HW #2 Solution

1.

(a) Midband gain: $v_o = G_m v_i R_L = G_m \cdot \frac{R_i}{R_s + R_i} v_{in} R_L$

$$\therefore A_M = \frac{v_o}{v_{in}} = \frac{R_i}{R_s + R_i} G_m \cdot R_L$$

(b)
$$\begin{cases} \omega_{PH} = \dfrac{1}{C_L R_L} \\ \omega_{PL} = \dfrac{1}{C_c (R_s + R_i)} \end{cases}$$

$$H(s) = \frac{A_M}{\left(1 + \frac{s}{\omega_{pL}}\right)\left(1 + \frac{s}{\omega_{pH}}\right)}$$

(c)
$$\frac{100k}{20k + 100k} \cdot G_m \cdot 10K = 10$$

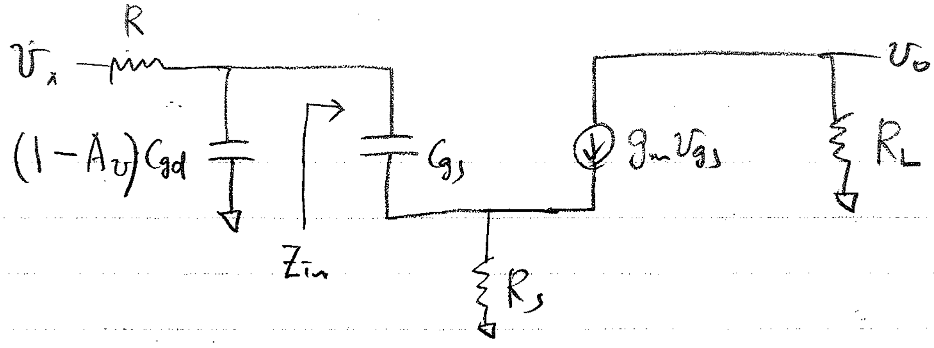
$$G_m = \frac{10}{10K} \cdot \frac{12\cancel{0}}{10\cancel{0}} = 1.2\,[mA/V]$$

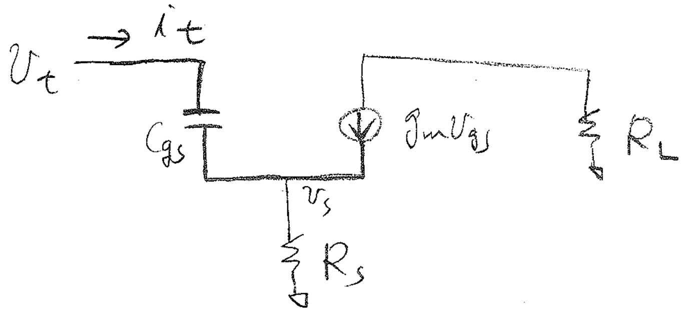
2.

(a)
$$v_o = -g_m v_{gs} R_L = -g_m \cdot \frac{1/g_m}{1/g_m + R_s} \cdot v_i R_L$$

$$\therefore \text{Low freq gain } A_v = -\frac{R_L}{1/g_m + R_s}$$

(b)





$$i_t = \frac{v_t - v_s}{(1/sC_{gs})}, \qquad i_t + g_m(v_t - v_s) = \frac{v_s}{R_s}$$

$$v_s = v_t - \left(\frac{1}{sC_{gs}}\right)(i_t)$$

$$i_t + g_m v_t = \left(g_m + \frac{1}{R_s}\right)\left(v_t - \left(\frac{1}{sC_{gs}}\right) i_t\right)$$

