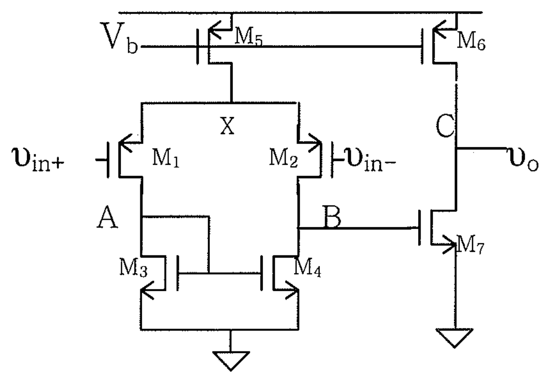
ECE 595A CMOS Analog IC Design
Exam 2

1. [25 points] Consider the differential input / single-ended output amplifier shown below. Assume the signal source has zero source impedance, node X is analog ground, and Cgd=0 for all transistors. Assume all transistors have finite r_o. ~~Assume~~ and $\boxed{r_o \gg \frac{1}{g_m}}$ assume
M1=M2, M3=M4, and M5=M6.



(a) [6 points] Derive the expression for the low frequency gain [$v_o/v_{in} = v_o/(v_{in+} - v_{in-})$].

$$\cancel{\frac{1}{2}v_{in} \rightarrow \frac{v_o}{v_{in+}} = -g_{m1}(r_{o2} // r_{o4})\, g_{m7}(r_{o6} // r_{o7})}$$

$$\cancel{-\frac{1}{2}v_{in} \quad \frac{v_o}{v_{in-}} = g_{m2}(r_{o2} // r_{o4})\, g_{m7}(r_{o6} // r_{o7})}$$

$$\cancel{v_o\left(\frac{1}{v_{in+}} - \frac{1}{v_{in-}}\right) = -2g_{m1}(r_{o2} // r_{o4})\, g_{m7}(r_{o6} // r_{o7})}$$

$$\cancel{v_o\left(\frac{1}{\frac{1}{2}v_{in}} + \frac{1}{\frac{1}{2}v_{in}}\right) = \frac{v_o}{v_{in}} 4}$$

$$i_o = +g_{m1} v_{in+} - g_{m2} v_{in-}$$

$$= +g_{m1}\frac{1}{2}v_{in} + g_{m1}\frac{1}{2}v_{in} = +g_{m1}v_{in}$$

$$v_{oB} = +g_{m1}v_{in}(r_{o2} // r_{o4}) = i_o(r_{o2} // r_{o4})$$

$$A_{v1} = g_{m1}(r_{o2} // r_{o4})$$

$$A_{v2} = -g_{m7}(r_{o6} // r_{o7})$$

$$\boxed{A_v = -g_{m1}(r_{o2} // r_{o4})\, g_{m7}(r_{o6} // r_{o7})}$$

(b) [6 points] Derive the expression for the positive-input to the output transfer function, H1(s)= $\upsilon_o/\upsilon_{in+}$. [Note: The low frequency gain is slightly different from the one in (a). Include Cgs, Cdb, and Csb in pole analysis. Find "POLES", but you don't need to find zero.]

$$i_o = g_{m1} v_{in+} = \frac{1}{2} g_{m1} v_{in}$$

$$v_{oB} = \frac{1}{2} g_{m1} (r_{o2} // r_{o4}) v_{in}$$

$$A_{v1} = \frac{1}{2} g_{m1} (r_{o2} // r_{o4})$$

$$A_{v2} = - g_{m7} (r_{o6} // r_{o7})$$

$$\boxed{A_{v+} = - \frac{1}{2} g_{m1} (r_{o2} // r_{o4}) g_{m7} (r_{o6} // r_{o7})}$$

$$\omega_{pA} = \frac{1}{(\cancel{C_{gd1}} + C_{db1} + C_{db3} + C_{gs3} + C_{gs4}) \left(\frac{1}{g_{m3}}\right)}$$

$$\omega_{pB} = \frac{1}{(C_{db2} + C_{db4} + C_{gs7}) (r_{o2} // r_{o4})}$$

$$\omega_{pC} = \frac{1}{(C_{db6} + C_{db4}) (r_{o6} // r_{o7})}$$

$$H1(s) = \frac{A_{v+}}{\left(1 + \frac{s}{\omega_{pA}}\right)\left(1 + \frac{s}{\omega_{pB}}\right)\left(1 + \frac{s}{\omega_{pC}}\right)}$$

