Lecture 21
Structural elements: beams, plates, shells

PURDUE UNIVERSITY | Mechanical Engineering
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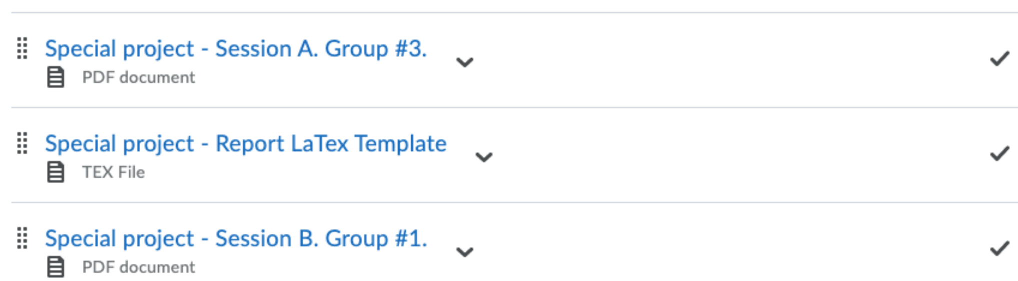
Announcements

Guidelines for special project. Final report:

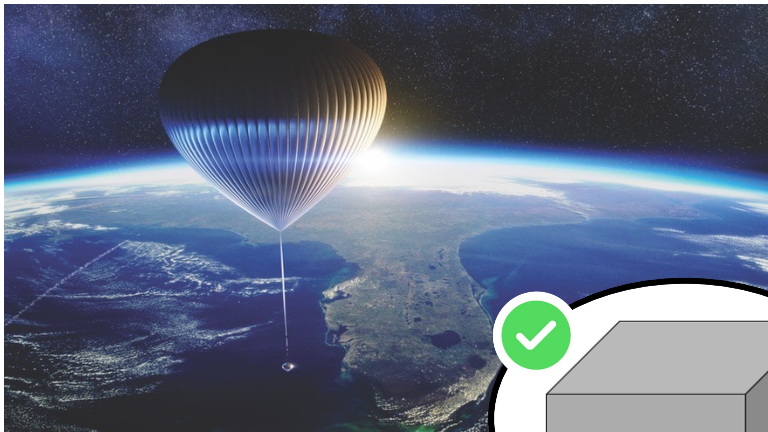
- Monday May 2nd at noon
- Technical report in LaTex, maximum of 4 pages (upload to Brightspace)
- Rewrite equations using the notation and nomenclature adopted in the class

- Grading: 5% of final grade (17% of project grade)

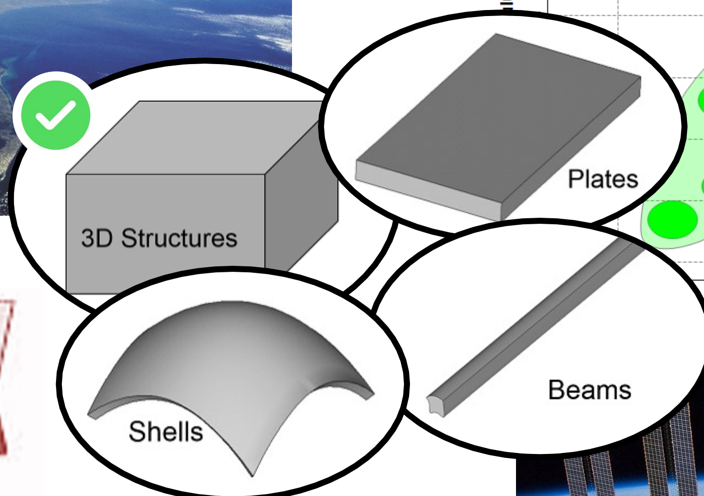


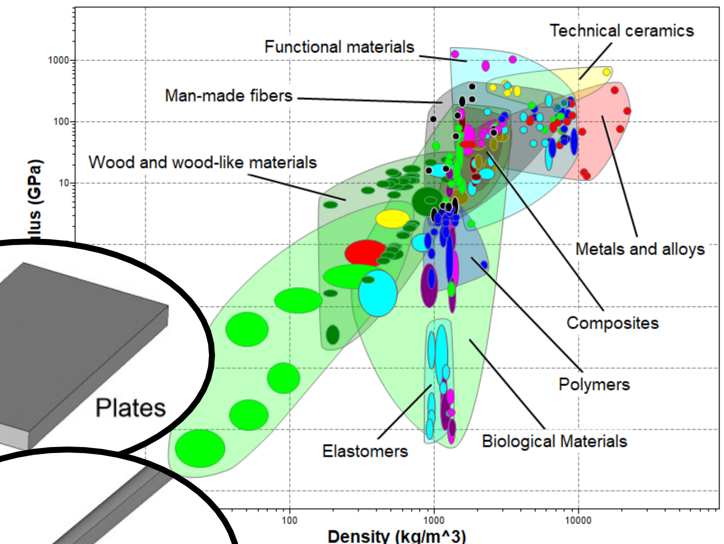
Structural elements: beams, plates, shells

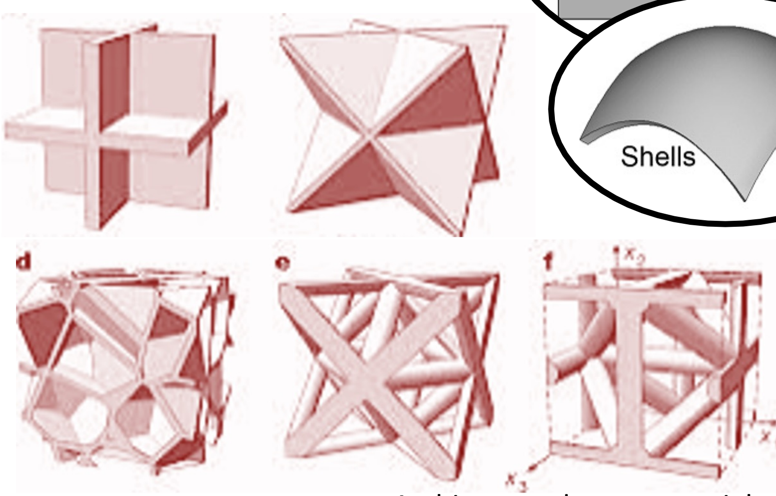
Formulation of **structural elements (beams, plates, shells)** as the analytical upscaling of continuum solids under kinematic assumptions.



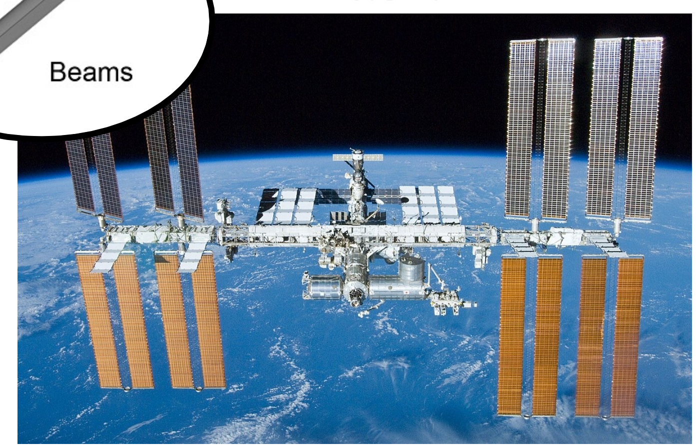
Stratospheric ballooning system







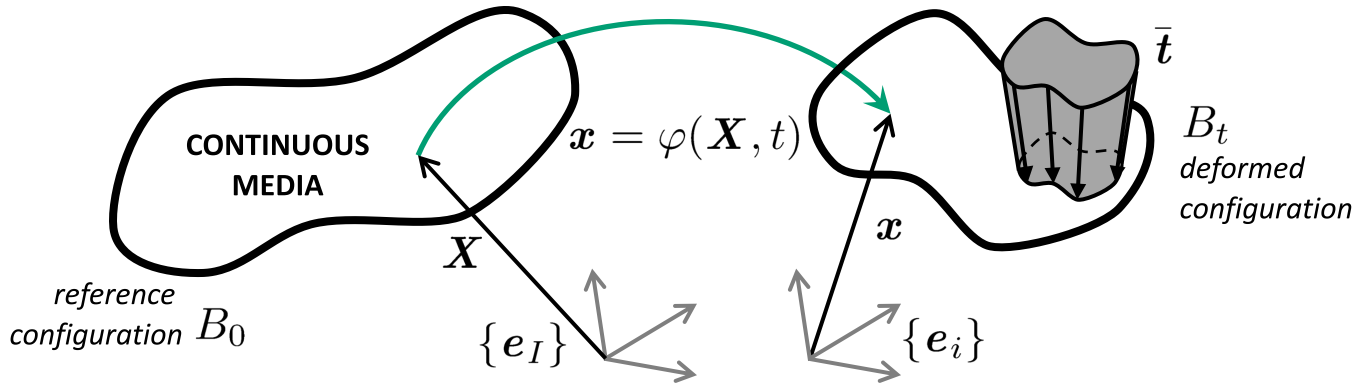
Architectured metamaterials



International space station

Structural elements: beams, plates, shells

Formulation of **structural elements (beams, plates, shells)** as the analytical upscaling of continuum solids under kinematic assumptions.



