Flavor Physics from Warped Extra Dimensions

Talk at Purdue University

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Yonsei University

November 14, 2007

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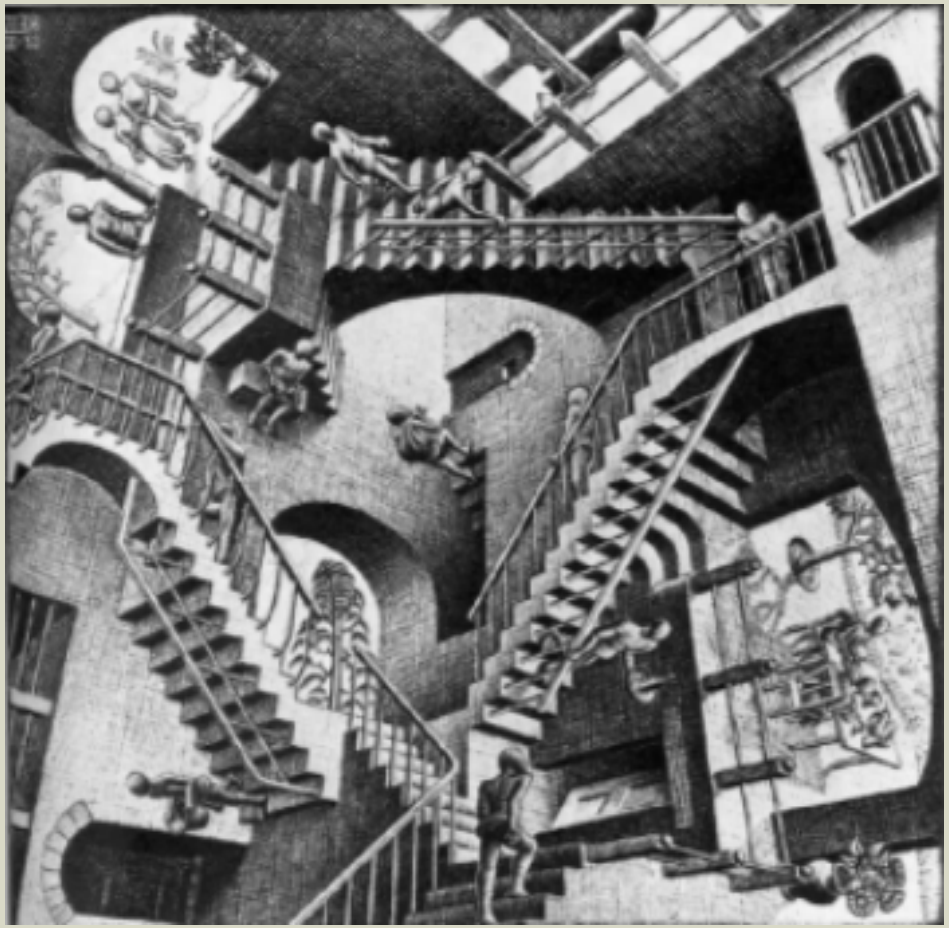
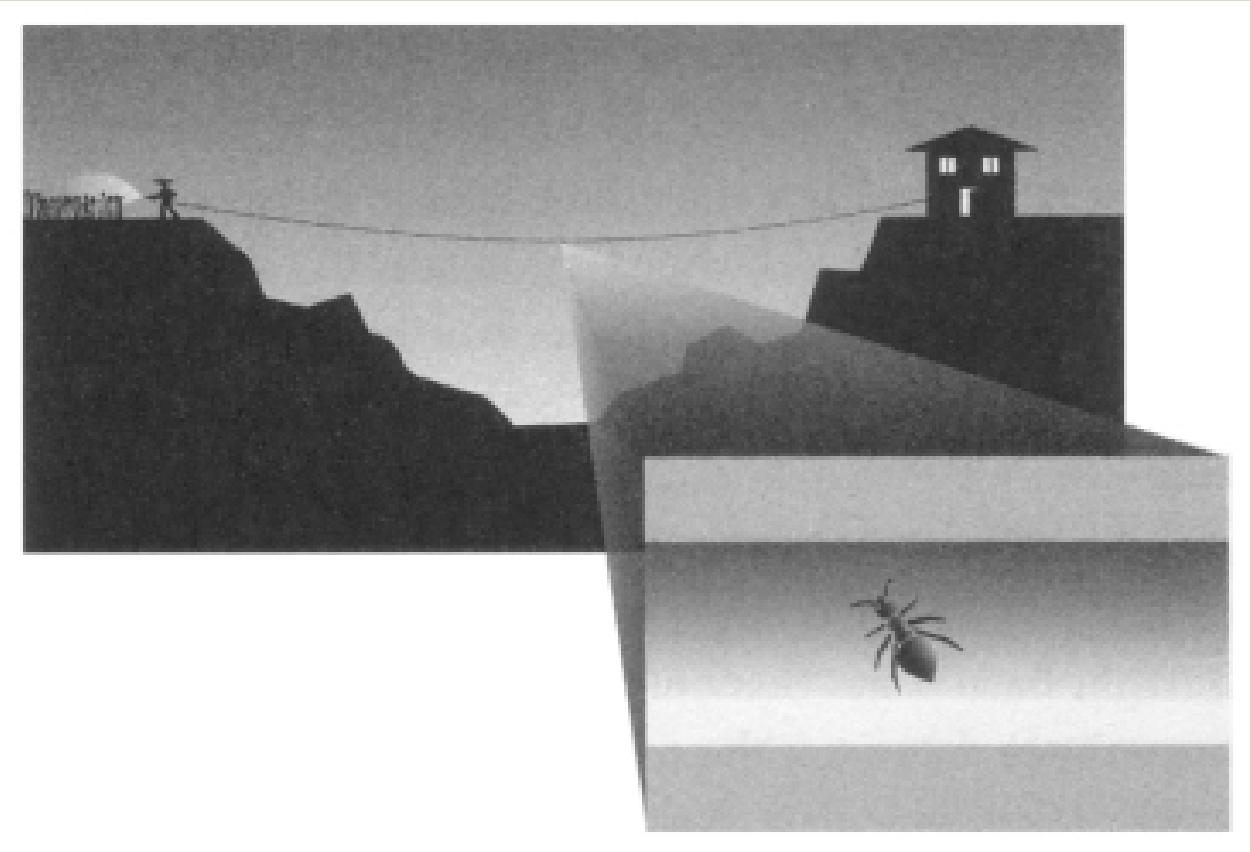
What is Extra Dimension?



