

Lesson 30 31, §16.2 Line Integrals

2. Definition If f is defined on a smooth curve C given by Equations 1, then the **line integral of f along C** is

$$\int_C f(x, y) ds = \lim_{n \rightarrow \infty} \sum_{i=1}^n f(x_i^*, y_i^*) \Delta s_i$$

if this limit exists.

$$\int_C f(x, y) ds = \int_a^b f(x(t), y(t)) \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt$$

Note:

$$\begin{aligned} \frac{ds}{dt} &= \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} \\ \Rightarrow ds &= \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt \end{aligned}$$

Equation of Line Segment A vector representation of the line segment that starts at \vec{r}_0 and ends at \vec{r}_1 is given by

$$\vec{r}(t) = (1-t)\vec{r}_0 + t\vec{r}_1 \quad 0 \leq t \leq 1$$

where this line starts at \vec{r}_0 and ends at \vec{r}_1 , (Hint: put $t = 0$ and $t = 1$.)

Line Integral along Vector Field Let \mathbb{F} be a continuous vector field defined on a smooth curve C given by a vector function $\vec{r}(t)$, $a \leq t \leq b$. Then the **line integral of \mathbb{F} along C** is

$$\int_C \mathbf{F} \cdot d\mathbf{r} = \int_a^b \mathbf{F}(\mathbf{r}(t)) \cdot \mathbf{r}'(t) dt = \int_C \mathbf{F} \cdot \mathbf{T} ds$$