Linearized kinematic – Small trains

- The linearization is evaluated in the undeformed configuration (i.e., $\boldsymbol{X} \to \boldsymbol{X} + \boldsymbol{u}(\boldsymbol{X})$ and $\boldsymbol{F} = \boldsymbol{I}$, $\nabla_0 \boldsymbol{u} = \nabla \boldsymbol{u}$):

$$\langle \nabla_\varphi \boldsymbol{E}; \boldsymbol{u} \rangle = \frac{1}{2}[\nabla \boldsymbol{u} + (\nabla \boldsymbol{u})^T] = \boldsymbol{\epsilon}$$

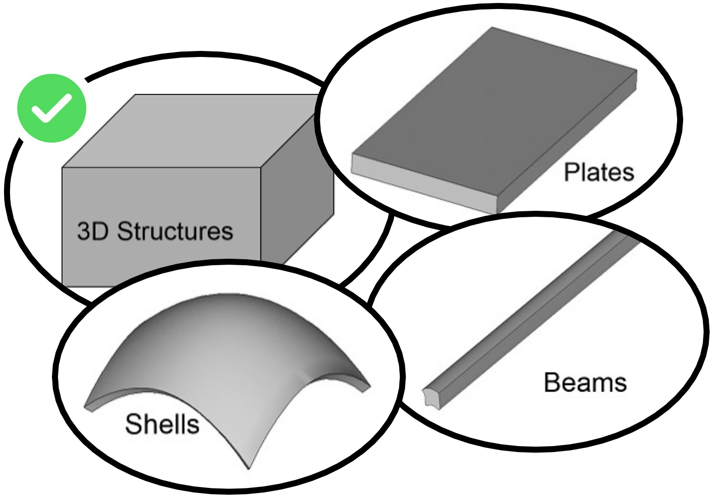
small-strain tensor
(*employed in elasticity theory*)

- Simplified geometry
- Simplifying kinematic assumptions (i.e., simplified displacement field $\boldsymbol{u} = (u_1, u_2, u_3)$)

Structural elements: beams, plates, shells

Simplified geometry

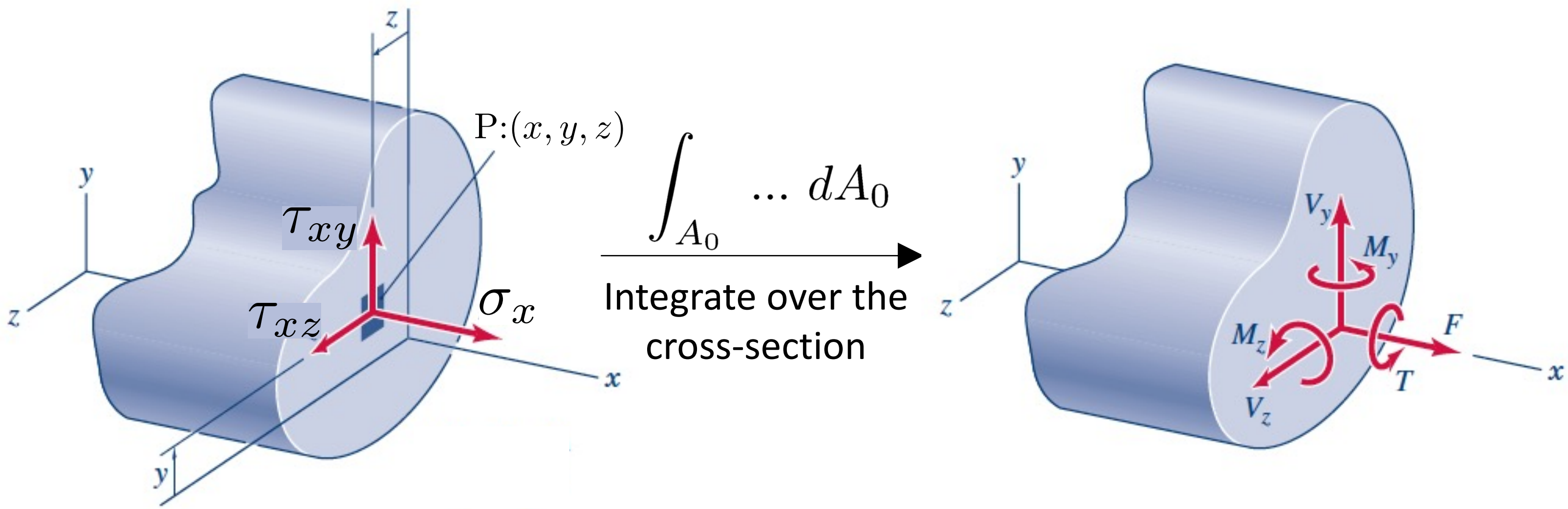
- Many everyday engineering applications utilize structural members such as rods, beams, cables, plates, and shells.
- Structural members can be idealized as one-dimensional (rods, beams, and cables) and two-dimensional (plates and shells) members.
- Assumption: two dimensions (in the case of beams) or one dimension (in the case of plates and shells) are significantly smaller than the other dimensions.



- Stress-strain behavior is upscaled to a relationship between **internal resultants** and **kinematic variables** (of the mid-plane).

Structural elements: beams, plates, shells

Review (undergrad): force and moment resultants



$$F = \int_A \sigma_x dA$$

$$V_y = \int_A \tau_{xy} dA$$

$$V_z = \int_A \tau_{xz} dA$$

$$T = \int_A y \; \tau_{xz} dA - \int_A z \; \tau_{xy} dA$$

$$M_y = \int_A z \; \sigma_x dA$$

$$M_z = -\int_A y \; \sigma_x dA$$

Structural elements: beams, plates, shells

Timoshenko beams

- Kinematic assumptions, with $\bar{w} \ll h$

$$u_1(x,z) = \bar{u}(x) + z\theta_2(x)$$
$$u_2 = 0$$
$$u_3(x) = \bar{w}(x)$$

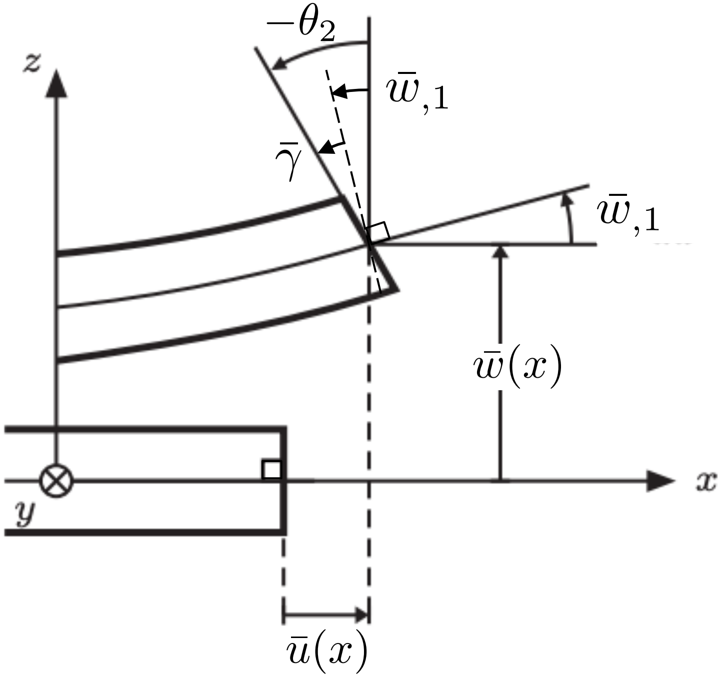
- Small-strain tensor, with $(\bar{w}_{,1})^2 \ll 1$

$$\epsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i})$$

$$\epsilon_{ij} = \begin{bmatrix} \bar{u}_{,1} + z\theta_{2,1} & 0 & (\theta_2 + \bar{w}_{,1})/2 \\ 0 & 0 & 0 \\ (\theta_2 + \bar{w}_{,1})/2 & 0 & 0 \end{bmatrix}$$

$$\epsilon_{ij} = \begin{bmatrix} \bar{\epsilon}_{11} + z\bar{\kappa}_{11} & 0 & \bar{\gamma}_{13}/2 \\ 0 & 0 & 0 \\ \bar{\gamma}_{13}/2 & 0 & 0 \end{bmatrix}$$

