Homework Assignment

Problem 1

Consider a potential

$$V(x) = \begin{cases} V_0 & \text{if} \quad |x| < \frac{a}{2} \\ 0 & \text{if} \quad |x| > \frac{a}{2} \end{cases} \tag{0.1}$$

with $V_0 > 0$. Compute the scattering matrix S and the reflection and transmission coefficients. Consider both cases, $0 < E < V_0$ and $V_0 < E$. Check the unitarity and symmetries of S.

Problem 2

Consider a particle in the one dimensional potential $V(x) = \lambda x^4$ such that the Hamiltonian is

$$H = \frac{p^2}{2m} + \lambda x^4 \tag{0.2}$$

- Write the corresponding Schrödinger equation for the (possible) wave function of energy E. Rescale the variable x by a constant a, namely define $z = ax$ and find a to eliminate λ from the equation (after appropriately rescaling E).

- Solve the resulting equation numerically for different values of E. [Use the boundary condition $\psi(0) = 1$, $\psi'(0) = 0$]. By considering the behavior of ψ at infinity determine the lowest eigenvalue of the energy. If you want compute other eigenvalues.

- Consider the wave function

$$\psi = Ae^{-\alpha x^2} \tag{0.3}$$

  Choose A such the $\psi(x)$ is normalized and then compute $E(\alpha) = \langle\psi|H|\psi\rangle$. Minimize $E(\alpha)$ with respect to α and compare the minimum value of $E(\alpha)$ with the result of the previous point to see how good the approximation is. Finally try to improve the approximation by considering a trial function $\psi = (A + Bx^2)e^{-\alpha x^2}$ and compare again with the numerical result.

Note: This problem requires the use of a compute algebra program such as Mathematica, Maple, Matlab etc. (or the coding the integration in some computer language such as C, fortran etc.). It is more difficult and requires some extra research but playing with the numerics is always useful to understand what one is doing analytically.

Problem 3

Using the same numerical method of the previous problem find the ground state energy for the potential

$$H = \frac{p^2}{2m} + \lambda x^N \tag{0.4}$$

for several values of N. After appropriately rescaling (as in the previous problem), extrapolate the result to $N \to \infty$. What should be the result?