# Lesson 36-37, §16.7 Surface Integrals

**Surface Integral of $f$ over the Surface $S$**

$$\iint_S f(x,y,z)\,dS = \lim_{m,n\to\infty}\sum_{i=1}^{m}\sum_{j=1}^{n} f(P_{ij}^*)\Delta S_i j \quad (1)$$

$$= \lim_{m,n\to\infty}\sum_{i=1}^{m}\sum_{j=1}^{n} f(x_i^*, y_j^*, g(x_i^*, y_j^*))\sqrt{[g_x(x_i,y_j)]^2+[g_y(x_i,y_j)]^2+1}\,\Delta A \quad (2)$$

$$= \iint_D f(x,y,g(x,y))\sqrt{[g_x(x_i,y_j)]^2+[g_y(x_i,y_j)]^2+1}\;\Delta A \quad (3)$$

$$= \iint_D f(x,y,g(x,y))\sqrt{\left(\frac{\partial z}{\partial x}\right)^2+\left(\frac{\partial z}{\partial y}\right)^2+1}\;dA \quad (4)$$

provided that $f$ is continuous on $S$ and $g$ has continuous derivatives.

**Parametric Surfaces** If a surface $S$ has a vector equation $\vec{r}(u,v) = x(u,v)\vec{i} + y(u,v)\vec{j} + z(u,v)\vec{k} \quad (u,v)\in D$

$$\iint_S f(x,y,z)\,dS = \iint_D f(\vec{r}(u,v))|\vec{r}_u \times \vec{r}_v|\,dA$$

**Note:** $\iint_S 1\,dS = \iint_D |\vec{r}_u \times \vec{r}_v|\,dA = A(S)$,
where $|\vec{r}_x \times \vec{r}_y| = \sqrt{1+\left(\frac{\partial z}{\partial x}\right)^2+\left(\frac{\partial z}{\partial y}\right)^2}$