Simplified geometry: $b \ll L \ , \ h \ll L$



kinematic variables

$\bar{\epsilon}_{11}(x) \qquad \bar{\kappa}_{11}(x) \qquad \bar{\gamma}_{13}(x)$

extensional, bending, and shear components of beam strain

Structural elements: beams, plates, shells

Timoshenko beams

- Small-strain tensor || Linear elasticity (generalized Hooke's law)

$$\epsilon_{ij} = \begin{bmatrix} \bar{\epsilon}_{11} + z\bar{\kappa}_{11} & 0 & \bar{\gamma}_{13}/2 \\ 0 & 0 & 0 \\ \bar{\gamma}_{13}/2 & 0 & 0 \end{bmatrix} \qquad \sigma_{ij} = \begin{bmatrix} E(\bar{\epsilon}_{11} + z\bar{\kappa}_{11}) & 0 & \frac{E}{2(1+\nu)}\bar{\gamma}_{13} \\ 0 & 0 & 0 \\ \frac{E}{2(1+\nu)}\bar{\gamma}_{13} & 0 & 0 \end{bmatrix}$$

… assuming $\sigma_{22} \ll \sigma_{11}$ and $\sigma_{33} \ll \sigma_{11}$

Since lateral surfaces are traction free, stresses in 22 and 33 directions must be zero on free surfaces.

incompatible with strain field

(Reissner's theory overcomes this issue)

$$\begin{Bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{12} \\ \sigma_{23} \\ \sigma_{13} \end{Bmatrix} = \frac{E}{(1+\nu)(1-2\nu)} \begin{bmatrix} (1-\nu) & \nu & \nu & 0 & 0 & 0 \\ \nu & (1-\nu) & \nu & 0 & 0 & 0 \\ \nu & \nu & (1-\nu) & 0 & 0 & 0 \\ 0 & 0 & 0 & (1-2\nu)/2 & 0 & 0 \\ 0 & 0 & 0 & 0 & (1-2\nu)/2 & 0 \\ 0 & 0 & 0 & 0 & 0 & (1-2\nu)/2 \end{bmatrix} \begin{Bmatrix} \epsilon_{11} \\ \epsilon_{22} \\ \epsilon_{33} \\ \gamma_{12} \\ \gamma_{23} \\ \gamma_{13} \end{Bmatrix}$$

Structural elements: beams, plates, shells

Timoshenko beams

- Small-strain tensor || Linear elasticity (generalized Hooke's law)

$$\epsilon_{ij} = \begin{bmatrix} \bar{\epsilon}_{11} + z\bar{\kappa}_{11} & 0 & \bar{\gamma}_{13}/2 \\ 0 & 0 & 0 \\ \bar{\gamma}_{13}/2 & 0 & 0 \end{bmatrix} \qquad \sigma_{ij} = \begin{bmatrix} E(\bar{\epsilon}_{11} + z\bar{\kappa}_{11}) & 0 & \frac{E}{2(1+\nu)}\bar{\gamma}_{13} \\ 0 & 0 & 0 \\ \frac{E}{2(1+\nu)}\bar{\gamma}_{13} & 0 & 0 \end{bmatrix}$$

DIY

$$\begin{Bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{12} \\ \sigma_{23} \\ \sigma_{13} \end{Bmatrix} = \frac{E}{(1+\nu)(1-2\nu)} \begin{bmatrix} (1-\nu) & \nu & \nu & 0 & 0 & 0 \\ \nu & (1-\nu) & \nu & 0 & 0 & 0 \\ \nu & \nu & (1-\nu) & 0 & 0 & 0 \\ 0 & 0 & 0 & (1-2\nu)/2 & 0 & 0 \\ 0 & 0 & 0 & 0 & (1-2\nu)/2 & 0 \\ 0 & 0 & 0 & 0 & 0 & (1-2\nu)/2 \end{bmatrix} \begin{Bmatrix} \epsilon_{11} \\ \epsilon_{22} \\ \epsilon_{33} \\ \gamma_{12} \\ \gamma_{23} \\ \gamma_{13} \end{Bmatrix}$$

Structural elements: beams, plates, shells

Timoshenko beams

- Linear elasticity (generalized Hooke's law) assuming $\sigma_{22} \ll \sigma_{11}$ and $\sigma_{33} \ll \sigma_{11}$ Since lateral surfaces are traction free, stresses in 22 and 33 directions must be zero on free surfaces.

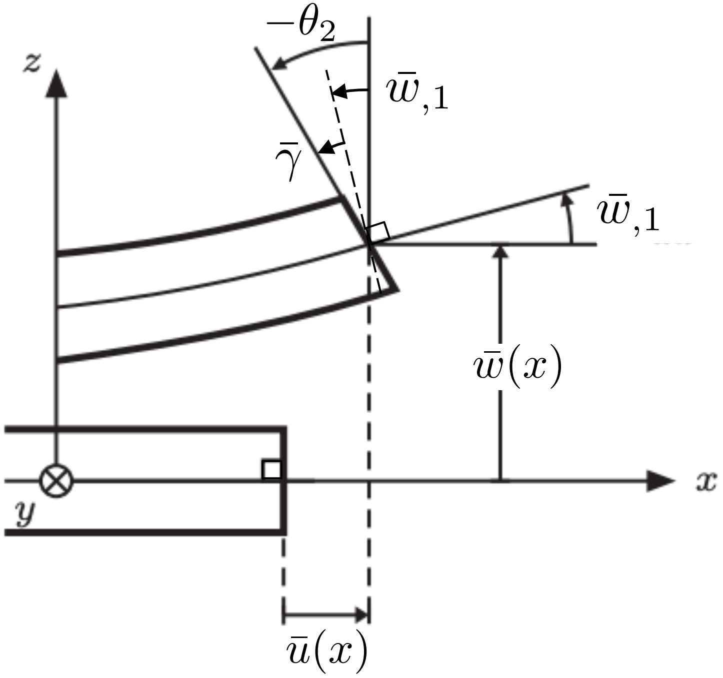
$$\sigma_{ij} = \begin{bmatrix} E(\bar{\epsilon}_{11} + z\bar{\kappa}_{11}) & 0 & \frac{E}{2(1+\nu)}\bar{\gamma}_{13} \\ 0 & 0 & 0 \\ \frac{E}{2(1+\nu)}\bar{\gamma}_{13} & 0 & 0 \end{bmatrix}$$

- Internal resultants

$$N_1 = \int_A \sigma_{11} \mathrm{d}A \qquad M_2 = \int_A z\sigma_{11} \mathrm{d}A$$

$$V_1 = K_s \int_A \sigma_{13} \mathrm{d}A$$

Simplified geometry: $b \ll L$, $h \ll L$



beam geometry

height (h), width (b)
cross-sectional area (A)
moment of inertia (I)
Shear correction factor (K_s)

