Flavor Physics from Warped Extra Dimensions

Talk at Purdue University

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Hierarchy Problem

Why Gravity is so weak?

Weak Scale \( (M_W) = 1000 \text{ GeV} \)

Planck Scale \( (M_{pl}) = 10^{19} \text{ GeV} \)
Hierarchy Problem

- Why Gravity is so weak?
  - Weak Scale \((M_W) = 1000\ \text{GeV}\)
  - Planck Scale \((M_{pl}) = 10^{19}\ \text{GeV}\)

- Weak scale: Standard Model
- Above Planck Scale: Quantum Gravity
- Between two scales: GUT, Supersymmetry, Superstring etc.
Hierarchical Problem

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- Weak scale: Standard Model
- Above Planck Scale: Quantum Gravity
- Between two scales: GUT, Supersymmetry, Superstring etc.

There is no experimental clue on the intermediate scale.
That’s not what they wanted to prove.
The Problem of Higgs Mass

In Standard Model (SM),

Higgs mass correction is quadratic divergent to cutoff scale.

\[ M_{Higgs}^2 \rightarrow M_{Higgs}^2 + c\Lambda_{cut}^2 \]
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In Standard Model (SM),
Higgs mass correction is quadratic divergent to cutoff scale.

\[ M_{Higgs}^2 \rightarrow M_{Higgs}^2 + c\Lambda_{cut}^2 \]

We need an upper bound of \( \Lambda_{cut} \) to be around 1 TeV.

In SM, \( M_{pl} \) is a natural cut-off.
Supersymmetry (SUSY)

To solve Hierarchy problem, SUSY was introduced.
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To solve Hierarchy problem, SUSY was introduced. SUSY is a symmetry between boson and fermion. Each particle in SM has a superpartner with the same quantum number but different spin. Each loop-correction by scalar loop is cancelled by higgsino loop.

(soft) SUSY breaking generates natural cut-off at $\sim$TeV.
Not exactly replacing SUSY, but a simple idea was brought by Arkani-Hamed, Dimopoulos and Dvali PLB 429 (1998).
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ADD suggest that the hierarchy problem can be solved by existence of extra spatial dimensions.
The FLATLAND
Flat Extra dimensions

If our Universe has a thickness $R$, we will find extra dimensions in a small scale $r < R$. 
Flat Extra dimensions

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Let the number of extra dimensions $n$.

Then the gravity become stronger in small range.
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Then the gravity become stronger in small range.

$$G \frac{1}{r}, \quad (r > R)$$

$$G (4+n) \frac{1}{r^{n+1}}, \quad (r < R)$$
The 4D gravity can be weakened by extra-dimensions.
A short tour into the Extra dimensions
How to compactify extra-dimensions is not yet understood.
How to compactify extra-dimensions is not yet understood.

It is hard to comprehend due to its “large” size.
Our (3 + 1)D universe

We are here
Flat extra dimension $S^1$ (Arkani-Hamed et al.)
RS1 Warped extra dimension $S^1/\mathbb{Z}_2$

Gravity is localized

We are here
All SM fields should be confined on the 3+1D brane except gravity.
This can be related with the string theory with brane solutions
As you know, string need to be compactified.
The object of this model is to solve the hierarchy problem.

To explicit, to keep the Higgs mass 1-loop contribution TeV which should be the QG scale.
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To explicit, to keep the Higgs mass 1-loop contribution TeV which should be the QG scale.

In short, \( \text{QG scale} = \text{TeV} \)

Existence of massive Kaluza-Klein (KK) graviton is cosmologically dangerous.
Randall-Sundrum model (RS1)
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Randall and Sundrum PRL (1999)
Randall-Sundrum model (RS1)

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RS1 is model with one warped extra-dimension.

\[ ds^2 = e^{-2\sigma(y)}(dt^2 - dx^2) - dy^2, \]

\[ \sigma(y) = k|y| \] is the warp factor

\[ k \sim M_{pl} \] is the curvature of warped space.
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\[ k \sim M_{pl} \] is the curvature of warped space.

The 5th direction \( y \) is bounded and the bulk is AdS\(_5\).

All SM fields are confined on TeV brane (IR boundary)

Gravity resides in the bulk.
Where do we live in RS1?
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UV(IR) brane is actually a UV(IR) cut-off (boundary).

There is no reason for SM to be confined.
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There is no reason for SM to be confined.

If the energy level is close to TeV,

SM fields behave more tamed in the bulk.
Where do we live in RS1?

UV(IR) brane is actually a UV(IR) cut-off (boundary).

There is no reason for SM to be confined.

If the energy level is close to TeV,

SM fields behave more tamed in the bulk.

