

HW #2 Solution

1.

(a) Midband gain: $V_o = G_m V_i R_L = G_m \cdot \frac{R_i}{R_s + R_i} V_{in} R_L$

$\therefore A_M = \frac{V_o}{V_{in}} = \frac{R_i}{R_s + R_i} G_m \cdot R_L$

(b)

$$\left\{ \begin{aligned} \omega_{PH} &= \frac{1}{C_L R_L} \\ \omega_{PL} &= \frac{1}{C_c (R_s + R_i)} \end{aligned} \right.$$

$$H(s) = \frac{A_M}{\left(1 + \frac{s}{\omega_{PL}}\right) \left(1 + \frac{s}{\omega_{PH}}\right)}$$

(c)

$$\frac{100k}{20k + 100k} \cdot G_m \cdot 10k = 10$$

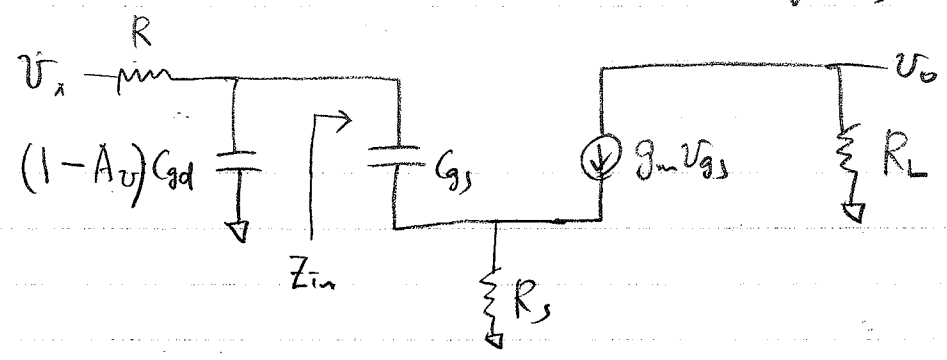
$$G_m = \frac{10}{10k} \cdot \frac{120}{100} = 1.2 \text{ [mA/V]}$$

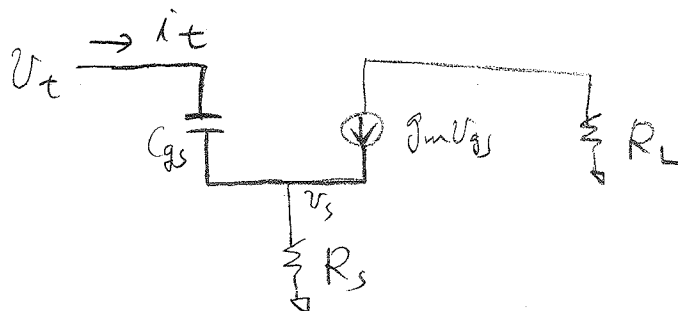
2.

(a) $V_o = -g_m V_{gs} R_L = -g_m \cdot \frac{1/g_m}{1/g_m + R_s} \cdot V_i R_L$

\therefore Low freq gain $A_v = - \frac{R_L}{1/g_m + R_s}$

(b)





$$i_t = \frac{V_t - V_s}{(1/sC_{gs})}$$

$$i_t + g_m(V_t - V_s) = \frac{V_s}{R_s}$$

$$V_s = V_t - \left(\frac{1}{sC_{gs}}\right) i_t$$

$$i_t + g_m V_t = \left(g_m + \frac{1}{R_s}\right) \left(V_t - \left(\frac{1}{sC_{gs}}\right) i_t\right)$$

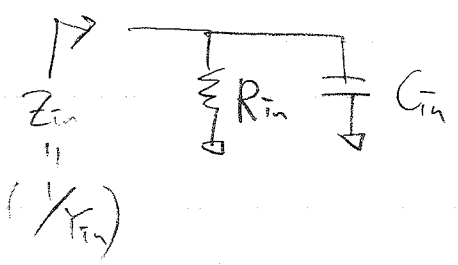
$$\left[1 + \left(g_m + \frac{1}{R_s}\right) \left(\frac{1}{sC_{gs}}\right)\right] i_t = \left[g_m + \left(g_m + \frac{1}{R_s}\right)\right] V_t$$