Structural elements: beams, plates, shells

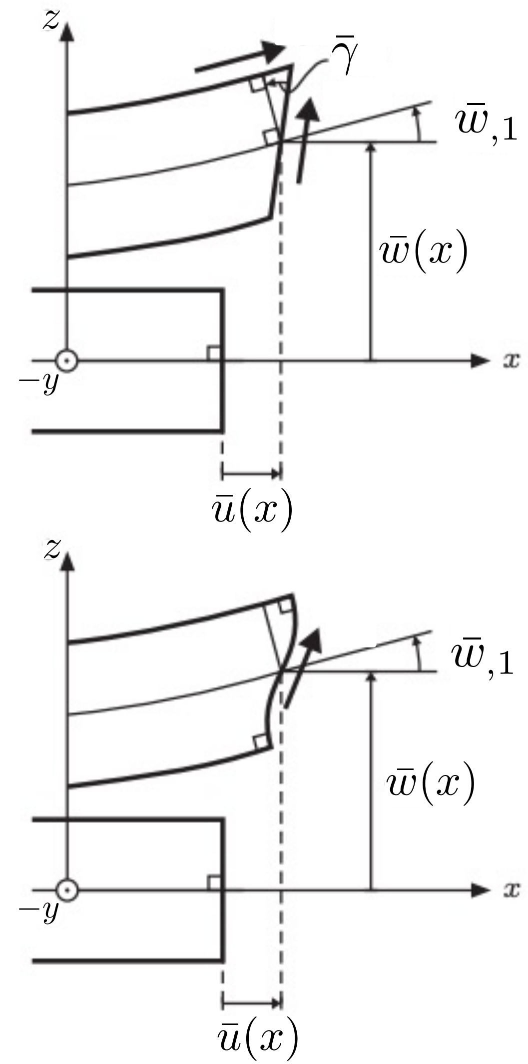
Timoshenko beams

- Internal resultants

$$N_1 = b\int_{-h/2}^{h/2} E(\bar{\epsilon}_{11} + z\bar{\kappa}_{11})\mathrm{d}z = EA\bar{\epsilon}_{11}$$

$$M_2 = b\int_{-h/2}^{h/2} E(z\bar{\epsilon}_{11} + z^2\bar{\kappa}_{11})\mathrm{d}z = EI_{22}\bar{\kappa}_{11}$$

$$V_1 = K_\mathrm{s} b\int_{-h/2}^{h/2} \frac{E}{2(1+\nu)}\bar{\gamma}_{13}\mathrm{d}z = \frac{K_\mathrm{s}AE}{2(1+\nu)}\bar{\gamma}_{13}$$



Note: The shear correction factor K_s is required in the formulation due to the absence of shear stress and strain at the top and bottom boundaries of the beam. K_s is computed such that resultant shear force V_1 creates the same strain energy as does the true transverse stresses predicted by the three-dimensional elasticity theory.

Structural elements: beams, plates, shells

Timoshenko beams

- Shear correction factor

$$V_1 = K_\mathrm{s} b \int_{-h/2}^{h/2} \frac{E}{2(1+\nu)} \bar{\gamma}_{13} \mathrm{d}z = \frac{K_\mathrm{s} A E}{2(1+\nu)} \bar{\gamma}_{13}$$

LXVI. *On the Correction for Shear of the Differential Equation for Transverse Vibrations of Prismatic Bars.* By Prof. S. P. Timoshenko *.

In studying the transverse vibrations of prismatic bars, we usually start from the differential equation

$$\mathrm{EI}\frac{\partial^4 y}{\partial x^4} + \frac{\rho\Omega}{g}\frac{\partial^2 y}{\partial t^2} = 0, \quad . \quad . \quad . \quad . \quad . \quad (1)$$

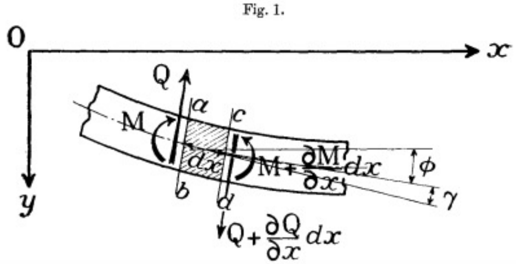
in which EI denotes the flexural rigidity of the bar,
Ω the area of the cross-section,
and $\frac{\rho}{g}$ the density of the material.

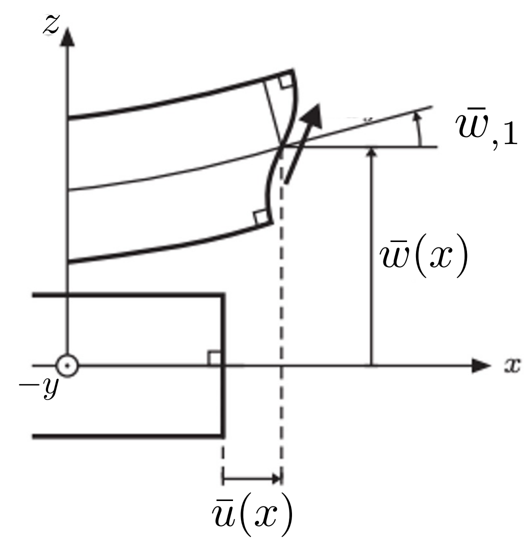
When the "rotatory inertia" is taken into consideration, the equation takes the form

$$\mathrm{EI}\frac{\partial^4 y}{\partial x^4} - \frac{\mathrm{I}\rho}{g}\frac{\partial^4 y}{\partial x^2 \partial t^2} + \frac{\rho\Omega}{g}\frac{\partial^2 y}{\partial t^2} = 0. \quad . \quad . \quad . \quad (2)$$

I now propose to show how the effect of the shear may be taken into account in investigating transverse vibrations, and I shall deduce the general equation of vibration, from which equations (1) and (2) may be obtained as special cases.

Fig. 1.





S.P. Timoshenko (**1921**), "On the correction for shear of the differential equation for transverse vibrations of prismatic bars", *Philosophical Magazine and Journal of Science*, 41:245, 744-746.

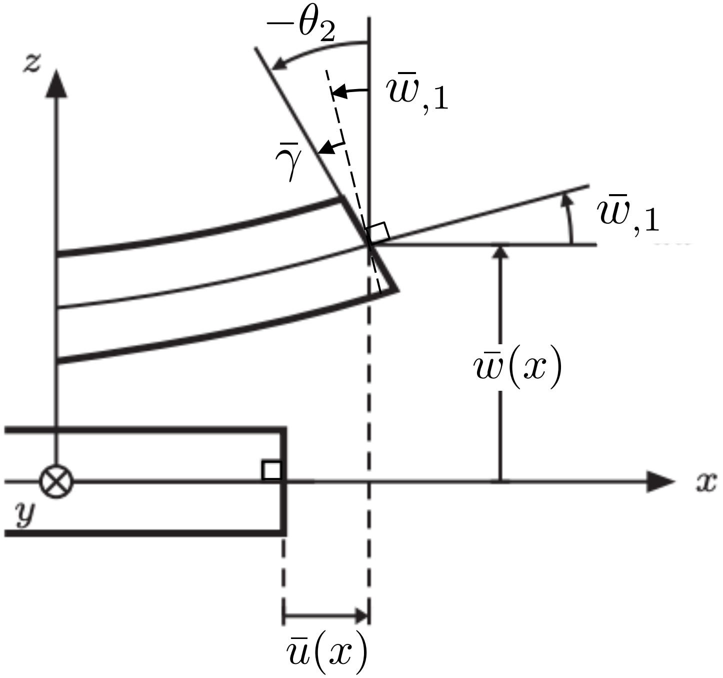
Dong, S. B., Alpdogan, C., & Taciroglu, E. (**2010**). "Much ado about shear correction factors in Timoshenko beam theory". International Journal of Solids and Structures, 47(13), 1651-1665.

Structural elements: beams, plates, shells

Timoshenko beams

- Constitutive relation
  (relationship between internal resultants and kinematic variables)

$$\begin{Bmatrix} N_1 \\ M_2 \\ V_1 \end{Bmatrix} = \begin{bmatrix} EA & 0 & 0 \\ 0 & EI_{22} & 0 \\ 0 & 0 & \frac{K_s EA}{2(1+\nu)} \end{bmatrix} \begin{Bmatrix} \bar{\epsilon}_{11} \\ \bar{\kappa}_{11} \\ \bar{\gamma}_{13} \end{Bmatrix}$$



Euler-Bernoulli beams

- Constitutive relation

$$\begin{Bmatrix} N_1 \\ M_2 \end{Bmatrix} = \begin{bmatrix} EA & 0 \\ 0 & EI_{22} \end{bmatrix} \begin{Bmatrix} \bar{\epsilon}_{11} \\ \bar{\kappa}_{11} \end{Bmatrix}$$