But, how can we develop bulk SM?
The conformal coordinate of $z \equiv e^\sigma/k$ is useful.

$$ds_5^2 = \frac{1}{(kz)^2} \left( dt^2 - d\vec{x}^2 - dz^2 \right).$$
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$$ds_5^2 = \frac{1}{(kz)^2} \left( dt^2 - d\vec{x}^2 - dz^2 \right).$$

$y$ is confined in $0 \leq y \leq L$

$z$ is also bounded in $1/k \leq z \leq 1/T$.

$$T \equiv e^{-kL}k \equiv \epsilon k.$$

With $kL \approx 35$, the warp factor $\epsilon \equiv e^{-kL}$ reduces $T$ at TeV scale from $k$ at Planck scale:
The conformal coordinate of $z \equiv e^{\sigma}/k$ is useful.

$$ds^2_5 = \frac{1}{(kz)^2} \left( dt^2 - d\vec{x}^2 - dz^2 \right).$$

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With this scaling the gauge hierarchy problem is answered.
Bulk fields in RS1

* Bulk scalar field was introduced at early stage of RS model.

Bulk fields in RS1

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- Then Bulk gauge field was introduced by
  
Bulk fields in RS1

- Bulk scalar field was introduced at early stage of RS model.

- Then Bulk gauge field was introduced by

- And bulk fermion was followed soon after.
  SC, Hisano, Nakano, Okada and Yamaguchi, (1999),
Bulk gauge bosons

The action for a 5D $U(1)$ gauge field

$$S_{\text{gauge}} = \int d^4x dz \sqrt{G} \left[ -\frac{1}{4} g^{MP} g^{NQ} F_{MN} F_{PQ} + \frac{1}{2} M^2 g^{MN} A_M A_N \right],$$

where $F_{MN} = \partial_M A_N - \partial_N A_M$. 

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where $F_{MN} = \partial_M A_N - \partial_N A_M$.

The general mass term $M^2(z)$,

$$M^2(z) = a_{UV}^2 k \delta(z - z_{UV}) + a_{IR}^2 k \delta(z - z_{IR}) + b^2 k^2 ,$$

the dimensionless $b$ and $a_{UV}$ ($a_{IR}$) are bulk mass and localized mass on the UV (IR) brane.
The KK expansion of the dimension 3/2 field $A^M(x, z)$ is

$$A_\nu(x, z) = \sqrt{k} \sum_n A_\nu^{(n)}(x) f_A^{(n)}(z)$$
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With the equation of motion for mode function

$$-z \partial_z \left( \frac{1}{z} \partial_z f_A^{(n)}(z) \right) + \frac{M^2(z)}{k^2 z^2} f_A^{(n)}(z) = m_A^{(n)} f_A^{(n)}(z),$$
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$$A_\nu(x,z) = \sqrt{k} \sum_n A^{(n)}_{\nu}(x) f^{(n)}_A(z)$$

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$$-z \partial_z \left( \frac{1}{z} \partial_z f^{(n)}_A(z) \right) + \frac{M^2(z)}{k^2 z^2} f^{(n)}_A(z) = m^{(n)}_A f^{(n)}_A(z),$$

The mode function can be obtained

$$f^{(n)}_A(z) = \frac{z}{N^{(n)}_A} \left[ J_\nu(m^{(n)}_A z) + \beta^{(n)}_A Y_\nu(m^{(n)}_A z) \right],$$

where $\nu = \sqrt{1 + b^2}$, $b = 0$ if the gauge symmetry is conserved in the bulk.
Fermions in RS1 Bulk

\[
S_{\text{fermion}} = \int d^4x dy \left[ \bar{\psi} e^\sigma i \gamma^\mu \partial_\mu \psi - \frac{1}{2} \bar{\psi} \gamma_5 \partial_y \psi + \frac{1}{2} (\partial_y \bar{\psi}) \gamma_5 \psi + m_D \bar{\psi} \psi \right]
\]
Fermions in RS1 Bulk

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Bulk Lagrangian should respect the \( \mathbb{Z}_2 \) parity

\[ \gamma_5 \Psi(x, -y) = \pm \Psi(x, y) \]
Fermions in RS1 Bulk

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The bulk fermion can be divided into two chiral components,

\[ \hat{\Psi} = \hat{\Psi}_L + \hat{\Psi}_R \]
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which can be expanded to KK modes

\[ \hat{\psi}(x, y)_{L(R)} = \sqrt{k} \sum_n \psi^{(n)}_{L(R)}(x) f^{(n)}_{L(R)}(y). \]
Parity of RS1 Warped extra dimension $S^1/\mathbb{Z}_2$
Parity of RS1 Warped extra dimension $S^1/\mathbb{Z}_2$
The 5-th D parity should be conserved.