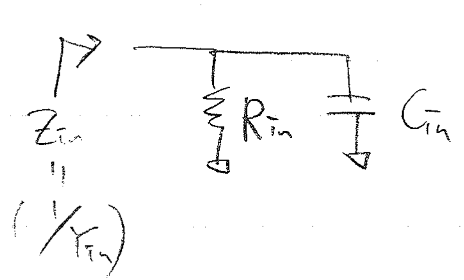
$$\left[1 + \left(g_m + \frac{1}{R_s}\right)\left(\frac{1}{sC_{gs}}\right)\right] i_t = \left[-\cancel{g_m} + \left(\cancel{g_m} + \frac{1}{R_s}\right)\right] v_t$$

$$\therefore Z_{in} = \frac{v_t}{i_t} = R_s\left[1 + \left(g_m + \frac{1}{R_s}\right)\left(\frac{1}{sC_{gs}}\right)\right] = R_s + (1 + g_m R_s)\left(\frac{1}{sC_{gs}}\right)$$

$$Y_{in} = \frac{1}{R_s + \dfrac{1 + g_m R_s}{sC_{gs}}} = \frac{1}{\dfrac{sC_{gs}R_s + 1 + g_m R_s}{sC_{gs}}} = \frac{sC_{gs}}{1 + g_m R_s + sC_{gs}R_s}$$

$$= \frac{sC_{gs}(1 + g_m R_s - j\omega C_{gs} R_s)}{(1 + g_m R_s)^2 + \omega^2 C_{gs}^2 R_s^2}$$

$$= \frac{\omega^2 C_{gs}^2 R_s + j\omega C_{gs}(1 + g_m R_s)}{(1 + g_m R_s)^2 + \omega^2 C_{gs}^2 R_s^2}$$



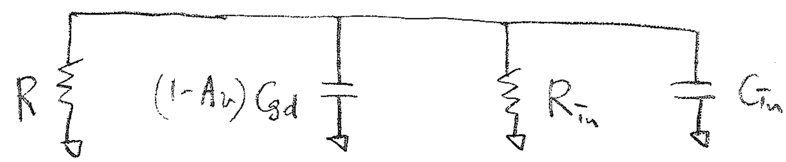
$$\therefore R_{in} = \frac{(1 + g_m R_s)^2 + \omega^2 C_{gs}^2 R_s^2}{\omega^2 C_{gs}^2 R_s}$$

$$= R_s + \frac{(1 + g_m R_s)^2}{\omega^2 C_{gs}^2 R_s}$$

$$\therefore \omega C_{in} = \frac{\omega C_{gs}(1 + g_m R_s)}{(1 + g_m R_s)^2 + \omega^2 C_{gs}^2 R_s^2}$$

$$\therefore C_{in} = \frac{C_{gs}(1 + g_m R_s)}{(1 + g_m R_s)^2 + \omega^2 C_{gs}^2 R_s^2}$$

input pole



$$\therefore \omega_{p,in} = \frac{1}{(R \parallel R_{in})[(1-A_v)C_{gd} + C_{in}]} \quad \Leftarrow \text{Main pole}$$

Output pole

$$\omega_{p,out} \cong \frac{1}{R_L \cdot C_{gd}}$$

$$\therefore H(s) = \frac{A_v}{\left(1+\frac{s}{\omega_{p,in}}\right)\left(1+\frac{s}{\omega_{p,out}}\right)}$$

(c)

R_s	A_v	$\omega_{p,in}$	$G.BW = A_v \times \frac{\omega_{p,in}}{2\pi}$
0			
100			
250			

3. Midband gain $A_v = -g_m R_D$

$$\begin{cases} \omega_p = \dfrac{1}{C_s \left(R_s \parallel \frac{1}{g_m} \right)} , \quad \omega_{p2} \approx \dfrac{1}{(C_{gd} + C_{db}) R_D} \\ \omega_Z \approx \dfrac{1}{C_s \cdot R_s} \end{cases}$$

optional

$$\therefore \boxed{H(s) \cong A_v \cdot \frac{\left(1 + \frac{s}{\omega_Z}\right)}{\left(1 + \frac{s}{\omega_p}\right)\left(1 + \frac{s}{\omega_{p2}}\right)}}$$