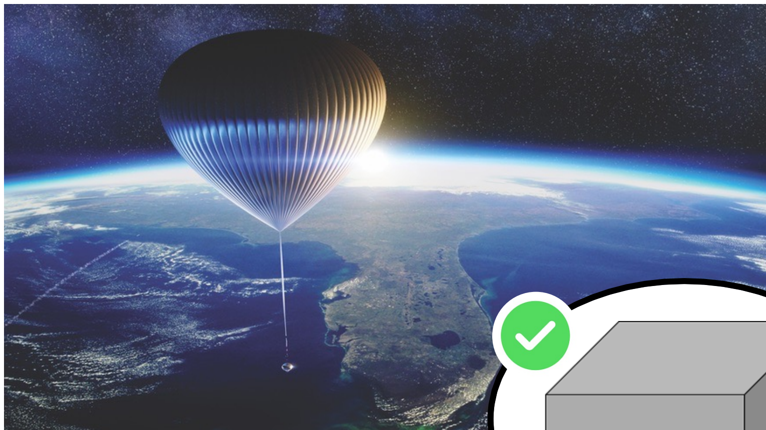
kinematic variables

$\bar{\epsilon}_{11}(x)$ $\quad \bar{\kappa}_{11}(x)$ $\quad \bar{\gamma}_{13}(x)$

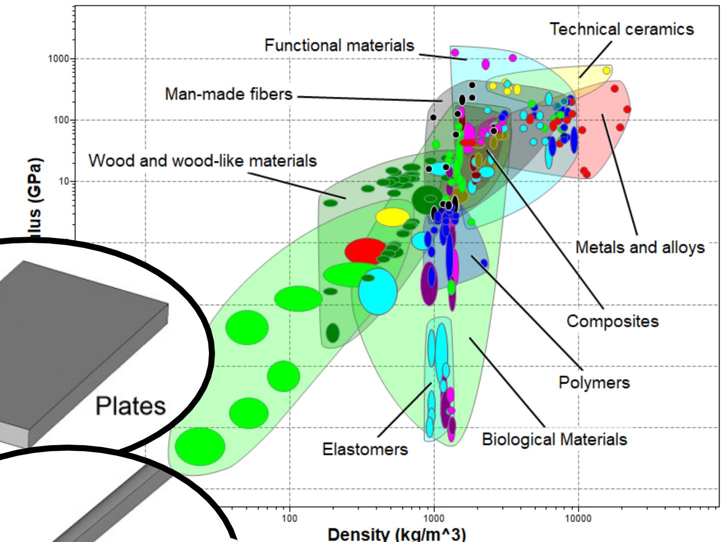
extensional, bending, and shear components of beam strain

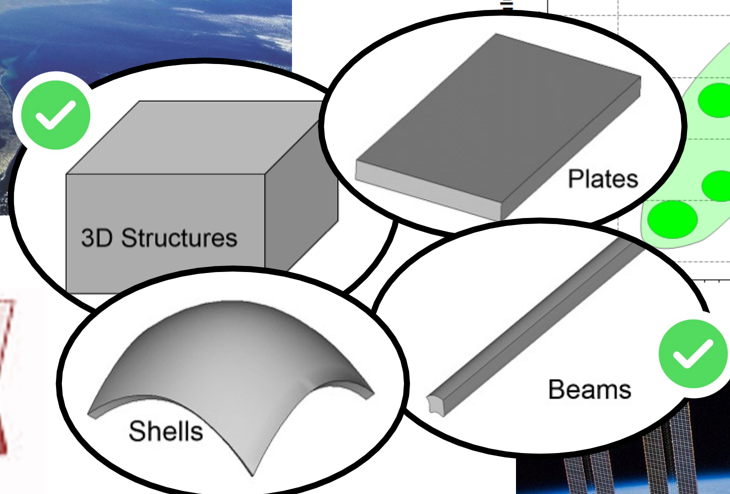
Structural elements: beams, plates, shells

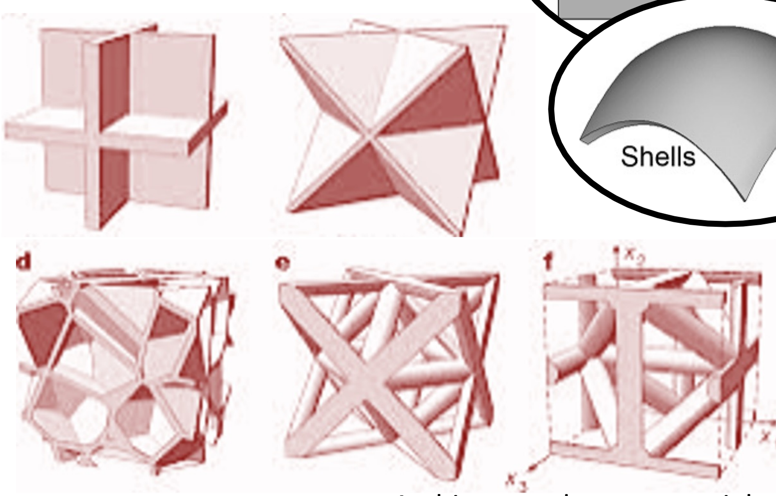
Formulation of **structural elements (beams, plates, shells)** as the analytical upscaling of continuum solids under kinematic assumptions.



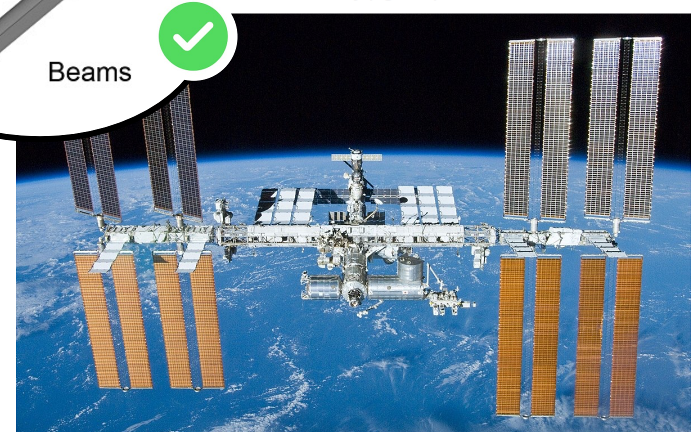
Stratospheric ballooning system







Architectured metamaterials



International space station

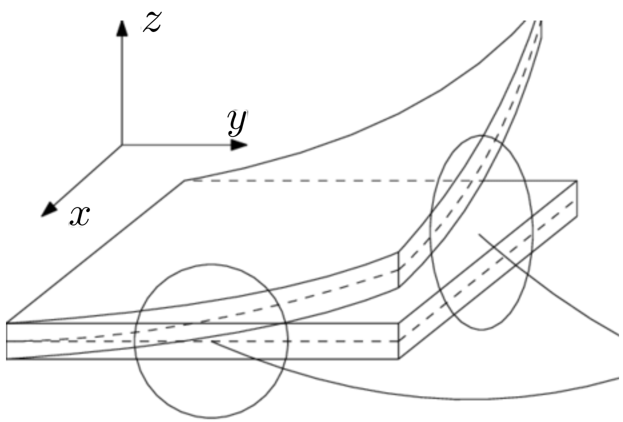
Structural elements: beams, plates, shells

Mindlin's plates

Simplified geometry: the plate is initially flat

- Kinematic assumptions, with $\bar{w} \ll h$

$$u_1(x,y,z) = \bar{u}(x,y) + z\theta_1(x,y)$$
$$u_2(x,y,z) = \bar{v}(x,y) + z\theta_2(x,y)$$
$$u_3(x,y,z) = \bar{w}(x,y)$$



analogy with beams

- Small-strain tensor, with $(\bar{w}_{,1})^2 \ll 1$

$$\epsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i})$$

$$\epsilon_{ij} = \begin{bmatrix} \bar{\epsilon}_{11} + z\bar{\kappa}_{11} & \bar{\gamma}_{12}/2 + z\bar{\kappa}_{12} & \bar{\gamma}_{13}/2 \\ \bar{\gamma}_{12}/2 + z\bar{\kappa}_{12} & \bar{\epsilon}_{22} + z\bar{\kappa}_{22} & \bar{\gamma}_{23}/2 \\ \bar{\gamma}_{13}/2 & \bar{\gamma}_{23}/2 & 0 \end{bmatrix}$$

kinematic variables

$\bar{\epsilon}_{11}$ $\bar{\epsilon}_{22}$ $\bar{\kappa}_{11}$ $\bar{\kappa}_{22}$

extensional strains, bending strains

kinematic variables (cont.)