Bulk fermion with odd parity cannot have a zero mode.

The zero mode is chiral (thus massless).

Chiral fermion is automatically localized on IR boundary.

All bulk gauge boson have (massless) zero mode.

(Massless) SM fields can be induced from bulk fields (except the Higgs).
Will KK fields survive experimental bounds?
Experimental signals

Energy scale is warped down at IR boundary \((y = y_0)\).

\[ M_{p\ell} e^{-ky_0} \sim \text{TeV} \]
Experimental signals

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\[ M_p e^{-k y_0} \sim \text{TeV} \]

The QG scale is about TeV.

Bulk field generates KK modes with about TeV scale mass gap.
Energy scale is warped down at IR boundary \((y = y_0)\).

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M_p e^{-ky_0} \sim \text{TeV}
\]

The QG scale is about TeV.

Bulk field generates KK modes with about TeV scale mass gap.

Quark KK mode contribution to \(M_W/M_Z\) conflicts with experimental data.
Custodial isospin

In 2003, Agashe et.al. suggested that if there is

$$SU(2)_L \times SU(2)_R \times U(1)_{B-L}$$

gauge symmetry on $S^1/\mathbb{Z}_2 \times \mathbb{Z}'_2$. 
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\[ SU(2)_L \times SU(2)_R \times U(1)_{B-L} \text{ gauge symmetry on } S^1/\mathbb{Z}_2 \times \mathbb{Z}_2'. \]

There exist Ads5/CFT conformal dual global $SU(2)$ custodial isospin.

Which protect $M_W/M_Z$ ratio from KK mode contributions.

Other than custodial isospin,

this model has another important feature in it.
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Two independent parities on each boundary (UV,IR).

\[(±, ±) \text{ and } (±, ∓)\]
Other than custodial isospin,
this model has another important feature in it.

Two independent parities on each boundary (UV,IR).

\[(\pm, \pm) \text{ and } (\pm, \mp)\]

For bulk fermions, this gives symmetry

\[
\gamma_5 \psi(x, -y) = \pm \psi(x, y), \quad \gamma_5 \psi(x, L - y) = \pm \psi(x, L + y)
\]
Parities of Agashe et.al. Warped extra dimension $S^1/\mathbb{Z}_2 \times \mathbb{Z}'_2$
Parities of Agashe et.al. Warped extra dimension $S^1/\mathbb{Z}_2 \times \mathbb{Z}'_2$

Gravity is localized

We are here

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Flavor Physics from Warped Extra Dimensions
Gauge boson KK mode mass on $S^1/\mathbb{Z}_2 \times \mathbb{Z}_2'$

$m_A^{(n)}/T$ 10

$a_{UV} \rightarrow \infty$

$a_{IR} \rightarrow \infty$

$a_{IR,UV} \rightarrow \infty$
Only $(++)$ parity bulk field can have zero mode.

The gauge symmetry in 4D is broken for all other parities.

Thus $W_R$ fields are massive without bulk mass term.
Only $(++)$ parity bulk field can have zero mode.

The gauge symmetry in 4D is broken for all other parities.

Thus $W_R$ fields are massive without bulk mass term.

The $(+-)$ field contains a very light KK mode.

If the boundary mass term turns on and start to increase,

each mode mass transform to the other parity
The boundary mass modifies the KK modes masses

\[ a_{UV} \rightarrow \infty \quad \text{and} \quad a_{IR} \rightarrow \infty \]
### KK mass of bulk gauge boson

<table>
<thead>
<tr>
<th>Mass</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_{A^{++}}^{(1)}$</td>
<td>$2.45 T$</td>
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<tr>
<td>$m_{A^{++}}^{(2)}$</td>
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<tr>
<td>$m_{A^{++}}^{(3)}$</td>
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</tr>
<tr>
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<td>$0.24 T$</td>
</tr>
<tr>
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</tr>
<tr>
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</tr>
<tr>
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If \(T \sim \text{TeV}\), \(m_{A(+-)}^{(1)}\) can be about 100 GeV.
**KK mass of bulk gauge boson**

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If $T \sim$ TeV, $m^{(1)}_{A^{(+ -)}}$ can be about 100 GeV. (Without Higgs)
Bulk fermion can have four different $\mathbb{Z}_2 \times \mathbb{Z}'_2$ parity sets,

$$\hat{\Psi}_i(x, y) = \sqrt{k} \sum_n [\psi^{(n)}(x)f^{(n)}_{iL}(y) + \psi^{(n)}(x)f^{(n)}_{iR}(y)]$$
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$i = 1, 2$ represent the parallel conditions $(\pm \pm)$