∴ $Z_{in} = \frac{V_t}{i_t} = R_s \left[1 + \left(g_m + \frac{1}{R_s}\right) \left(\frac{1}{sC_{gs}}\right)\right] = R_s + (1 + g_m R_s) \left(\frac{1}{sC_{gs}}\right)$

$$Y_{in} = \frac{1}{R_s + \frac{1 + g_m R_s}{sC_{gs}}} = \frac{1}{\frac{sC_{gs} R_s + 1 + g_m R_s}{sC_{gs}}} = \frac{sC_{gs}}{1 + g_m R_s + sC_{gs} R_s}$$

$$= \frac{sC_{gs} (1 + g_m R_s - j\omega C_{gs} R_s)}{(1 + g_m R_s)^2 + \omega^2 C_{gs}^2 R_s^2}$$

$$= \frac{\omega^2 C_{gs}^2 R_s + j\omega C_{gs} (1 + g_m R_s)}{(1 + g_m R_s)^2 + \omega^2 C_{gs}^2 R_s^2}$$



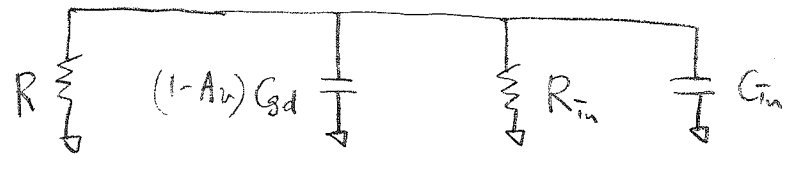
$$\therefore R_{in} = \frac{(1 + g_m R_s)^2 + \omega^2 C_{gs}^2 R_s^2}{\omega^2 C_{gs}^2 R_s}$$

$$= R_s + \frac{(1 + g_m R_s)^2}{\omega^2 C_{gs}^2 R_s}$$

$$\therefore \omega C_{in} = \frac{\omega C_{gs} (1 + g_m R_s)}{(1 + g_m R_s)^2 + \omega^2 C_{gs}^2 R_s^2}$$

$$\therefore C_{in} = \frac{C_{gs} (1 + g_m R_s)}{(1 + g_m R_s)^2 + \omega^2 C_{gs}^2 R_s^2}$$

Input pole



$$\therefore \omega_{p, in} = \frac{1}{(R \parallel R_{in}) [(1-A_v)G_{sd} + C_{in}]} \quad \Leftarrow \text{Main pole}$$

Output pole

$$\omega_{p, out} \approx \frac{1}{R_L C_{gd}}$$

$$\therefore H(s) = \frac{A_v}{\left(1 + \frac{s}{\omega_{p, in}}\right) \left(1 + \frac{s}{\omega_{p, out}}\right)}$$

(c)

R_s	A_v	$\omega_{p, in}$	$GBW = A_v \times \frac{\omega_{p, in}}{2\pi}$
0			
100			
250			

3. Midband gain $A_v = -g_m R_D$

$$\left[\begin{array}{l} \omega_p = \frac{1}{C_s (R_s \parallel \frac{1}{g_m})} \\ \omega_z \approx \frac{1}{C_s \cdot R_s} \end{array} \right. , \quad \omega_{p2} \approx \frac{1}{(C_{gd} + C_{cb}) R_D}$$

↓ optional

$$\infty H(s) \cong A_v \frac{(1 + \frac{s}{\omega_z})}{(1 + \frac{s}{\omega_p}) (1 + \frac{s}{\omega_{p2}})}$$