$\bar{\gamma}_{12}$ $\bar{\kappa}_{12}$ $\bar{\gamma}_{13}$ $\bar{\gamma}_{23}$

contribution of the mid-plane displacement to in-plane shear, contribution of normal rotations to in-plane shear, and shear components

Structural elements: beams, plates, shells

Mindlin's plates

- Small-strain tensor

$$\epsilon_{ij} = \begin{bmatrix} \bar{\epsilon}_{11} + z\bar{\kappa}_{11} & \bar{\gamma}_{12}/2 + z\bar{\kappa}_{12} & \bar{\gamma}_{13}/2 \\ \bar{\gamma}_{12}/2 + z\bar{\kappa}_{12} & \bar{\epsilon}_{22} + z\bar{\kappa}_{22} & \bar{\gamma}_{23}/2 \\ \bar{\gamma}_{13}/2 & \bar{\gamma}_{23}/2 & 0 \end{bmatrix}$$

- Linear elasticity (generalized Hooke's law) assuming $\sigma_{33} \ll \sigma_{22}$ and $\sigma_{33} \ll \sigma_{11}$ Since top/bottom surfaces are traction free, the stress in 33 direction must be zero on free surfaces.

solution of a mixed boundary condition problem

$$\begin{Bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{12} \\ \sigma_{23} \\ \sigma_{13} \end{Bmatrix} = \frac{E}{(1+\nu)(1-2\nu)} \begin{bmatrix} (1-\nu) & \nu & \nu & 0 & 0 & 0 \\ \nu & (1-\nu) & \nu & 0 & 0 & 0 \\ \nu & \nu & (1-\nu) & 0 & 0 & 0 \\ 0 & 0 & 0 & (1-2\nu)/2 & 0 & 0 \\ 0 & 0 & 0 & 0 & (1-2\nu)/2 & 0 \\ 0 & 0 & 0 & 0 & 0 & (1-2\nu)/2 \end{bmatrix} \begin{Bmatrix} \epsilon_{11} \\ \epsilon_{22} \\ \epsilon_{33} \\ \gamma_{12} \\ \gamma_{23} \\ \gamma_{13} \end{Bmatrix}$$

Structural elements: beams, plates, shells

Aside: Kirchhoff plates

- Small-strain tensor, with $\theta_2 = -\bar{w}_{,1}\ ,\ \theta_1 = -\bar{w}_{,2}$

$$\epsilon_{ij} = \begin{bmatrix} \bar{\epsilon}_{11} + z\bar{\kappa}_{11} & \bar{\gamma}_{12}/2 + z\bar{\kappa}_{12} & \bcancel{\bar{\gamma}_{13}/2} \\ \bar{\gamma}_{12}/2 + z\bar{\kappa}_{12} & \bar{\epsilon}_{22} + z\bar{\kappa}_{22} & \bcancel{\bar{\gamma}_{23}/2} \\ \bcancel{\bar{\gamma}_{13}/2} & \bcancel{\bar{\gamma}_{23}/2} & 0 \end{bmatrix}$$

- Linear elasticity (generalized Hooke's law) assuming $\sigma_{33} \ll \sigma_{22}$ and $\sigma_{33} \ll \sigma_{11}$ Since top/bottom surfaces are traction free, the stress in 33 direction must be zero on free surfaces.

solution of a mixed boundary condition problem

$$\begin{Bmatrix} \sigma_{11} \\ \sigma_{22} \\ \bcancel{\sigma_{33}} \\ \sigma_{12} \\ \sigma_{23} \\ \sigma_{13} \end{Bmatrix} = \frac{E}{(1+\nu)(1-2\nu)} \begin{bmatrix} (1-\nu) & \nu & \nu & 0 & 0 & 0 \\ \nu & (1-\nu) & \nu & 0 & 0 & 0 \\ \nu & \nu & (1-\nu) & 0 & 0 & 0 \\ 0 & 0 & 0 & (1-2\nu)/2 & 0 & 0 \\ 0 & 0 & 0 & 0 & (1-2\nu)/2 & 0 \\ 0 & 0 & 0 & 0 & 0 & (1-2\nu)/2 \end{bmatrix} \begin{Bmatrix} \epsilon_{11} \\ \epsilon_{22} \\ \epsilon_{33} \\ \bcancel{\gamma_{12}} \\ \bcancel{\gamma_{23}} \\ \gamma_{13} \end{Bmatrix}$$

Structural elements: beams, plates, shells

Mindlin's plates

- Internal resultants

normal forces

$$N_1 = \int_{-h/2}^{h/2} \frac{E}{1-\nu^2}(\bar{\epsilon}_{11} + z\bar{\kappa}_{11} + \nu\bar{\epsilon}_{22} + \nu z\bar{\kappa}_{22})\mathrm{d}z = \frac{Eh}{1-\nu^2}(\bar{\epsilon}_{11} + \nu\bar{\epsilon}_{22})$$

$$N_2 = \int_{-h/2}^{h/2} \frac{E}{1-\nu^2}(\bar{\epsilon}_{22} + z\bar{\kappa}_{22} + \nu\bar{\epsilon}_{11} + \nu z\bar{\kappa}_{11})\mathrm{d}z = \frac{Eh}{1-\nu^2}(\bar{\epsilon}_{22} + \nu\bar{\epsilon}_{11})$$

in-plane shear forces

$$V_{12} = \int_{-h/2}^{h/2} \frac{E}{2(1+\nu)}(\bar{\gamma}_{12} + 2z\bar{\kappa}_{12})\,\mathrm{d}z = \frac{Eh}{2(1+\nu)}\bar{\gamma}_{12}$$

membrane state

bending moment

$$M_1 = \int_{-h/2}^{h/2} \frac{E}{1-\nu^2}(\bar{\epsilon}_{11} + z\bar{\kappa}_{11} + \nu\bar{\epsilon}_{22} + \nu z\bar{\kappa}_{22})z\mathrm{d}z = \frac{Eh^3}{12(1-\nu^2)}(\bar{\kappa}_{11} + \nu\bar{\kappa}_{22})$$

$$M_2 = \int_{-h/2}^{h/2} \frac{E}{1-\nu^2}(\bar{\epsilon}_{22} + z\bar{\kappa}_{22} + \nu\bar{\epsilon}_{11} + \nu z\bar{\kappa}_{11})z\mathrm{d}z = \frac{Eh^3}{12(1-\nu^2)}(\bar{\kappa}_{22} + \nu\bar{\kappa}_{11})$$

twisting moment

$$Q_{12} = \int_{-h/2}^{h/2} \frac{E}{2(1+\nu)}(\bar{\gamma}_{12} + 2z\bar{\kappa}_{12})\,z\mathrm{d}z = \frac{Eh^3}{24(1+\nu)}\bar{\kappa}_{12}$$

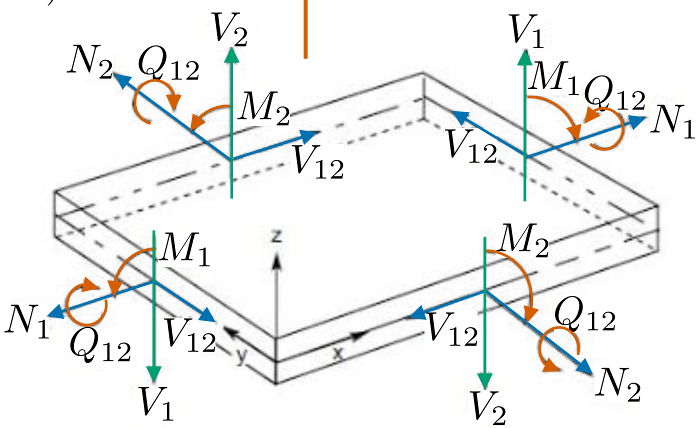
flexural state

transverse shear forces

$$V_1 = K_s \int_{-h/2}^{h/2} \frac{E}{2(1+\nu)}\bar{\gamma}_{13}\mathrm{d}z = \frac{K_s Eh}{2(1+\nu)}\bar{\gamma}_{13}$$

$$V_2 = K_s \int_{-h/2}^{h/2} \frac{E}{2(1+\nu)}\bar{\gamma}_{23}\mathrm{d}z = \frac{K_s Eh}{2(1+\nu)}\bar{\gamma}_{23}$$

