Lecture 1
Introduction to vectors and tensors

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What is a tensor?

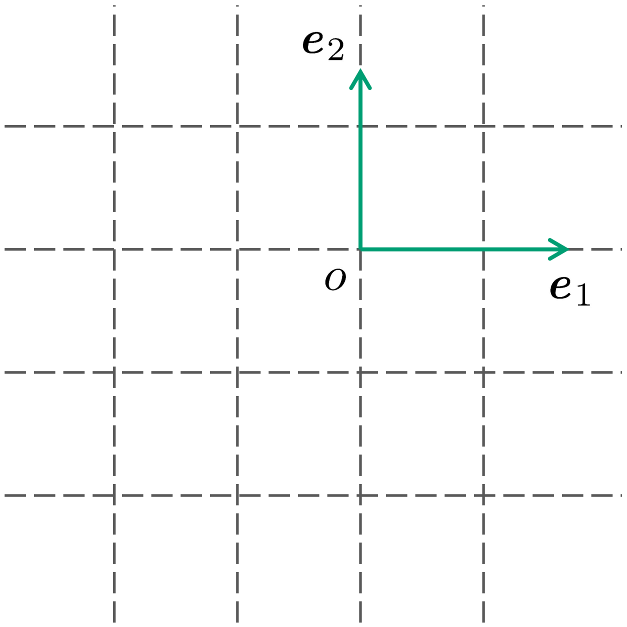
- Tensors are abstract mathematical entities.
- Vectors are *first order* tensors.
- Vectors and tensors exist separately of a particular coordinate system (i.e., they are coordinate invariant).

… so let's start by reviewing Vector Algebra.

Vector algebra

Coordinate systems

- An origin (relative to which positions are measured)
- A set of coordinate curves (that correspond to paths through space along which all but one of the coordinates are constant)



Vector algebra

Coordinate systems

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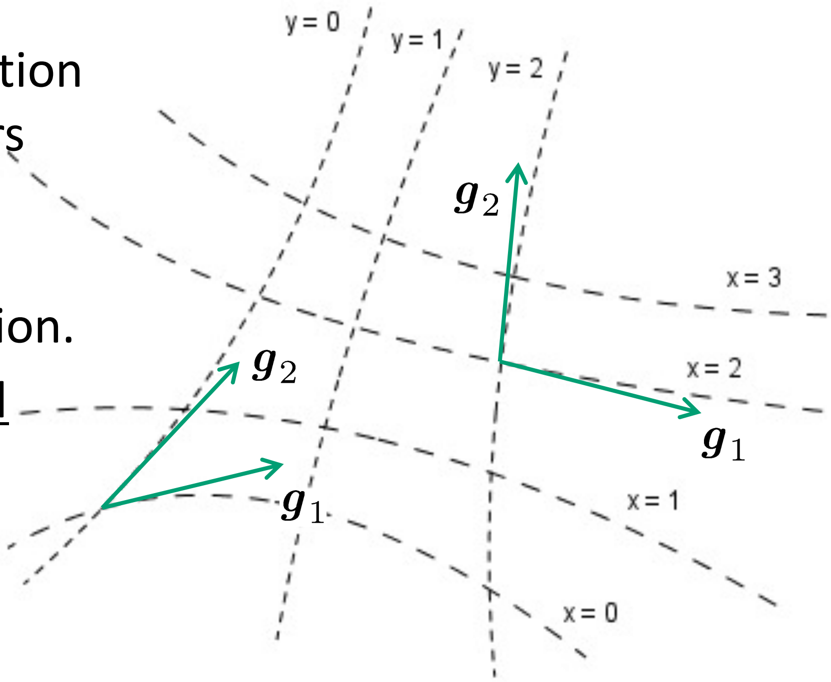

- <u>Basis</u> are defined at each position in space as the tangent vectors to the coordinate curves. Therefore, basis vectors $\{\boldsymbol{g}_i\}$ change from position to position.
- Basis $\{\boldsymbol{g}_i\}$ are <u>non-orthogonal</u> in general.

Vector algebra

Coordinate systems

- If the coordinate curves are straight lines, then the system is a rectilinear coordinate system.
- If the basis vectors of a rectilinear coordinate system are orthogonal then the system is a Cartesian coordinate system.
- If the basis vectors (or axes) of a Cartesian coordinate system are unit vectors $\{e_i\}$ then the basis are called orthonormal and follow the the condition

$$\boldsymbol{e}_i \cdot \boldsymbol{e}_j = \delta_{ij}$$

- We will choose basis vectors that form a right-handed triad

$$\boldsymbol{e}_i \times \boldsymbol{e}_j = \epsilon_{ijk} \boldsymbol{e}_k$$

DIY

What is the component of a vector (or the coordinate of a position vector) along a basis vector direction?

Vector algebra

Change of basis

- Two orthonormal bases $\{\boldsymbol{e}_\alpha\}$ and $\{\boldsymbol{e}'_i\}$
- The goal is to change bases from $\{\boldsymbol{e}_\alpha\}$ to $\{\boldsymbol{e}'_i\}$, that is to write $\{\boldsymbol{e}'_i\}$ as a linear combination of $\{\boldsymbol{e}_\alpha\}$.
- A linear transformation matrix $\boldsymbol{Q}$ is then defined as

$$\boldsymbol{e}'_i = \sum_{\alpha=3}^{3} Q_{i\alpha}\boldsymbol{e}_\alpha \qquad Q_{i\alpha} = \boldsymbol{e}'_i \cdot \boldsymbol{e}_\alpha$$

DIY

Let's write the transformation using a column-matrix notation.

Show the following properties:

$$\boldsymbol{Q}^T = \boldsymbol{Q}^{-1}$$

$$\det(\boldsymbol{Q})$$

Any questions?