(c) [6 points] Derive the expression for the negative-input to the output transfer function, H2(s)= $\upsilon_o/\upsilon_{in-}$. [Note: The low frequency gain is slightly different from the one in (a). Include Cgs, Cdb, and Csb in pole analysis. Find "POLES", but you don't need to find zero.]

$$A_{v2} = +\frac{1}{2} g_{m2} (r_{o2} \parallel r_{o4})\, g_{m7} (r_{o6} \parallel r_{o7}) = -A_{v1}$$

$$H_2(s) = \frac{A_{v2}}{\left(1 + \frac{s}{\omega_{pB}}\right)\left(1 + \frac{s}{\omega_{pC}}\right)}$$

(d) [7 points] Using H1(s) and H2(s), calculate the differential input to single-ended output transfer function, $H(s)=\upsilon_o/\upsilon_{in}(s)=\upsilon_o/(\upsilon_{in+}-\upsilon_{in-})(s)$ [Note: $\upsilon_{in+} = 0.5\upsilon_{in}$, $\upsilon_{in-} = -0.5\upsilon_{in}$][Hint: Although H1(s) and H2(s) do not have zero, H(s) could have zero.]

$$\cancel{H(s) = \frac{A_1}{(1+\frac{s}{\omega_{pA}})(1+\frac{s}{\omega_{pB}})(1+\frac{s}{\omega_{pC}})} + \frac{A_2}{(1+\frac{s}{\omega_{pB}})(1+\frac{s}{\omega_{pC}})}}$$

$$\cancel{= \frac{A_1 + A_2(1+\frac{s}{\omega_{pA}})}{(1+\frac{s}{\omega_{pA}})(1+\frac{s}{\omega_{pB}})(1+\frac{s}{\omega_{pC}})}}$$

$$v_{o1} = H1(s)\, v_{in+} = H1(s)\frac{v_{in}}{2}$$

$$+\; v_{o2} = H2(s)\, v_{in-} = H2(s)(-)\frac{v_{in}}{2}$$

$$v_o = \frac{v_{in}}{2}\left[H1(s) - H2(s)\right]$$

$$= \frac{v_{in}}{2}\left[\frac{A_1}{(1+\frac{s}{\omega_{pA}})(1+\frac{s}{\omega_{pB}})(1+\frac{s}{\omega_{pC}})} - \frac{-A_1}{(1+\frac{s}{\omega_{pB}})(1+\frac{s}{\omega_{pC}})}\right]$$

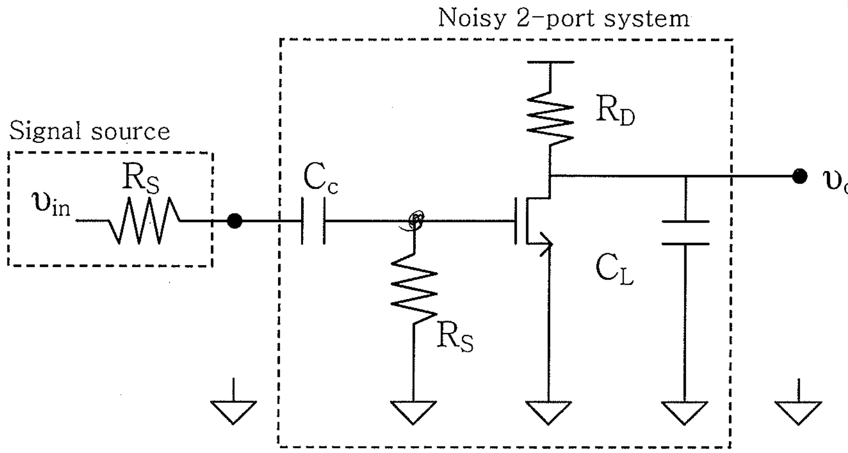
$$= \frac{v_{in}}{2}A_1\left[\frac{1 + 1 + \frac{s}{\omega_{pA}}}{(1+\frac{s}{\omega_{pA}})(1+\frac{s}{\omega_{pB}})(1+\frac{s}{\omega_{pC}})}\right]$$