$$K_s = 5/6$$



Structural elements: beams, plates, shells

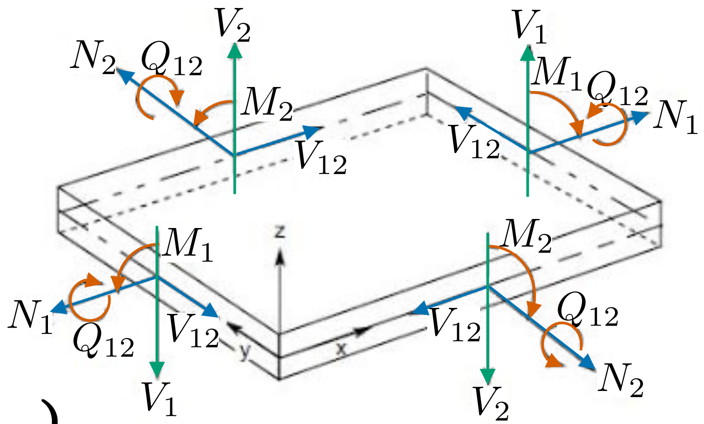
Mindlin's plates

- Constitutive relation
  (relationship between internal resultants and kinematic variables)

$$\begin{Bmatrix} N_1 \\ N_2 \\ V_{12} \end{Bmatrix} = \frac{Eh}{1-\nu^2} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} \bar{\epsilon}_{11} \\ \bar{\epsilon}_{22} \\ \bar{\gamma}_{12}/2 \end{Bmatrix}$$

$$\begin{Bmatrix} M_1 \\ M_2 \\ Q_{12} \end{Bmatrix} = \frac{Eh^3}{12(1-\nu^2)} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & 1-\nu \end{bmatrix} \begin{Bmatrix} \bar{\kappa}_{11} \\ \bar{\kappa}_{22} \\ \bar{\kappa}_{12}/2 \end{Bmatrix}$$

$$\begin{Bmatrix} V_1 \\ V_2 \end{Bmatrix} = \frac{K_s Eh}{2(1+\nu)} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{Bmatrix} \bar{\gamma}_{13} \\ \bar{\gamma}_{23} \end{Bmatrix}$$



kinematic variables

$\bar{\epsilon}_{11}$ $\bar{\epsilon}_{22}$ $\bar{\kappa}_{11}$ $\bar{\kappa}_{22}$

extensional strains, bending strains

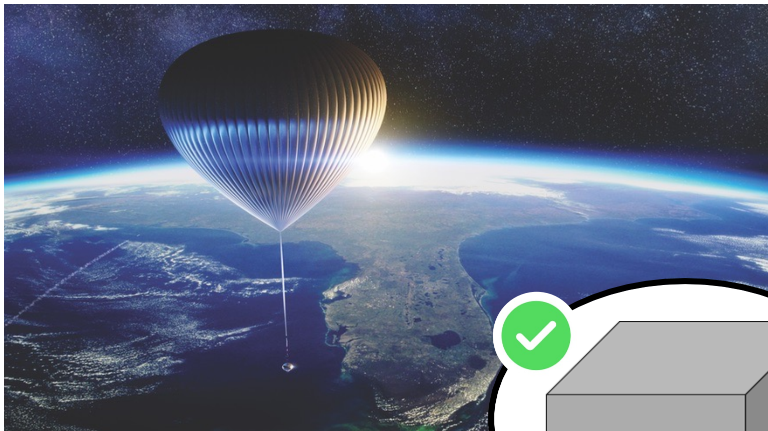
kinematic variables (cont.)

$\bar{\gamma}_{12}$ $\bar{\kappa}_{12}$ $\bar{\gamma}_{13}$ $\bar{\gamma}_{23}$

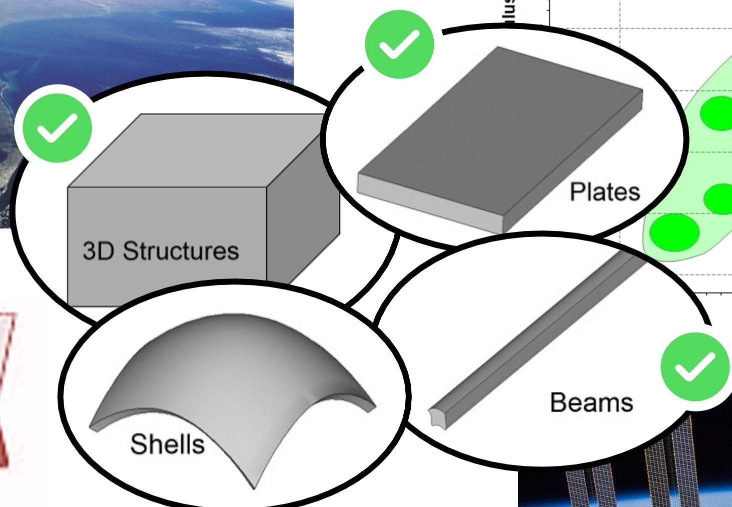
contribution of the mid-plane displacement to in-plane shear, contribution of normal rotations to in-plane shear, and shear components

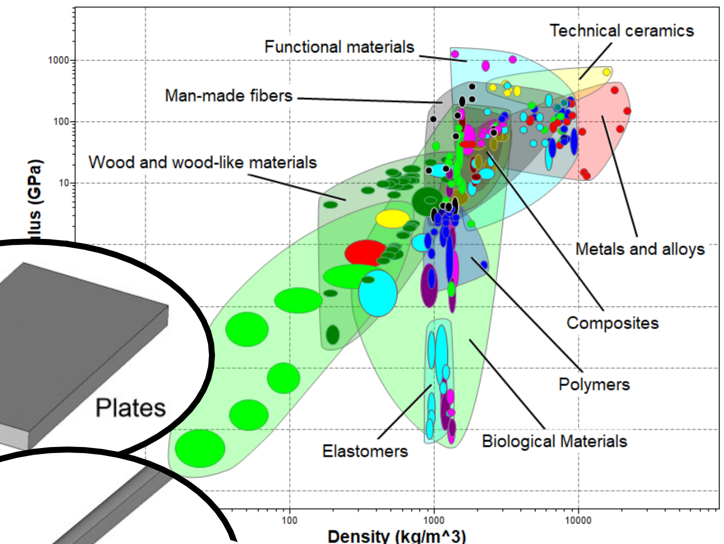
Structural elements: beams, plates, shells

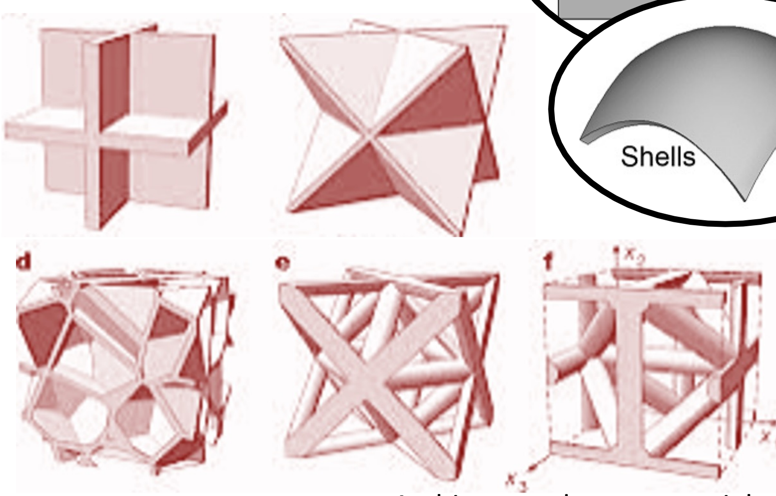
Formulation of **structural elements (beams, plates, shells)** as the analytical upscaling of continuum solids under kinematic assumptions.



