

Lesson 35, §16.6 Parametric Surfaces and Their Areas

Motivation

$$\vec{r}(t) = x(t)\vec{i} + y(t)\vec{j} + z(t)\vec{k} \implies \vec{r}(u, v) = x(u, v)\vec{i} + y(u, v)\vec{j} + z(u, v)\vec{k}$$

Glossary to Know

parametric surface

grid curves

Note: In general, a surface given as the graph of a function of x and y , that is, with an equation of the form $z = f(x, y)$, can always be regarded as a parametric surface by taking x and y as parameters and writing the parametric equations as

$$x = x \quad y = y \quad z = f(x, y)$$

Parametric representations (also called parametrizations) of surfaces are not unique.

Tangent Planes Start with $\vec{r}(u, v) = x(u, v)\vec{i} + y(u, v)\vec{j} + z(u, v)\vec{k}$ and find the normal vector \vec{n} using \vec{r}_u and \vec{r}_v (Cross Product.)

Definition If a smooth parametric surface S is given by the equation

$$\vec{r}(u, v) = x(u, v)\vec{i} + y(u, v)\vec{j} + z(u, v)\vec{k} \quad (u, v) \in D$$

and S is covered just once as (u, v) ranges throughout the parameter domain D , then the **surface area** of S is

$$A(S) = \iint_D |\vec{r}_u \times \vec{r}_v| dA$$

$$\text{where } \vec{r}_u = \frac{\partial x}{\partial u}\vec{i} + \frac{\partial y}{\partial u}\vec{j} + \frac{\partial z}{\partial u}\vec{k} \quad \vec{r}_v = \frac{\partial x}{\partial v}\vec{i} + \frac{\partial y}{\partial v}\vec{j} + \frac{\partial z}{\partial v}\vec{k}$$

For $z = f(x, y)$ this definition becomes

$$A(S) = \iint_D \sqrt{1 + \left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2} dA$$

Note: See 2) of page 1020.