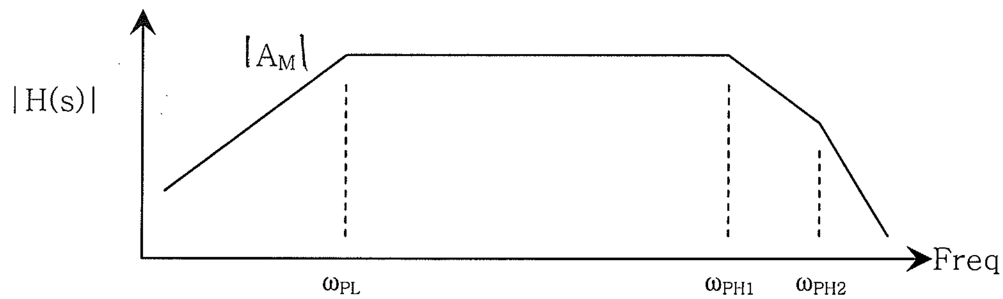
$$\frac{v_o}{v_{in}} = \frac{A_1}{2}\left[\frac{2+\frac{s}{\omega_{pA}}}{(1+\frac{s}{\omega_{pA}})(1+\frac{s}{\omega_{pB}})(1+\frac{s}{\omega_{pC}})}\right]$$

2. [25 points] Consider the common source amplifier shown below. It has band pass style transfer function. Cc=100nF, CL=5pF, Rs=50Ω, R_D=2KΩ.

Assume $C_{gd}=0$

$r_o \gg R_D$





(a) [5 points] Derive the expression for the mid-band gain, A_M.

$$\boxed{A_M = -\frac{1}{2} g_m R_D}$$

v_{in} to the gate

zero

$$\frac{R_s}{R_s + \frac{1}{sC_c} + R_s} = \frac{R_s}{2R_s + \frac{1}{sC_c}} = \frac{sC_cR_s}{1 + s2R_sC_c}$$

pole

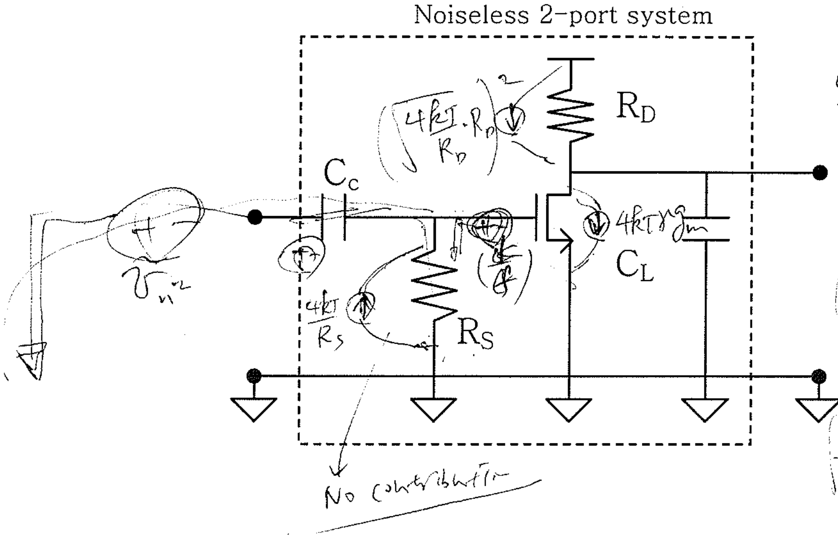
(b) [10 points] Derive the expressions for ω_{PL}, ω_{PH1}, and ω_{PH2}. [Hint: Cc is much bigger than Cgs. You can ignore Cgs, Cgd, CL and Cdb for low frequency analysis, and ignore Cc for high frequency analysis. You don't need to calculate zero.]

$$\omega_{PL} = \frac{1}{C_c(R_s + R_s)}$$

$$\omega_{PH1} = \frac{1}{C_{gs}(\frac{1}{2}R_s)}$$

$$\omega_{PH2} = \frac{1}{C_L R_D}$$

(c) [10 points] Calculate the input-referred noise voltage at mid-band. You don't need to calculate the input-referred noise current. Assume there are drain current noise and flicker noise in the transistor. [Hint: For mid-band analysis, you can ignore Cc, Cgs, Cgd, CL and Cdb.]



