Economics 690
Problem Set: Regression Model

(1) The lecture notes for the linear regression model provided an example related to the returns to education using a sample of wage, education and ability data. This analysis employed the improper prior

$$p(\beta, \sigma^2) \propto \sigma^{-2}.$$

Using this data and a small MATLAB program, obtain the marginal posterior distribution for the intercept parameter β_0. *Note:* You do not need to derive anything analytically here, but simply state the form of this marginal distribution. You answer should be similar to our result for the slope β_1:

$$\beta_1|y \sim t(.091, .0066^2, 1215)$$

with the appropriate mean, scale parameter and degree of freedom parameter inserted.

(2) In the lecture notes related to the linear regression model, we derived posterior distributions under two different priors. First, an improper prior of the form

$$p(\beta, \sigma^2) \propto \sigma^{-2}$$

and second, a conjugate prior of the form

$$p(\beta|\sigma^2) \sim N(\mu, \sigma^2 V_\beta)$$

$$\sigma^2 \sim IG(a, b).$$

For the first prior, the lecture notes were reasonably comprehensive, and derived $p(\beta|y)$, $p(\sigma^2|y)$ and $p(\beta|\sigma^2, y)$. For the second prior, only $p(\beta|\sigma^2, y)$ was derived.

Using similar steps to those provided in the derivation under the first prior, derive:

the marginal posterior for σ^2, $p(\sigma^2|y)$, under the conjugate prior

$$p(\beta|\sigma^2) \sim N(\mu, \sigma^2 V_\beta), \quad \sigma^2 \sim IG(a, b).$$

Hint: Write out the joint posterior $p(\beta, \sigma^2|y)$ (as done in your lecture notes). Then, complete the square on β and note that $p(\sigma^2|y) = \int p(\beta, \sigma^2|y) d\beta$. When performing this integration, note that all terms involving only σ^2 can be moved outside the integral.