4. Midband gain $A_v = \left(-g_{m1} R_D\right)\left(-g_{m2}\left(R_D \parallel R_L\right)\right)$

$$= g_{m1} \cdot g_{m2} \, R_D \cdot (R_D \parallel R_L)$$

Poles

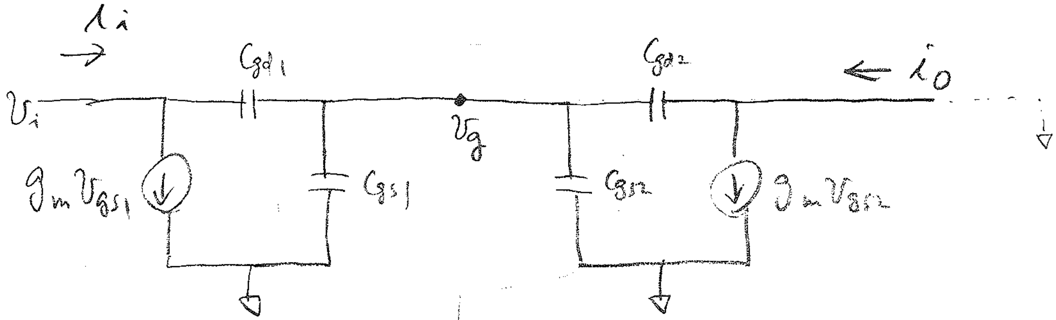
$$\begin{cases} \omega_{p1} \cong \dfrac{1}{R_s\left[(1 + g_{m1} R_D) C_{gd1} + C_{gs1}\right]} \\ \omega_{p2} \cong \dfrac{1}{R_D\left[(1 + g_{m2} R_D) C_{gd2} + C_{gs2}\right]} \\ \omega_{p3} \cong \dfrac{1}{(R_D \parallel R_L)(C_{gd2} + C_{db2})} \end{cases}$$

dominant !

; can be ignored

$$H(s) = \frac{A_v}{\left(1 + \frac{s}{\omega_{p1}}\right)\left(1 + \frac{s}{\omega_{p2}}\right)\left(1 + \frac{s}{\omega_{p3}}\right)}$$

5.



"$C_{gs1} + C_{gs2}$" $= 2C_{gs}$

$$\begin{pmatrix} C_{gs1} = C_{gs2} = C_{gs} \\ C_{gd1} = C_{gd2} = C_{gd} \\ g_{m1} = g_{m2} = g_m \end{pmatrix}$$

$$\left[\begin{aligned} & i_i = g_m v_g + \frac{v_i - v_g}{(1/sC_{gd1})} \\ & \frac{v_i - v_g}{(1/sC_{gd1})} = \frac{v_g}{1/s(C_{gs1} + C_{gs2} + C_{gd2})} \\ & \frac{v_g}{1/sC_{gd2}} + i_o = g_m v_g \end{aligned}\right.$$

$$\Rightarrow \begin{cases} (sC_{gd} - g_m) v_g = -i_o \\ \boxed{v_g = \frac{i_o}{g_m - sC_{gd}}} \end{cases}$$

$$\begin{aligned} \Rightarrow sC_{gd} v_i &= (sC_{gd} + sC_{gs} + sC_{gs} + sC_{gd}) v_g \\ &= 2s(C_{gd} + C_{gs}) \frac{i_o}{g_m - sC_{gd}} \end{aligned} \Rightarrow \boxed{v_i = \frac{2(C_{gd} + C_{gs})}{C_{gd}} \cdot \frac{i_o}{g_m - sC_{gd}}}$$