$f_{iL}$ has $(\pm \pm)$ parity and $f_{iR}$ has $(\mp \mp)$,

$i = 3, 4$ represent the crossed conditions, where $(\pm \mp)$

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$f_{iL}$ has $(\pm \mp)$ parity and $f_{iR}$ has $(\mp \pm)$.

$$f_{iL}^{(n)}(z) = \frac{\sqrt{z}}{N_i^{(n)}} \left[ J_{c_i} + \frac{1}{2} (m_i^{(n)} z) + \beta_i^{(n)} Y_{c_i} + \frac{1}{2} (m_i^{(n)} z) \right]$$

$$f_{iR}^{(n)}(z) = \frac{\sqrt{z}}{N_i^{(n)}} \left[ J_{c_i} - \frac{1}{2} (m_i^{(n)} z) + \beta_i^{(n)} Y_{c_i} - \frac{1}{2} (m_i^{(n)} z) \right]$$

$c_i$ is the bulk fermion mass.
Figure: KK mass spectra of bulk fermion without Higgs in unit of $T$
Only $(++)$ parity bulk mode function can be a zero mode.

The gauge field with other parity is massive in 4D.

$(+-)$ parity mode has a light gauge boson without Higgs.
Only $(++)$ parity bulk mode function can be a zero mode.

The gauge field with other parity is massive in 4D.

$(+−)$ parity mode has a light gauge boson without Higgs.

The fermions with other than $(++)$ parity become massive in 4D.

$(+−)$ and $(−+)\) mode can have very light KK modes.

Higgs should be confined on IR boundary

Bulk Higgs scalar brings back hierarchy problem again.
Higgsless Standard Model

If the VEV of Higgs become very large, SM fields

((++) mode) simulate the (+−) KK modes without Higgs field.
Higgsless Standard Model

If the VEV of Higgs become very large, SM fields 

((+++) mode) simulate the (+−) KK modes without Higgs field.

Since (+−) has light gauge boson, and can have light fermions 

There is a possible Higgsless SM (Higgsless gauge symmetry breaking).
Higgsless Standard Model

If the VEV of Higgs become very large, SM fields
((++) mode) simulate the (+−) KK modes without Higgs field.

Since (+−) has light gauge boson, and can have light fermions
There is a possible Higgsless SM (Higgsless gauge symmetry breaking).

Unfortunately, due to the parity conservation,
one of the fermion doublet component should have zero mass.
Basic assumptions

1. All mass hierarchy are generated from bulk mass structure:
   All 5D SM parameters are considered $\mathcal{O}(1)$. 
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   All 5D SM parameters are considered $O(1)$.

2. There should not be a order changing cancellation during the matrix algebra between mixing and mass matrices.
Basic assumptions

1. All mass hierarchy are generated from bulk mass structure:

All 5D SM parameters are considered $O(1)$.

2. There should not be a order changing cancellation during the matrix algebra between mixing and mass matrices.

With this two simple assumptions, we construct a bulk SM in warped RS1.
Universal Yukawa coupling

The SM fermion mass and mixing is decided by both bulk fermion masses and Yukawa coupling with Higgs. We assume “(almost) universal Yukawa coupling”.
Universal Yukawa coupling

The SM fermion mass and mixing is decided by both bulk fermion masses and Yukawa coupling with Higgs. We assume “(almost) universal Yukawa coupling”.

There is no hierarchy within Yukawa coupling matrix.

\[
y_f \approx \begin{pmatrix}
1 & 1 & 1 \\
1 & 1 & 1 \\
1 & 1 & 1 \\
\end{pmatrix}.
\]
The fermions mass matrix is generated from,

\[ \frac{M_{ij}}{v_W} \simeq F_L(c_i) \times F_R(c_j), \]
The fermions mass matrix is generated from,

\[ \frac{M_{ij}}{\nu_W} \simeq F_L(c_i) \times F_R(c_j), \]

where

\[ F_L(c_i) = \epsilon^{c_i-1/2} \sqrt{\frac{2c_i-1}{1-\epsilon^2c_i-1}}, \quad F_R(c_i) = \epsilon^{-c_i-1/2} \sqrt{\frac{2c_i+1}{\epsilon^{-2c_i-1}-1}}, \]

\( c_i \) is mass of bulk fermion \( f_i \).
The fermions mass matrix is generated from,

\[ M_{ij}/\nu_W \simeq F_L(c_i) \times F_R(c_j), \]

where

\[ F_L(c_i) = \epsilon^{c_i-1/2} \sqrt{\frac{2c_i-1}{1-\epsilon^{2c_i-1}}}, \quad F_R(c_i) = \epsilon^{-c_i-1/2} \sqrt{\frac{2c_i+1}{\epsilon^{-2c_i-1}-1}}. \]

\( c_i \) is mass of bulk fermion \( f_i \).