For us, a rope is 1-D, but for ant, it is 2-D space.



Hierarchy Problem

- Why Gravity is so weak?

Weak Scale (M_W) = 1000 GeV

Planck Scale (M_{pl}) = 10^{19} GeV

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- Weak scale : Standard Model
- Above Planck Scale : Quantum Gravity
- Between two scales : GUT, Supersymmetry, Superstring etc.

  There is no experimental clue on the intermediate scale.

That's not what they wanted to prove.

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The Problem of Higgs Mass

In Standard Model(SM),

Higgs mass correction is quadratic divergent to cutoff scale.

$$M^2_{Higgs} \longrightarrow M^2_{Higgs} + c\Lambda^2_{cut}$$

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We need an upper bound of Λ_{cut} to be around 1 TeV.

In SM, M_{pl} is a natural cut-off.

Supersymmetry(SUSY)

To solve Hierarchy problem, SUSY was introduced.

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Each loop-correction by scalar loop is cancelled by higgsino loop.

(soft) SUSY breaking generates natural cut-off at $\sim$TeV.

Not exactly replacing SUSY, but a simple idea was brought by

Arkani-Hamed, Dimopoulos and Dvali PLB 429 (1998).

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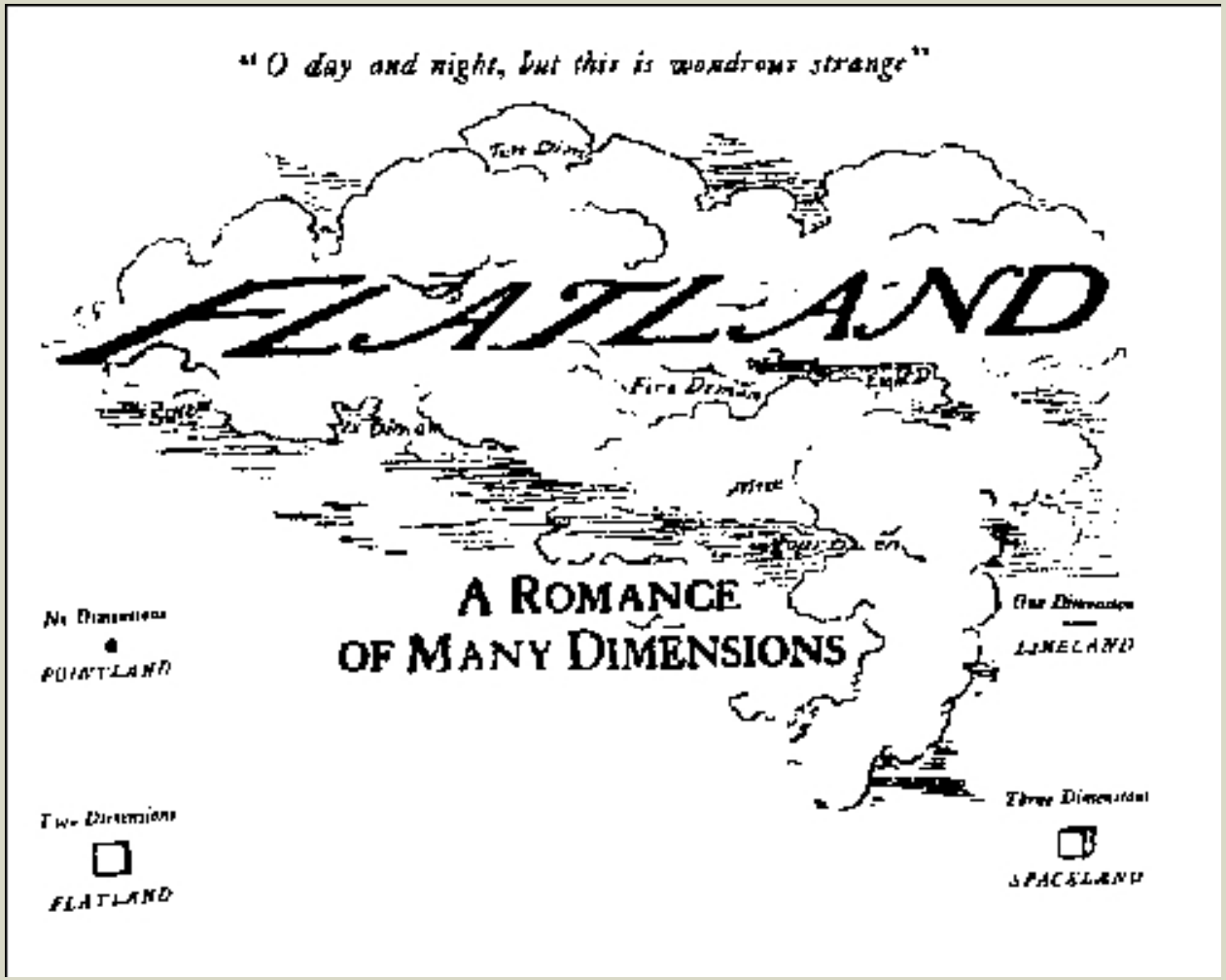
ADD suggest that the hierarchy problem can be solved by

existence of extra spatial dimensions.