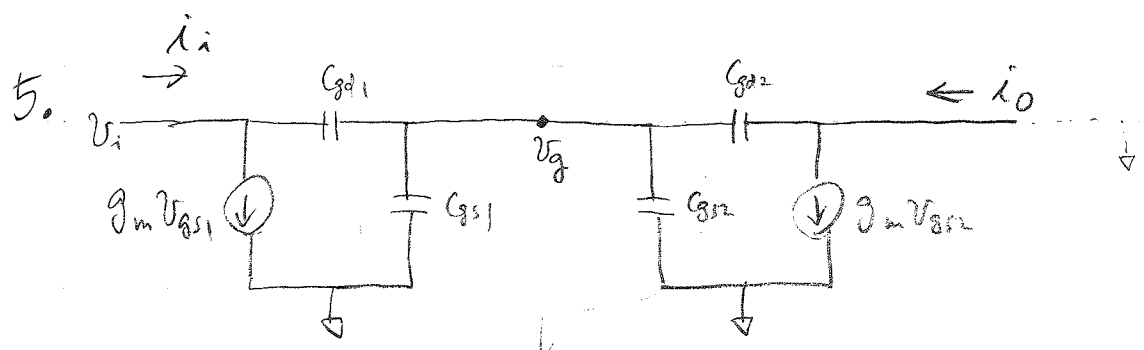
4. Midband gain $A_v = (-g_{m1} R_D) (-g_{m2} (R_D \parallel R_L))$
 $= g_{m1} g_{m2} R_D \cdot (R_D \parallel R_L)$

Poles

$$\left\{ \begin{array}{l} \omega_{p1} \cong \frac{1}{R_s [(1 + g_{m1} R_D) C_{gd1} + C_{gs1}]} \\ \omega_{p2} \cong \frac{1}{R_D [(1 + g_{m2} R_D) (C_{gd2} + C_{gs2})]} \end{array} \right\} \text{ dominant!}$$

$$\omega_{p3} \cong \frac{1}{(R_D \parallel R_L) (C_{gd2} + C_{cb2})} \quad ; \text{ can be ignored}$$

$$H(s) = \frac{A_v}{(1 + \frac{s}{\omega_{p1}}) (1 + \frac{s}{\omega_{p2}}) (1 + \frac{s}{\omega_{p3}})}$$



" $C_{gs1} + C_{gs2} = 2C_{gs}$ "

$$\begin{cases} C_{gs1} = C_{gs2} = C_{gs} \\ C_{gd1} = C_{gd2} = C_{gd} \\ g_{m1} = g_{m2} = g_m \end{cases}$$

$$i_i = g_m V_g + \frac{V_i - V_g}{1/sC_{gd1}}$$

$$\frac{V_i - V_g}{1/sC_{gd1}} = \frac{V_g}{1/s(C_{gs1} + C_{gs2} + C_{gd2})}$$

$$\frac{V_g}{1/sC_{gd2}} + i_o = g_m V_g \Rightarrow \begin{cases} (sC_{gd} - g_m) V_g = -i_o \\ V_g = \frac{i_o}{g_m - sC_{gd}} \end{cases}$$