Left-handed and right handed SM fermion comes from different bulk fermions.
\( F_L(c) \) as a function of bulk mass \( c \).
If we increase \((-c_i, \ F_{L(R)}(c_i)\) decrease slowly until \((-c_i = 1/2).

For \((-c_i > 1/2,\) it decrease fast in power of \(\epsilon(-c_i)\).

Without a large hierarchy in bulk fermion mass \(c_i,\)

All SM fermion masses can be generated.
If we increase \((-c_i)\), \(F_L(R)(c_i)\) decrease slowly until \((-c_i) = 1/2\).

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Without a large hierarchy in bulk fermion mass \(c_i\),

All SM fermion masses can be generated.

If \((-c_i) < 0\), the 1st KK mode can be very light. \(m^{(1)} \ll \text{TeV}\).

Model with larger symmetry might contains light neutral
and stable KK mode, i.e. WIMPS.
WIMPS are ...
Bulk fermions

$Q_i$ contains left handed quark

$U_i, D_i$ contains right handed quark

$L_i$ contains lepton doublet

$E_i$ contains lepton singlet

$N_i$ contains right handed neutrino
Quark mass and mixing

\[(M_a)_{ij} \simeq v_W F_L (c_{Q_i}) F_R (c_{A_j}),\]

where \(a = u, d\) and \(A = U, D\).

\[U_{qL}^T M_q U_{qR} = M_q^{\text{diag}} \quad \text{for} \ q = u, d.\]
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\[U_{qL}^T M_q U_{qR} = M_q^{\text{diag}} \quad \text{for } q = u, d.\]

The CKM matrix is defined as \(K = U_{uL}^T U_{dL}\).

\[K \simeq \begin{pmatrix}
1 & \lambda & \lambda^3 \\
\lambda & 1 & \lambda^2 \\
\lambda^3 & \lambda^2 & 1
\end{pmatrix},\]

For simplified Wolfenstein parameterization and \(\lambda \simeq 0.22\).
We also assume that there is no order changing cancellation in matrix diagonalization or other algebras.
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i.e. 1 is a approximation of a number between $\sqrt{\lambda}$ and $1/\sqrt{\lambda}$.

Thus the order $\lambda$ in matrix will not change after the algebra.

This simple assumption leads to a unique bulk mass structure for SM fermions.
Since $u_L$ and $d_L$ has almost the same matrix form,

$$U_{uL} \simeq U_{dL} \simeq \begin{pmatrix} 1 & \lambda & \lambda^3 \\ \lambda & 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix}.$$
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$$U_{uL} \sim U_{dL} \sim \begin{pmatrix} 1 & \lambda & \lambda^3 \\ \lambda & 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix}.$$ 

Write the quark masses with $\lambda$

$$M_{u}^{\text{diag}} = \text{diag}(m_u, m_c, m_t) \sim v_W \ \text{diag}(\lambda^8, \lambda^{3.5}, 1),$$
$$M_{d}^{\text{diag}} = \text{diag}(m_d, m_s, m_b) \sim v_W \ \text{diag}(\lambda^7, \lambda^5, \lambda^{2.5}).$$
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With known masses of SM fermions,
Following hierarchical relations are obtained:

\[ F_L(c_{Q_1}) : F_L(c_{Q_2}) : F_L(c_{Q_3}) \simeq \lambda^3 : \lambda^2 : 1, \]
\[ F_R(c_{A_1}) : F_R(c_{A_2}) : F_R(c_{A_3}) \simeq \lambda^3 : \lambda^2 : 1, \]
\[ F_R(c_{D_1})^2 + F_R(c_{D_2})^2 + F_R(c_{D_2})^2 \simeq \lambda^5 \left[ F_R(c_{U_1})^2 + F_R(c_{U_2})^2 + F_R(c_{U_2})^2 \right]. \]

The SM quark mixing matrices can be approximated as

\[ (U_{qL})_{ij}(i \leq j) \approx \frac{F_L(c_{Q_i})}{F_L(c_{Q_j})}, \quad (U_{qR})_{jj}(i \leq j) \approx \frac{F_R(c_{A_i})}{F_R(c_{A_j})}, \]

where \( A = U, D \).
From various experimental constraints, (e.g. $Z \rightarrow b\bar{b}$)

The bulk quark masses which satisfy all SM parameters can be determined for $T \sim$ TeV.

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The bulk quark masses which satisfy all SM parameters can be determined for $T \sim \text{TeV}$.