The FLATLAND



Flat Extra dimensions

If our Universe has a thickness R,

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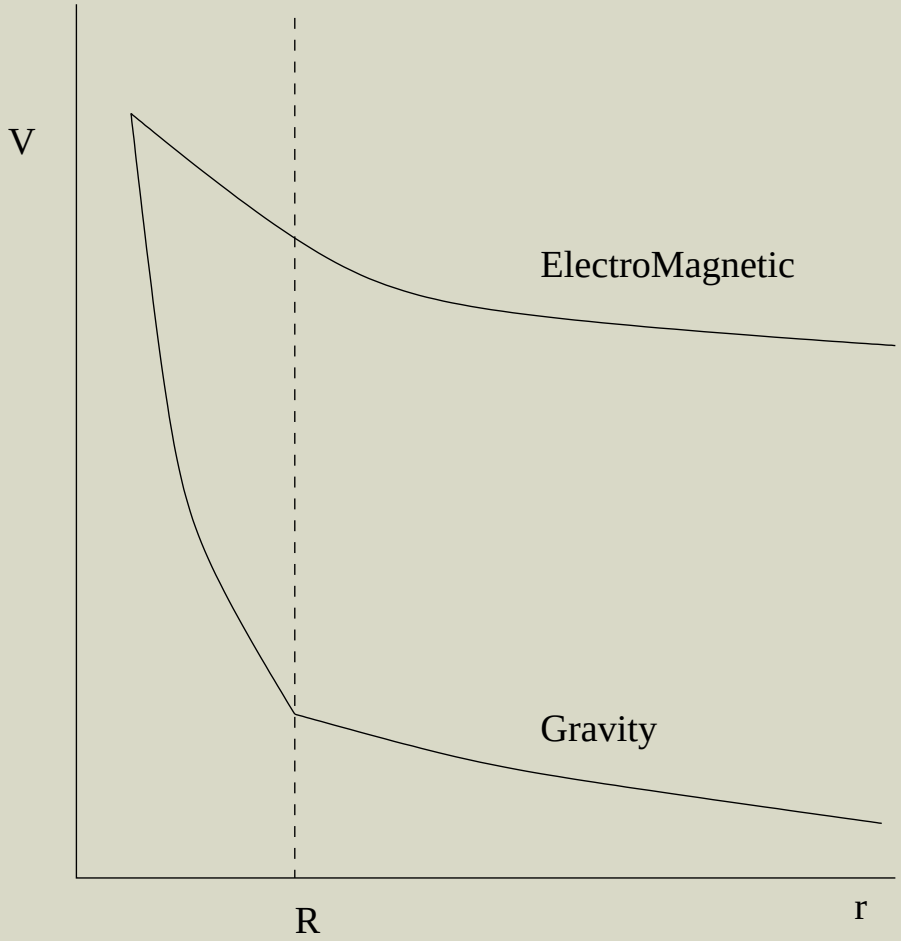
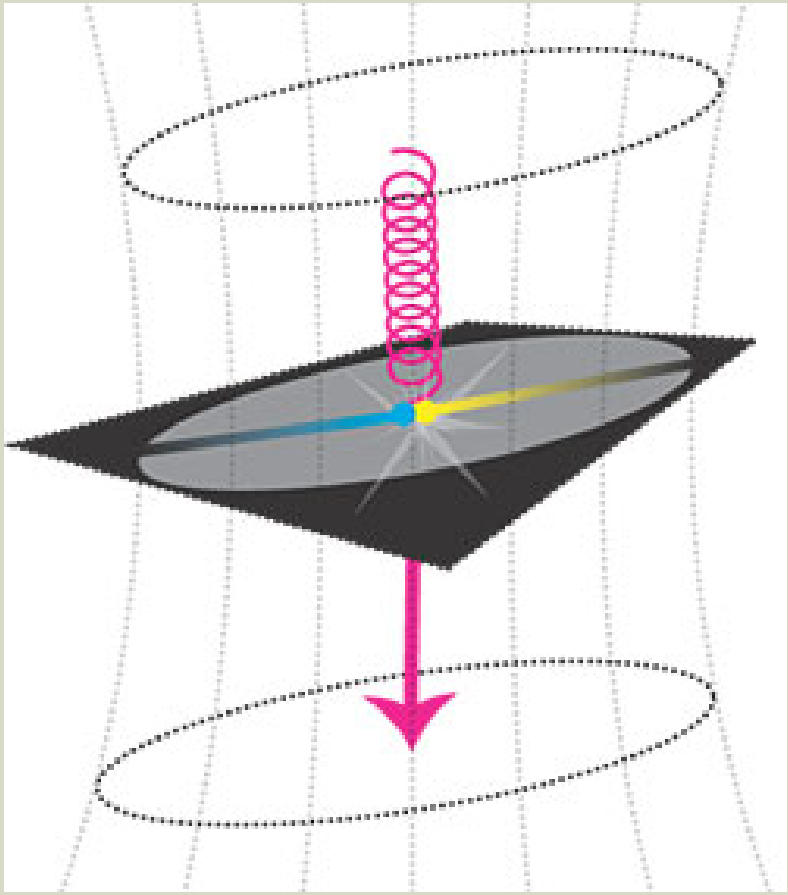
$$G\frac{1}{r}, \quad (r > R)$$

$$G_{(4+n)}\frac{1}{r^{n+1}}, \quad (r < R)$$



The 4D gravity can be weakened by extra-dimensions.

