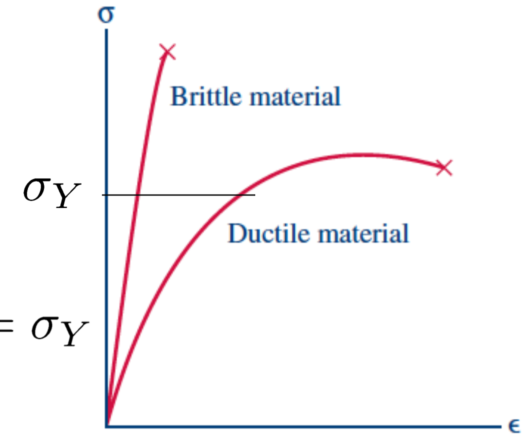
ME 323 within the engineering that moves the world



Failure theories - Ductile materials

Failure criteria for ductile materials

- Maximum-distortion-energy theory



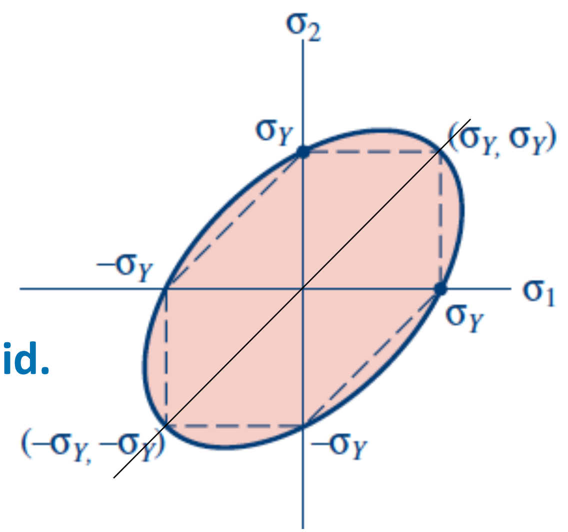
Failure criterion (for a triaxial state of stress)

$$\sigma_M = \frac{1}{\sqrt{2}}[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_1 - \sigma_3)^2]^{1/2} = \sigma_Y$$

For plane stress, using $\sigma_3 = 0$ in this analysis!)

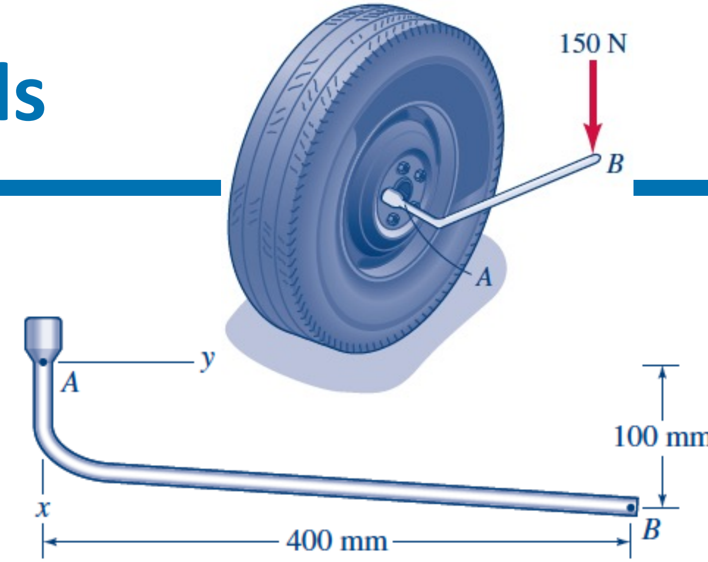
$$\sigma_M = [\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2]^{1/2} = \sigma_Y \quad \text{failure criterion}$$

$$\sigma_M = [\sigma_x^2 + \sigma_y^2 - \sigma_x\sigma_y + 3\tau_{xy}^2]^{1/2} = \sigma_Y$$



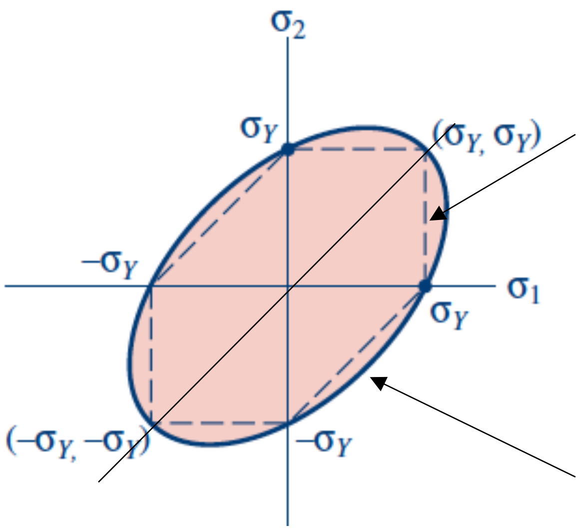
The failure envelope is an ellipsoid.
+ inside: no failure
+ outside: failure

Failure theories - Ductile materials



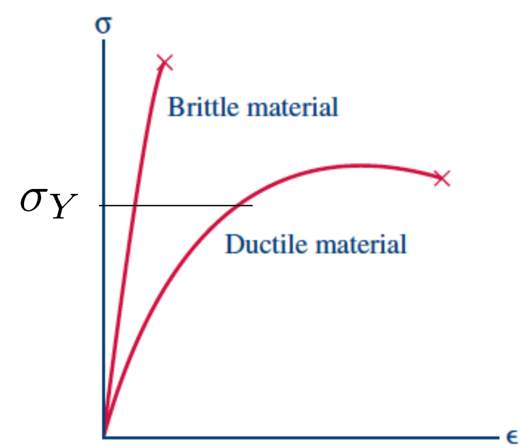
Problem 82:

Determine the principal stresses and the maximum shear stress at point A (i.e., the point on top of the wrench handle). The diameter of the circular cross section is 12.5 mm. **The material is ductile.**
The yield stress is 300 MPa.



Maximum shear Stress theory

Maximum Distortion Energy theory



Failure theories

Any questions?