

Quiz 9. Nov. 7th

1. Evaluate the integral

$$\iiint_E z \, dV$$

where E is the solid bounded above by the sphere $x^2 + y^2 + z^2 = 9$, below by the plane $z = 0$, and on the sides by the planes $x = 0, x = 1, y = 0$, and $y = 1$.

2. Use a triple integral in cylindrical coordinates to find the volume of the solid bounded above by the paraboloid $z = 2 - x^2 - y^2$ and below by the plane $z = 1$.

3. Consider the integral $\iiint_E x \, dV$, where E is the solid in the first octant, bounded above by the sphere $x^2 + y^2 + z^2 = 4$, below by the cone $z = \frac{1}{\sqrt{3}}\sqrt{x^2 + y^2}$, and on the sides by the planes $y = 0$ and $y = x$. Express the integral as an iterated integral in **spherical** coordinates. Do **not** evaluate the integral. (Hint: First express all surfaces in spherical coordinates).