A short tour into the Extra dimensions



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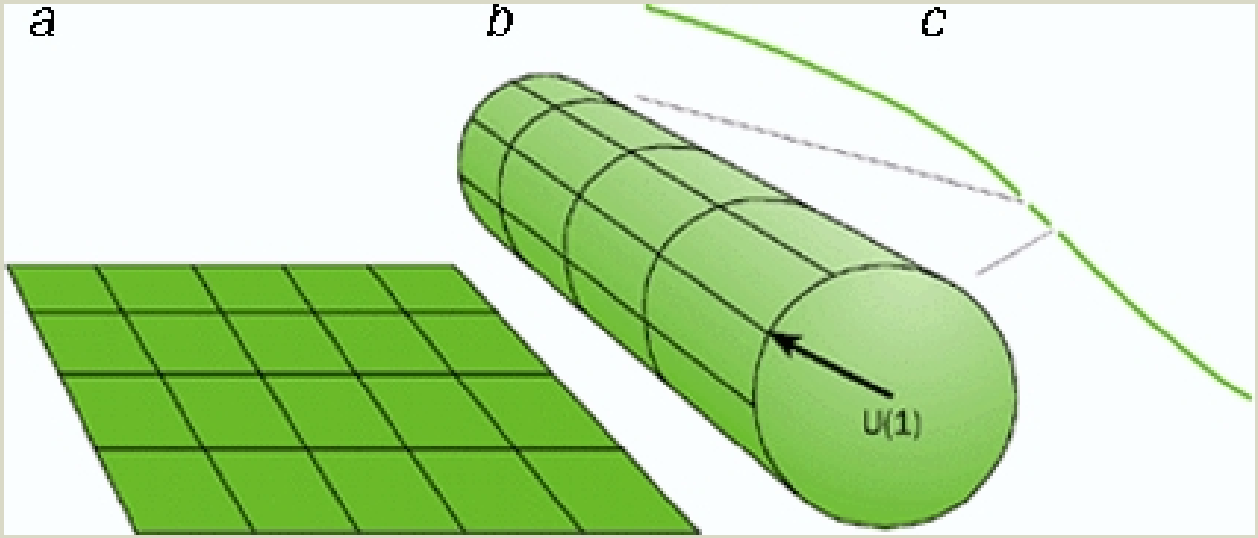
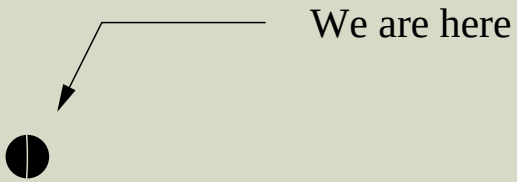
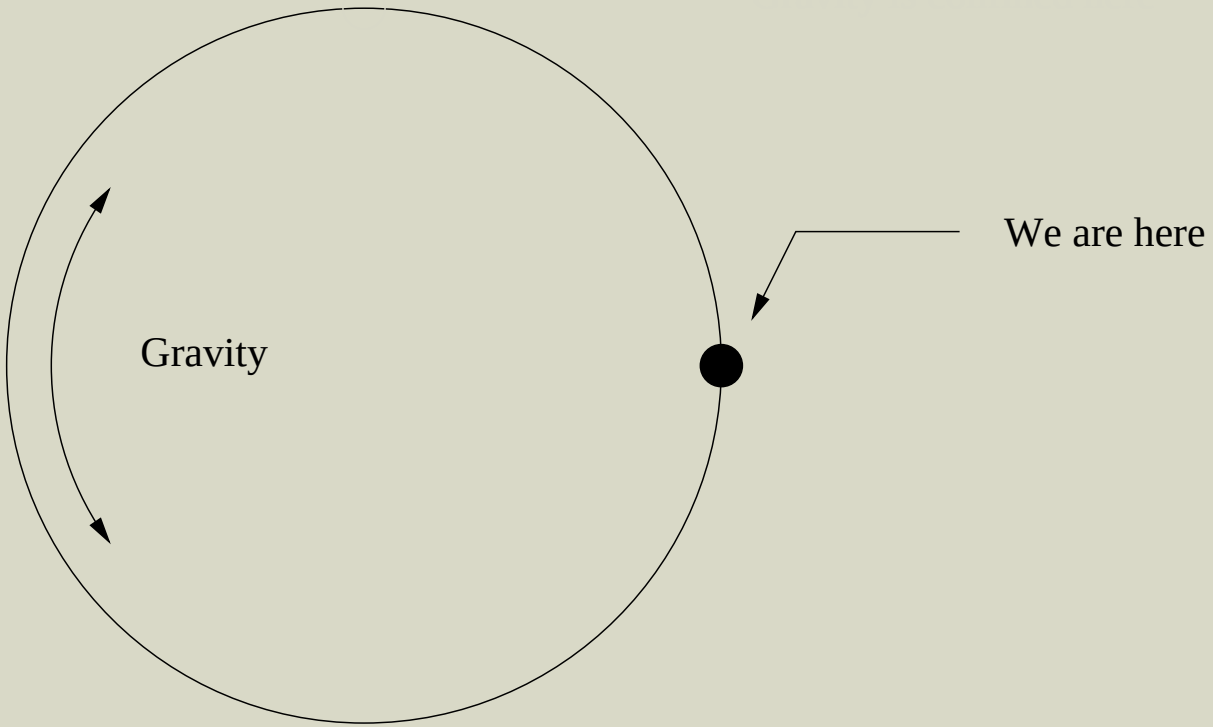
It is hard to comprehend due to its "*large*" size.