(SC, Kim, Yamaguchi)

\[ c_{Q1} \approx 0.61, \quad c_{Q2} \approx 0.56, \quad c_{Q3} \approx 0.3 \pm 0.03. \]

\[ c_{U1} \approx -0.70, \quad c_{U2} \approx -0.52, \quad 0 \lesssim c_{U3} \lesssim 0.2, \]

\[ c_{D1} \approx -0.66, \quad c_{D2} \approx -0.61, \quad c_{D3} \approx -0.56. \]
Lepton mass matrix

\[ U_{\text{MNS}} = U_e^\dagger U_\nu, \]

where

\[ M_\nu^\dagger M_\nu = U_\nu (M_\nu^\text{diag})^2 U_\nu^\dagger, \quad M_e^\dagger M_e = U_e (M_e^\text{diag})^2 U_e^\dagger. \]

MNS matrix can be approximated as

\[ |U_{\text{MNS}}| \sim \begin{pmatrix} 1 & 1 & \lambda^m \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \]

where the experimental constraint on \( U_{e3} \) gives \( m > 1.3 \).
Universal lepton mass matrices are

$$(M_{\nu})_{ij} \simeq v_W F_L(c_{Li}) F_R(c_{Nj}),$$
$$(M_e)_{ij} \simeq v_W F_L(c_{Li}) F_R(c_{Ej}).$$

Individual neutrino masses are not yet measured.
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But from the neutrino oscillation

$$\Delta m^2_{sol} = m^2_2 - m^2_1 \simeq 7.5 \times 10^{-5} \text{ eV}^2,$$

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Neutrino mass can be either Normal Hierarchy (NH) \((m_1 \approx 0)\)
Or Inverse Hierarchy (IH) \((m_3 = 0)\)
Assumption of no order changing cancellation in matrix product allows only NH for neutrino mass.
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\[ M_\nu^T M_\nu \propto \begin{pmatrix} \lambda^{2n} & \lambda^n & \lambda^n \\ \lambda^n & 1 & 1 \\ \lambda^n & 1 & 1 \end{pmatrix} \] (NH).

Also we parameterize lepton masses

\[ M_\nu^{\text{diag}} = \text{diag}(m_1, m_2, m_3) = v_W \text{ diag}(\lambda^{20.5}, \lambda^{20.5}, \lambda^{19}), \]
\[ M_e^{\text{diag}} = \text{diag}(m_e, m_\mu, m_\tau) = v_W \text{ diag}(\lambda^{8.5}, \lambda^5, \lambda^3). \]
Maximal mixing between 2 and 3 flavors suggests the mixing

\[ U_f \approx \begin{pmatrix} 1 & \lambda^{af} & \lambda^n \\ \lambda^{bf} & 1 & 1 \\ \lambda^{cf} & 1 & 1 \end{pmatrix} , \]

with \( f = e, \nu \).
Maximal mixing between 2 and 3 flavors suggests the mixing

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with \( f = e, \nu \).

To have a desired mass hierarchy,

\[ 1.5 + a_\nu \gtrsim n, 2 + a_e \gtrsim n. \]

Unlike quark mass, lepton mass has some ambiguity.
With MNS matrix $U_{\text{MNS}} = U_e^T U_\nu$

\[
\lambda^{a\nu} \sim \lambda^{b\nu} + \lambda^{c\nu} \sim 1, \quad \lambda^{b\nu} + \lambda^{c\nu} \lesssim \lambda^{m}, \quad \lambda^{n} \lesssim \lambda^{m}.
\]
With MNS matrix $U_{\text{MNS}} = U_e^T U_\nu$

$$\lambda^{a\nu} \sim \lambda^{b\nu} + \lambda^{c\nu} \sim 1, \quad \lambda^{be} + \lambda^{ce} \lesssim \lambda^m, \quad \lambda^n \lesssim \lambda^m.$$ 

The allowed values of $n$ and $m$ in a narrow range

$$1.3 \lesssim m \lesssim n \lesssim 1.5.$$
With MNS matrix $U_{\text{MNS}} = U_e^T U_\nu$

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\]

The allowed values of $n$ and $m$ in a narrow range

\[
1.3 \lesssim m \lesssim n \lesssim 1.5.
\]

Thus, as a representative value, we expect

\[
U_{e3} \sim \lambda^m \approx 0.10 - 0.14,
\]

which is almost marginal to experimental bound.
Lepton flavor violation

If we focus the KK mass within detectable range $2 - 3$ TeV, The Lepton flavor violation (LFV) processes gives a strong bound.