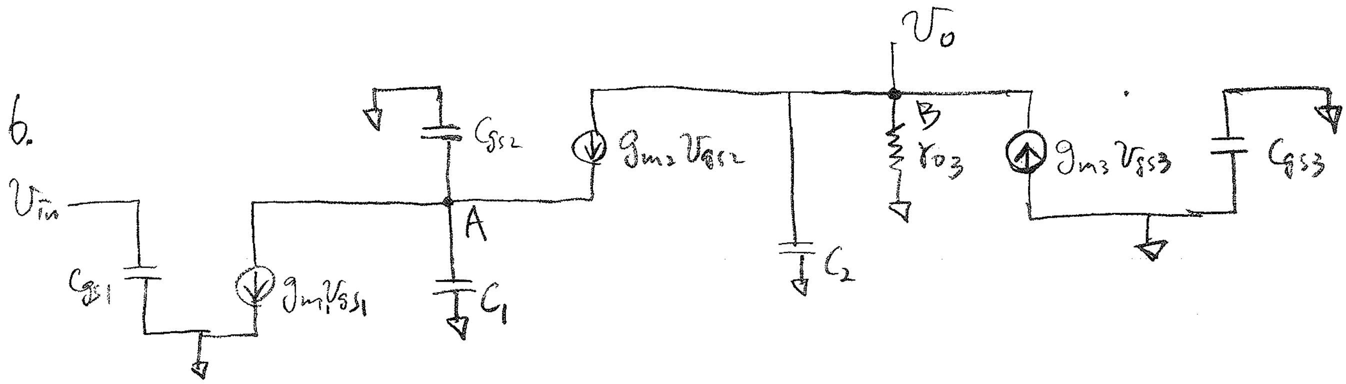
$$\therefore i_i = \cancel{(g_m - sC_{gd})} \frac{i_o}{\cancel{g_m - sC_{gd}}} + s\cancel{C_{gd}} \cdot \frac{2(C_{gd} + C_{gs})}{\cancel{C_{gd}}} \frac{i_o}{g_m - sC_{gd}}$$

$$= \left[1 + \frac{2s(C_{gd} + C_{gs})}{g_m - sC_{gd}}\right] i_o = \frac{g_m + 2s(C_{gd} + C_{gs}) - sC_{gd}}{g_m - sC_{gd}} i_o$$

$$\therefore H(s) = \frac{i_o(s)}{i_i(s)} = \frac{g_m - sC_{gd}}{g_m + 2s(C_{gs} + C_{gd}) - sC_{gd}}$$

$$= \frac{g_m - sC_{gd}}{g_m + sC_{gd} + 2sC_{gs}} = \boxed{\frac{g_m - sC_{gd}}{g_m + s(C_{gd} + 2C_{gs})}}$$

6.



$$A_v = -g_{m1} \cdot r_{o3}$$

\# Poles

$$\omega_{pA} = \frac{1}{\left(\frac{1}{g_{m2}}\right)(C_1 + C_{gs2})}$$

$$\omega_{pB} = \frac{1}{r_{o3} \cdot C_2}$$

$$H(s) = \frac{A_v}{\left(1 + \frac{s}{\omega_{p1}}\right)\left(1 + \frac{s}{\omega_{p2}}\right)}$$

7.

(a) $$Z_{out} = \frac{1}{2}(r_{o2} \cdot g_{m2} \cdot r_{o1})$$

$$\therefore\ A_v = -g_{m1} \cdot \frac{1}{2} \cdot r_{o1} \cdot r_{o2} \cdot g_{m2} = \boxed{-\frac{1}{2} g_{m1} \cdot g_{m2} \cdot r_{o1} \cdot r_{o2}}$$

(b) $$\omega_p = \frac{1}{\frac{1}{2} \cdot (r_{o1} \cdot r_{o2} \cdot g_{m2})(C_L + C_{gd2} + C_{db2})}$$

$$\approx \boxed{\frac{1}{\frac{1}{2}(r_{o1} \cdot r_{o2} \cdot g_{m2})\, C_L}}$$

8.

(a) R_{in} is extremely large

(b)

$$\omega_{p1} = \frac{1}{\left(\frac{1}{g_{m1}}\right)\left[(1+g_{m2} r_{o2}) C_{gd2} + C_{gs2}\right]}$$

2nd most significant

$$\omega_{p2} = \frac{1}{r_{o2} \cdot C_L}$$

dominant