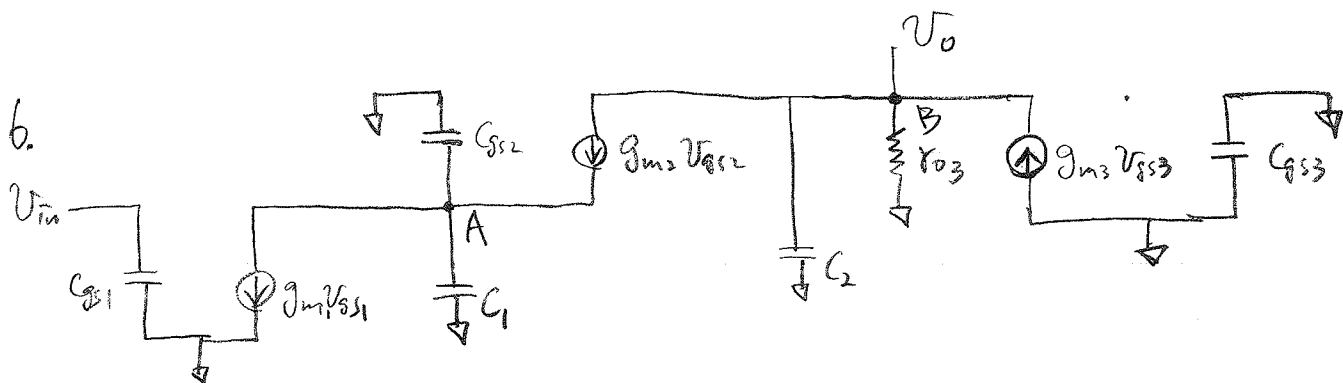
$$\begin{aligned} sC_{gd} V_i &= (sC_{gd} + sC_{gs} + sC_{gs} + sC_{gd}) V_g \\ &= 2s(C_{gd} + C_{gs}) \frac{i_o}{g_m - sC_{gd}} \end{aligned} \Rightarrow V_i = \frac{2(C_{gd} + C_{gs})}{C_{gd}} \cdot \frac{i_o}{g_m - sC_{gd}}$$

$$\begin{aligned} \therefore i_i &= \left(g_m - sC_{gd} \right) \frac{i_o}{g_m - sC_{gd}} + sC_{gd} \cdot \frac{2(C_{gd} + C_{gs})}{C_{gd}} \cdot \frac{i_o}{g_m - sC_{gd}} \\ &= \left[1 + \frac{2s(C_{gd} + C_{gs})}{g_m - sC_{gd}} \right] i_o = \frac{g_m + 2s(C_{gd} + C_{gs}) - sC_{gd}}{g_m - sC_{gd}} i_o \end{aligned}$$

$$\therefore H(s) = \frac{i_o(s)}{i_i(s)} = \frac{g_m - sC_{gd}}{g_m + 2s(C_{gs} + C_{gd}) - sC_{gd}}$$

$$= \frac{g_m - sC_{gd}}{g_m + sC_{gd} + 2sC_{gs}} = \frac{g_m - sC_{gd}}{g_m + s(C_{gd} + 2C_{gs})}$$

(6)



$$A_v = -g_{m1} \cdot r_{o3}$$

Poles

$$\left(\begin{aligned} \omega_{PA} &= \frac{1}{\left(\frac{1}{g_{m2}}\right) (C_1 + C_{gs2})} \\ \omega_{PB} &= \frac{1}{r_{o3} \cdot C_2} \end{aligned} \right.$$

$$H(s) = \frac{A_v}{\left(1 + \frac{s}{\omega_{p1}}\right) \left(1 + \frac{s}{\omega_{p2}}\right)}$$

7.

(a) $Z_{out} = \frac{1}{2} (r_{o2} \cdot g_{m2} \cdot r_{o1})$

$\therefore A_v = -g_{m1} \cdot \frac{1}{2} \cdot r_{o1} \cdot r_{o2} \cdot g_{m2} = \boxed{-\frac{1}{2} g_{m1} \cdot g_{m2} \cdot r_{o1} \cdot r_{o2}}$

(b)

$$\omega_p = \frac{1}{\frac{1}{2} (r_{o1} \cdot r_{o2} \cdot g_{m2}) (C_L + C_{gd2} + C_{db2})}$$

$$\approx \boxed{\frac{1}{\frac{1}{2} (r_{o1} \cdot r_{o2} \cdot g_{m2}) C_L}}$$

8.

(a) R_{in} is extremely large

(b)

$$\omega_{p1} = \frac{1}{\left(\frac{1}{g_{m1}}\right) \left[(1 + g_{m2} R_{o2}) C_{gd2} + C_{gs2} \right]}$$
$$\omega_{p2} = \frac{1}{R_{o2} \cdot C_L}$$

← 2nd most significant

← dominant