\[
\frac{\Gamma(\tau \rightarrow 3e)}{\Gamma(\mu \rightarrow e\nu_\mu \bar{\nu}_e)} \approx (K_{L11}^{(1)} K_{L12}^{(1)})^2 \left( \frac{m_Z}{M_A^{(1)}} \right)^4 \lesssim 1.0 \times 10^{-12},
\]

\[
\frac{\Gamma(\tau \rightarrow 3\mu)}{\Gamma(\tau \rightarrow \mu\nu_\tau \bar{\nu}_\mu)} \approx (K_{L22}^{(1)} K_{L23}^{(1)})^2 \left( \frac{m_Z}{M_A^{(1)}} \right)^4 \lesssim 10^{-6},
\]
The $K^{(1)}_{Lij}$ factor represents $l_i{-}l_j{-}A^{(1)}$ vertex,

$$K^{(n)}_{Lij} = \sum_{k=1}^{3} (U_{eL})_{ik} \hat{g}^{(n)}(c_{L_k}) \left( U_{eL}^\dagger \right)_{kj},$$

where effective coupling of KK gauge mode is,

$$\hat{g}^{(n)}_L(c_{f_i}) = \sqrt{kL} \int dk \left[ f_L^{(0)}(z, c_{f_i}) \right]^2 f_A^{(n)}(z) \equiv \tilde{g}^{(n)}(c_{f_i}).$$
\( \hat{g}^{(n)}(c) \) saturates around -0.2 if \( c > 0.55 \).
if KK gauge boson is experimentally testable, i.e. $M_A^{(1)} \approx 2\text{-}3$ TeV, we can specify its mixing matrix

$$U_{eL} \approx \begin{pmatrix} 1 & \delta & \delta \\ \delta & 1 & 1 \\ \delta & 1 & 1 \end{pmatrix},$$

where $\delta = \lambda^m \approx 0.1$. 
if KK gauge boson is experimentally testable, i.e. $M_A^{(1)} \sim 2-3$ TeV, we can specify its mixing matrix

$$U_{eL} \approx \begin{pmatrix} 1 & \delta & \delta \\ <\delta^2 & 1 & 1 \\ \delta & 1 & 1 \end{pmatrix},$$

where $\delta = \lambda^m \sim 0.1$, and bulk lepton masses (SC, Kim, Song)

$$c_{L1} \sim 0.59, \quad c_{L2} \sim 0.5, \quad c_{L3} \sim 0.5,$$

$$c_{E1} \sim -0.74, \quad c_{E2} \sim -0.65, \quad c_{E3} \sim -0.55.$$

$$c_{N2} \sim -1.2, \quad c_{N3} \sim -1.1.$$
Is there experimental application of this model?

Theoretical or Experimental?
$B_q^0 - \bar{B}_q^0$ mixing data

The current experimental results are well measured

\[
\Delta M_d^{\text{exp}} = (0.507 \pm 0.004) \text{ ps}^{-1},
\]
\[
\Delta M_s^{\text{exp}} = \left[17.33^{+0.42}_{-0.21}(\text{stat}) \pm 0.07(\text{syst})\right] \text{ ps}^{-1}.
\]

There is no tree level contribution to $B_q^0 - \bar{B}_q^0$ in SM.

Can be a good test for the RS bulk SM.
$B^0_q - \bar{B}^0_q$ mixing at tree level by bulk gluon

The KK gluon contribution to $B^0_q - \bar{B}^0_q$ is tree level while SM contribution is from box diagram.
For the $B^0_q - \bar{B}^0_q$ transition amplitude $M^q_{bb}$ defined by

$$\langle B^0_q | \mathcal{H}_{\Delta B=2}^{\text{eff}} | \bar{B}^0_q \rangle = 2 M_{B_q} M^q_{bb},$$

where

$$\Delta M_q = 2 |M^q_{bb}|.$$

“mixing-induced” CP violation by phase

$$\phi_q = \arg \left( M^q_{bb} \right).$$
$B^0_q - \bar{B}^0_q$ mixing from the SM box diagrams and the RS KK gluons:

$$M_{qb} = M_{qb}^{\text{SM}} \left( 1 + \frac{M_{qb}^{\text{RS}}}{M_{qb}^{\text{SM}}} \right).$$

We parameterize the new physics effect by

$$r_q e^{i\sigma_q} = \frac{M_{qb}^{\text{RS}}}{M_{qb}^{\text{SM}}},$$

where $r_q \geq 0$ and $\sigma_q$ is real.
The $r_q$ and $\sigma_q$ are constrained by the experimental result for $\Delta M_q$ and the theoretical calculation of the $\Delta M_q^{\text{SM}}$, of which the ratio is defined by $\rho_q$:

$$
\rho_q \equiv \frac{\Delta M_q}{\Delta M_q^{\text{SM}}} = \frac{M_{q,\text{SM}}^{q,\bar{b}b} + M_{q,\text{RS}}^{q,\bar{b}b}}{M_{q,\text{SM}}^{q,\bar{b}b}} = \sqrt{1 + 2r_q \cos \sigma_q + r_q^2}.
$$
\( \phi_d \) can be divided into the SM phase and that from New Physics (NP) contributions.

