Economics 671
Problem Set #1

(1) Consider the simple regression model

$$y_i = \beta_1 + x_i\beta_2 + \epsilon_i.$$

where x_i is a scalar. You may recall, from a previous econometrics course, that in this simple regression model, the OLS estimator of the slope is:

$$\hat{\beta}_2 = \frac{\sum_i (x_i - \overline{x})(y_i - \overline{y})}{\sum_i (x_i - \overline{x})^2}.$$

Verify that application of the general formula

$$\hat{\beta} = (X'X)^{-1}X'y,$$

in the context of the simple regression model gives you this same answer.

(2) (a) When running a regression, a researcher faces an arbitrary and seemingly inconsequential choice: Suppose that she wants to run a regression of a dependent variable of y on a set of variables x_1 through x_k. The researcher could include these variables in the given chronological order, and construct the $n \times k$ X matrix accordingly, or could interchange the order of the x_j variables in any way she chooses, construct a different X matrix (with columns interchanged) and the run the regression again.

Intuitively, the order she chooses should not really change anything: if the columns of X are permuted, then the elements of the $\hat{\beta}$ vector should be correspondingly permuted; if this were not the case, then regression would be ... weird, and decidedly unscientific.

Using the methods discussed in class (which will require linear algebra), prove that any permutation of the columns of X will not alter values of the coefficient estimates themselves, but will simply permute the elements of $\hat{\beta}$.

Related to part (a) above, consider the following regression, where w, x and z are all scalars:

$$y_i = \beta_0 + \beta_1 w_i + \beta_2 x_i + \beta_3 z_i + u_i$$

and a similar regression

$$y_i = \pi_0 + \pi_1 r_i + \pi_2 x_i + \pi_3 z_i + v_i$$

where

$$r_i = w_i + x_i.$$

(b) Since x_i is contained in r_i and is also included as a right-hand side covariate in the second regression, does this violate our full-rank condition on the $n \times 4$ matrix X?

(c) If it does not, establish (using linear algebra) the relationship between $\hat{\pi}$ and $\hat{\beta}$.