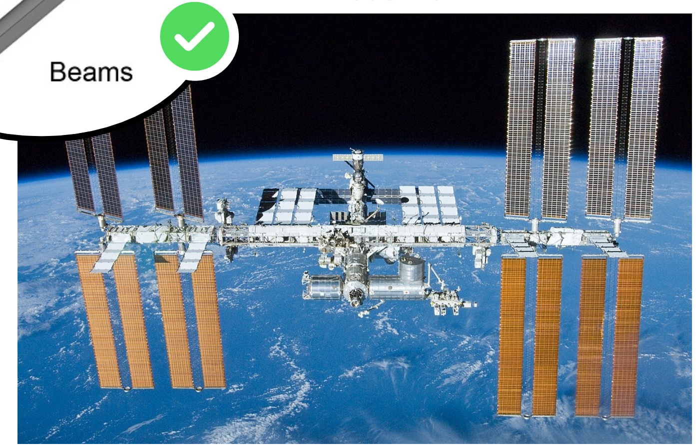
Stratospheric ballooning system







Architectured metamaterials

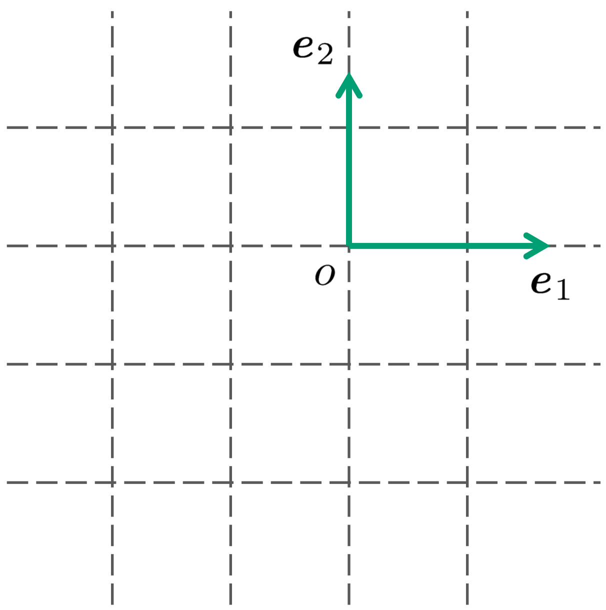


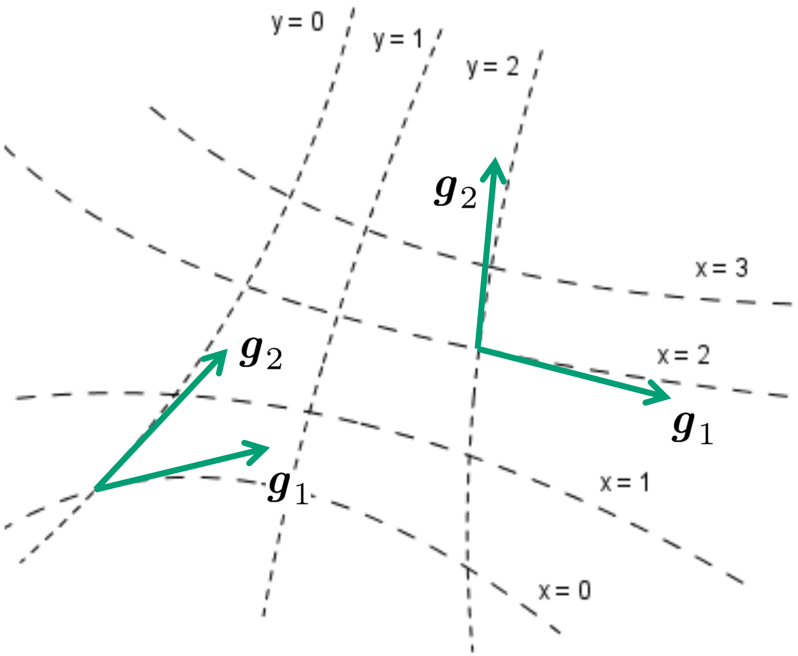
International space station

Review Lecture 1: Coordinate system

Curvilinear coordinate system

- An <u>origin</u> (relative to which positions are measured)
- A <u>set of coordinate curves</u>
- <u>Basis</u> are defined at each position in space as the tangent vectors $\{\boldsymbol{g}_i\}$ to the coordinate curves. Therefore, basis vectors change from position to position. Basis $\{\boldsymbol{g}_i\}$ are <u>non-orthogonal</u> in general.

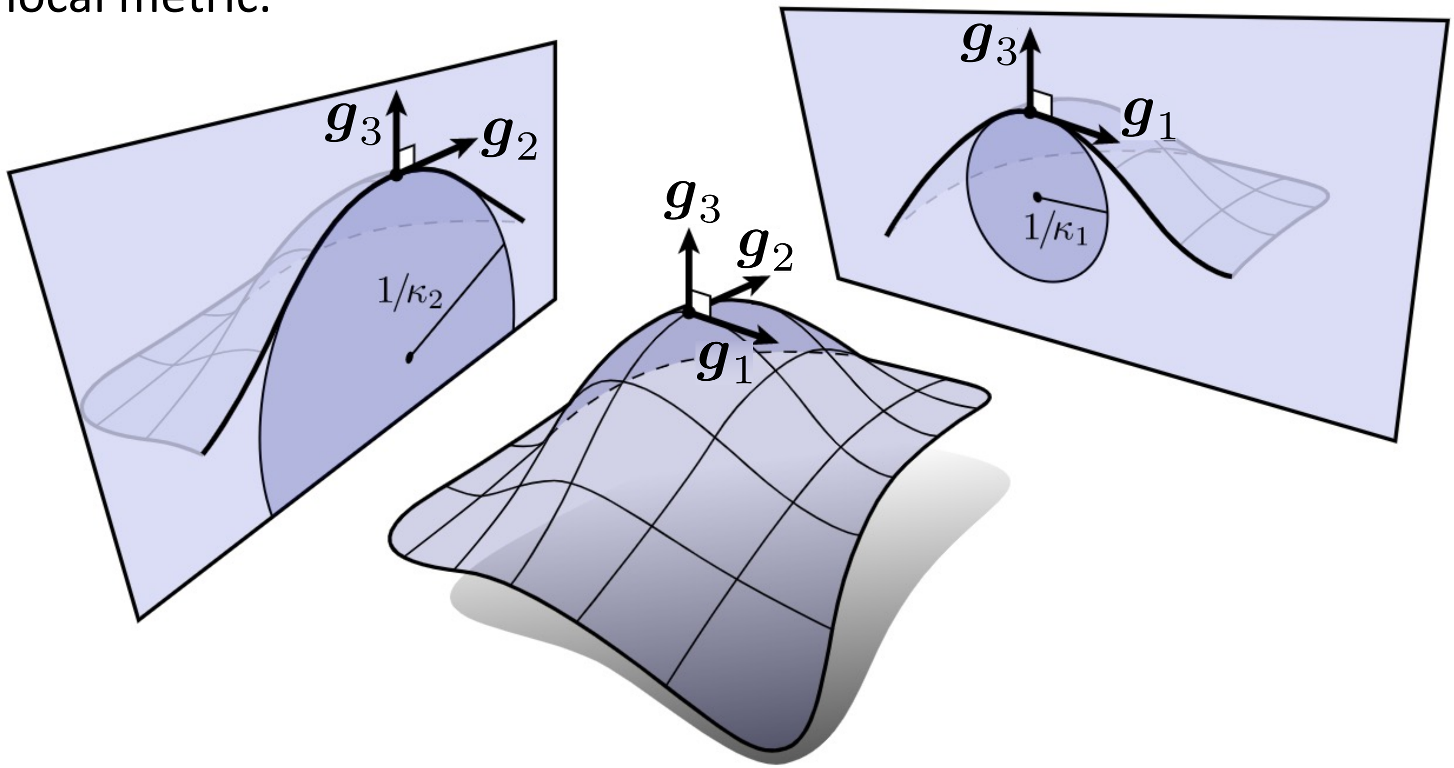




Structural elements: beams, plates, shells

Mindlin's shells – Kirchhoff shells

- Like plate formulations but using an orthogonal curvilinear coordinate system (lines of principal curvature on a smooth surface are orthogonal).
- Small-strain tensor and kinematic variables now involve local basis and local metric.

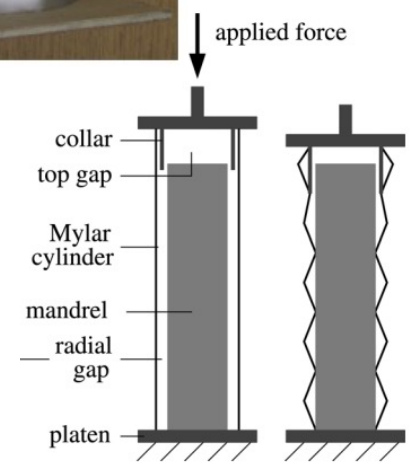


Structural elements: beams, plates, shells

Shell buckling

"Yoshimura" pattern

Progressive formation of a surface texture during axial buckling of a thin-walled cylinder fitted onto a mandrel core



Structural elements: beams, plates, shells

Any questions?