ME 270 – Spring 2011
Examination No. 3
Please review the following statement:
I certify that I have not given unauthorized aid nor have I received aid in the completion of this exam.

Signature: ________________________________

INSTRUCTIONS
Begin each problem in the space provided on the examination sheets. If additional space is required, use the yellow paper provided to you.

Work on one side of each sheet only, with only one problem on a sheet.

Each problem is worth 20 points.

Please remember that for you to obtain maximum credit for a problem, it must be clearly presented, i.e.

- The coordinate system must be clearly identified.
- Where appropriate, free body diagrams must be drawn. These should be drawn separately from the given figures.
- Units must be clearly stated as part of the answer.
- You must carefully delineate vector and scalar quantities.

If the solution does not follow a logical thought process, it will be assumed in error.

When handing in the test, please make sure that all sheets are in the correct sequential order and make sure that your name is at the top of every page that you wish to have graded.

Instructor's Name and Section:

Section 1: J. Jones 9:30 – 10:20 a.m.  
Section 2: V. Kumar 2:30 – 3:20 p.m.

Problem 1 _________

Problem 2 _________

Problem 3 _________

Total ____________
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PROBLEM 1 (20 points) – Prob. 1 questions are all or nothing.

PROBLEM 1A. (5 points)

FIND: The particle P moves in the circular path shown. On the artwork provided, sketch the acceleration vector \( \vec{a} \) (both cases – 2 pts.) and determine its magnitude (\(|\vec{a}|\)) for the following cases:

- i) the speed \( v \) is 1.2 m/s and is constant
- ii) the speed \( v \) is 1.2 m/s and decreasing at a rate of 4.8 m/s². In each case the particle is in the position shown in the figure.

\[
\begin{align*}
&i) \ |\vec{a}| = 2.4 \text{ m/s}^2 & (1 \text{ point}) \\
&ii) \ |\vec{a}| = 5.37 \text{ m/s}^2 & (2 \text{ points})
\end{align*}
\]

PROBLEM 1B. (5 points)

FIND: The cannonball A is launched from the ground at a 50 degree angle with initial velocity 100 m/s. The jet plane B is flying in a circular arc (\( r = 500 \) m) with a constant velocity 100 m/s. The plane reaches the bottom of its path at the same instant that the ball reaches the top of its path.

- i) Determine the relative velocity of the ball as observed by the pilot in plane B (i.e., \( \vec{v}_{A/B} \)) in cartesian coordinates.
- ii) Determine the relative acceleration of the ball as observed by the pilot in plane B (i.e., \( \vec{a}_{A/B} \)) in cartesian coordinates.

\[
\begin{align*}
&i) \ \vec{v}_{A/B} = -16.4 \hat{i} \text{ m/s} & (2 \text{ points}) \\
&ii) \ \vec{a}_{A/B} = -29.8 \hat{j} \text{ m/s}^2 & (3 \text{ points})
\end{align*}
\]
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PROBLEM 1C. (5 points)

FIND: A man pulls himself up the 15° incline by the method shown. The combined weight of the man and the cart is 250 lbs. Assume all of the rope, pulley and wheels are frictionless.

i) If the man pulls the rope in at a rate of 6 ft/s, what will be the velocity of the cart?
ii) Determine the magnitude of the acceleration of the cart if the man exerts a force of 60 lbs on the rope.

\[ i) \ v = \frac{-2 \text{ m/s}}{} \quad \text{or} \quad \frac{2 \text{ m/s \ (up \ incline)}}{} \]
\[ ii) \ a = \frac{14.9 \text{ m/s}^2}{\text{}} \]

PROBLEM 1D. (5 points)

FIND: The spring constant \( k = 200 \text{ N/m} \) is attached to both the support and the 2 kg cylindrical collar. A constant 10-N force begins to be applied to the collar at time \( t = 0 \) when the spring is undeformed and at rest.

i) Determine an expression for the velocity (\( v \)) as a function of position (\( x \)).
ii) With the 10 N applied force, what is the maximum displacement of the collar?

\[ i) \ v = \left[\left(10 \frac{\text{N}}{m} - \frac{\text{N} \cdot \text{m}}{\text{m}^2} x^2\right) / m\right]^{\frac{1}{2}} \quad \text{or} \quad \sqrt{\left(5x - 50x^2\right)}^{\frac{1}{2}} \]
\[ ii) \ x_{\text{Max}} = 0.1 \text{ m} \]
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PROBLEM 2. (20 points)

GIVEN: At the bottom of a loop in the vertical \((r - \theta)\) plane at an altitude of 400 m, the airplane \(P\) has a constant horizontal velocity of \(200 \text{ m/s}\). The radius of curvature of the circular loop is 1500 m.

FIND:

a) Determine the velocity and acceleration in path variables \((\bar{u}_t, \bar{u}_n)\). (6 points)
b) On the artwork provided, sketch the path variable unit vectors \((\bar{u}_t, \bar{u}_n)\) and the polar coordinate unit vectors \((\bar{u}_r, \bar{u}_\theta)\). (2 points)
c) Write an expression for \(\bar{u}_t\) and an expression for \(\bar{u}_n\) in terms of \(\bar{u}_r, \bar{u}_\theta\). (4 points)
d) Determine the value of \(\dot{\theta}\) at the instant shown. (4 points)
e) Determine the value of \(\ddot{r}\) at the instant shown. (4 points)

\[
\bar{v}_p = 200 \bar{u}_t \text{ m/s}
\]

\[
\bar{a}_p = 26.7 \bar{u}_n \text{ m/s}^2
\]

\[
\bar{u}_t = 0.928 \bar{u}_r - 0.371 \bar{u}_\theta
\]

\[
\bar{u}_n = 0.371 \bar{u}_r + 0.928 \bar{u}_\theta
\]

\[
\dot{\theta} = -0.0689 \text{ rad/s}
\]

\[
\ddot{r} = 15.0 \text{ m/s}^2
\]
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PROBLEM 3. (20 points)

GIVEN: The horizontal force $F$ moves the 3 kg package up a frictionless hill whose profile is given by $y = x^2/4$, where $x$ and $y$ are in meters. If it is known that the horizontal component of the velocity of the package is constant at 5 m/s.

FIND:

a) Determine the velocity of the package in cartesian coordinates when $x = 1$ m. (5 points)

b) Determine the acceleration of the package in cartesian coordinates when $x = 1$ m. (5 points)

c) Draw a free body diagram of the box when $x = 1$ m. (4 points)

d) Write the equations of motion of the box in cartesian coordinates and determine the normal force ($N$) from the hill and the applied force ($F$) when $x = 1$ m. (6 points)

\[
\begin{align*}
\overline{V}_p &= 5\hat{i} + 2.5\hat{j} \text{ m/s} \\
\overline{a}_p &= 12.5\hat{j} \text{ m/s}^2 \\
N &= 74.9 \text{ N} \\
F &= 33.5 \text{ N}
\end{align*}
\]