

Lesson 27, §15.8 Triple Integrals in Cylindrical and Spherical Coordinates

1 Cylindrical Coordinates

Formula for Triple Integration in Cylindrical Coordinates

$$\iiint_E f(x, y, z) dV = \iint_D \left[\int_{u_1(x, y)}^{u_2(x, y)} f(x, y, z) dz \right] dA$$

$$\iiint_E f(x, y, z) dV = \int_{\alpha}^{\beta} \int_{h_1(\theta)}^{h_2(\theta)} \int_{u_1(r \cos \theta, r \sin \theta)}^{u_2(r \cos \theta, r \sin \theta)} f(r \cos \theta, r \sin \theta, z) r dz dr d\theta$$

$$x \Rightarrow r \cos \theta \quad (1)$$

$$y \Rightarrow r \sin \theta \quad (2)$$

$$z \Rightarrow z \quad (3)$$

$$dV \Rightarrow r dz dr d\theta \quad (4)$$

2 Spherical Coordinates

Formula for Triple Integration in Spherical Coordinates

$$\iiint_E f(x, y, z) dV = \int_c^d \int_{\alpha}^{\beta} \int_a^b f(\rho \sin \phi \cos \theta, \rho \sin \phi \sin \theta, \rho \cos \phi) \rho^2 \sin \phi d\rho d\theta d\phi$$

where E is a spherical wedge given by

$$E = \{(\rho, \theta, \phi) | a \leq \rho \leq b, \alpha \leq \theta \leq \beta, c \leq \phi \leq d\}$$

$$x \Rightarrow \rho \sin \phi \cos \theta \quad (5)$$

$$y \Rightarrow \rho \sin \phi \sin \theta \quad (6)$$

$$z \Rightarrow \rho \cos \phi \quad (7)$$

$$dV \Rightarrow \rho^2 \sin \phi d\rho d\theta d\phi \quad (8)$$