$\frac{4kT}{R_D} R_D^2 = \boxed{4kTR_D}$

$\left(\sqrt{4kT\gamma g_m} \cdot R_D\right)^2$

$4kT\gamma g_m R_D^2$

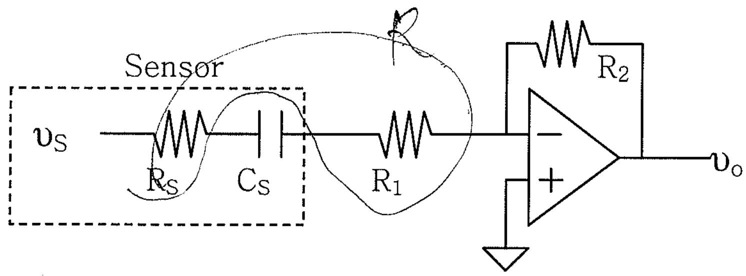
$\frac{K}{C_{ox}WL} \frac{1}{f} \cdot g_m R_D$

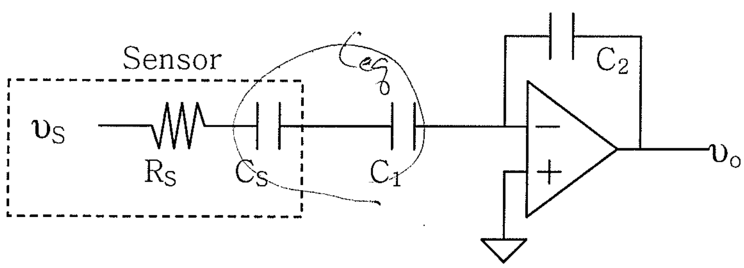
$$\begin{cases} \overline{v_{n,out}^2} = 4kTR_D + 4kT\gamma g_m R_D^2 + \frac{K}{C_{ox}WL} \frac{1}{f} g_m^2 R_D^2 \\ A_v = -g_m \cdot R_D \end{cases}$$

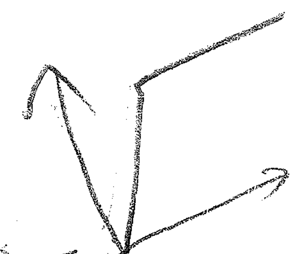
$$\overline{v_{n,in}^2} = \frac{4kT\cancel{R_D}}{g_m^2 R_D^{\cancel{2}}} + \frac{4kT\gamma \cancel{g_m} \cancel{R_D^2}}{g_m^{\cancel{2}} \cancel{R_D^2}} + \frac{K}{C_{ox}WL} \cdot \frac{1}{f} \cdot \frac{g_m^2 R_D^2}{g_m^2 R_D^2}$$

$$= \boxed{\frac{4kT}{g_m^2 R_D} + \frac{4kT\gamma}{g_m} + \frac{K}{C_{ox}WL} \cdot \frac{1}{f}}$$

4. [25 points] Assume there is a sensor that has resistive and capacitive source impedance. Two different types of amplifiers are used to amplify the sensor signal. Calculate the voltage transfer function, H(s)= υ_o/υ_s, for each case [12.5 points each]. Assume the op-amp is ideal.







$\cancel{R + \frac{1}{j\omega C_s}}$ $$\frac{\upsilon_s}{R + \frac{1}{sC_s}} = \frac{0 - \upsilon_o}{R_2}$$

$$-\frac{R_2 s C_s}{1 + sC_s R}$$

$$\boxed{\frac{\upsilon_o}{\upsilon_s} = \frac{-R_2}{R + \frac{1}{sC_s}} \quad (R = R_s + R_1)}$$

$$\frac{\upsilon_s}{R_s + \frac{1}{sC_{eq}}} = \frac{0 - \upsilon_o}{(1/sC_2)}$$

$$\frac{\upsilon_o}{\upsilon_s} = \frac{\left(-\frac{1}{sC_2}\right)}{R_s + \frac{1}{sC_{eq}}} = \frac{-1}{sC_2\left(R_s + \frac{1}{sC_{eq}}\right)} = \frac{-1}{sC_2 R_s + \frac{C_2}{C_{eq}}}$$