\[
\phi_q = \phi_q^{\text{SM}} + \phi_q^{\text{NP}} = \phi_q^{\text{SM}} + \arg(1 + r_q e^{i\sigma_q}).
\]

\( \phi_q^{\text{NP}} \) is determined by \( r_q \) and \( \sigma_q \).

\[
\begin{align*}
\sin \phi_q^{\text{NP}} &= \frac{r_q \sin \sigma_q}{\sqrt{1 + 2r_q \cos \sigma_q + r_q^2}}, \\
\cos \phi_q^{\text{NP}} &= \frac{1 + r_q \cos \sigma_q}{\sqrt{1 + 2r_q \cos \sigma_q + r_q^2}}.
\end{align*}
\]
SM amplitude is

\[ M_{\tilde q \tilde b}^{q,SM} = \frac{G_F^2 m_W^2}{12\pi^2} M_{Bq} \hat{\eta}^B \hat{B}_{Bq} f_{Bq}^2 (V^*_t V_{tb})^2 S_0(x_t), \]

where \( x_t = m_{top}^2 / m_W^2 \), and \( S_0 \) is an “Inami–Lim” function.

CKM components are

\[ |V^*_{td} V_{tb}| = (8.6 \pm 1.3) \times 10^{-3}, |V^*_{ts} V_{tb}| = (41.3 \pm 0.7) \times 10^{-3}. \]

Short-distance QCD correction \( \hat{\eta}^B = 0.552 \)
\( \hat{B}_{B_{d,s}} f_{B_{d,s}}^2 \) is determined from lattice simulation by JLQCD collaboration.
New physics effect becomes

\[ r_q e^{i\sigma_q} \equiv \frac{M_{q,RS}^{b\bar{b}}}{M_{q,SM}^{b\bar{b}}} = \frac{16\pi^2}{N_C} \frac{8g_s^2}{g^4S_0(x_t)} m_W^2 \kappa_{33}^2 \kappa_{q3}^2 \sum_{n=1} \left( \hat{g}^{(n)}(c_{Q3}) \right)^2. \]

\( \mathcal{O}(1) \) ambiguity and phases in mixings are absorbed in

\[ \sqrt{\lambda} < \kappa_{ij} < \frac{1}{\sqrt{\lambda}}. \]

\[ (U_{qL})_{ij} = \kappa_{ij} V_{ij}^{CKM}, \]
\( \phi_d \) associated with \( B_d^0 - \bar{B}_d^0 \) is well measured,

\[
\langle \sin \phi_d \rangle_{cc\bar{s}} = \sin(2\beta + \phi_d^{NP}) = 0.687 \pm 0.032, 
\]

where \( \phi_d^{SM} = 2\beta \).

\( \phi_d^{NP} \) is estimated by (Ball and Fleischer, 2006)

\[
\phi_d^{NP} \bigg|_{incl} = -(10.1 \pm 4.6)^\circ, \quad \phi_d^{NP} \bigg|_{excl} = -(2.5 \pm 8.0)^\circ. 
\]

New Physics parameters are constrained by the CP phases.
Figure: Allowed parameter space of $(\sigma_d, M_{KK})$. Red lines satisfy the observed $\rho_d$ and blue lines for $\phi_d^{NP}|_{incl}$, with 1σ uncertainty.
**Figure:** Allowed parameter space of $(\sigma_s, M_{KK})$ from the observed $\Delta M_s$. 
Results

For the choice of optimistic parameters for bulk SM,

\[ c_{Q3} = 0.32, \quad \kappa = 1/\sqrt{2} \]

\( B_d^0 - \bar{B}_d^0 \) bounds allow KK mode mass as low as 3 TeV.

\( B_s \) mixing gives a weaker bound than \( B_d \) mixing.
Summary

1. SM field can be resides on $S^1/\mathbb{Z}_2 \times \mathbb{Z}'_2$ bulk space.
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Summary

1. SM field can be resides on $S^1/\mathbb{Z}_2 \times \mathbb{Z}_2'$ bulk space.

2. Fermion bulk masses can be estimate from SM parameters.

3. In universal Yukawa coupling model, $U_{e3} \sim 0.1$ in the neutrino mixing matrix.

4. $B^0_d - \bar{B}^0_d$ constrains 1st KK mode $M_A^{(1)} \gtrsim 3$ TeV.

The last one is rather disappointing, since it will be impossible to detect for LC and will be hard for LHC.
Credit for the movie files

- NBC TV series *Law & Order*
  Episode: Big Bang

- WB TV series *Angel*
  Episode: Supersymmetry