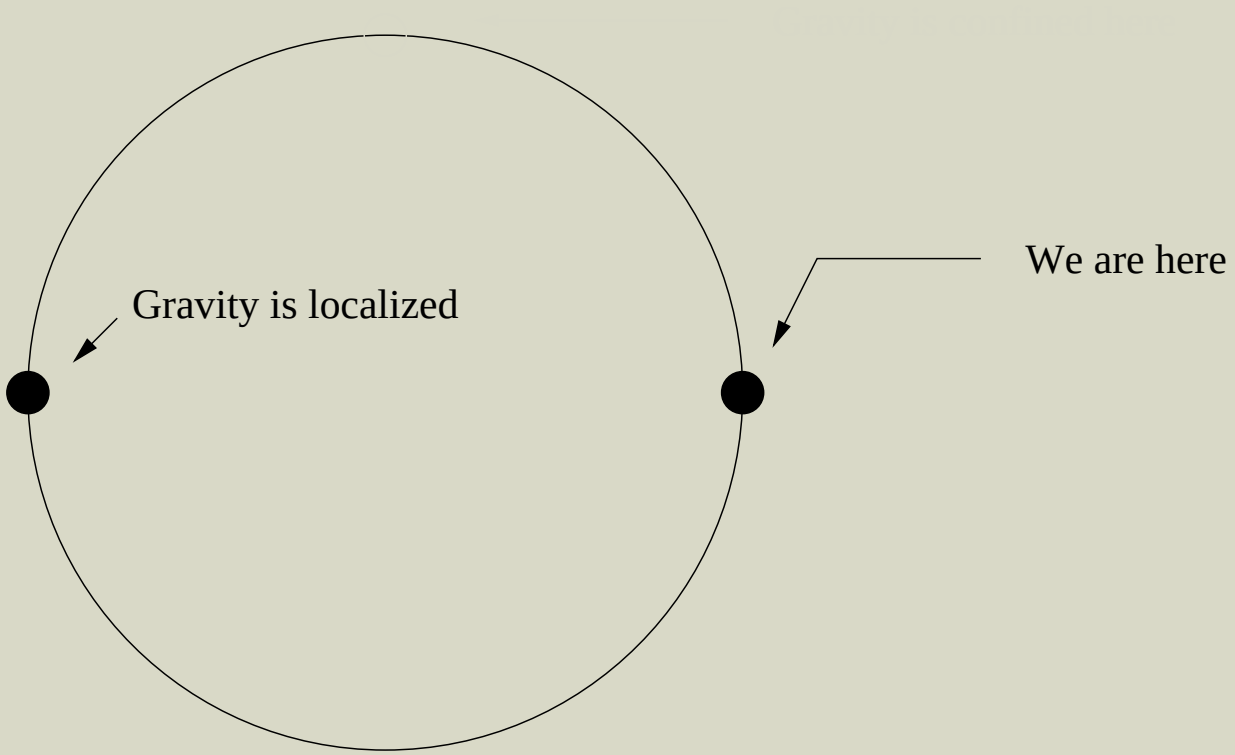




Our $(3+1)$D universe

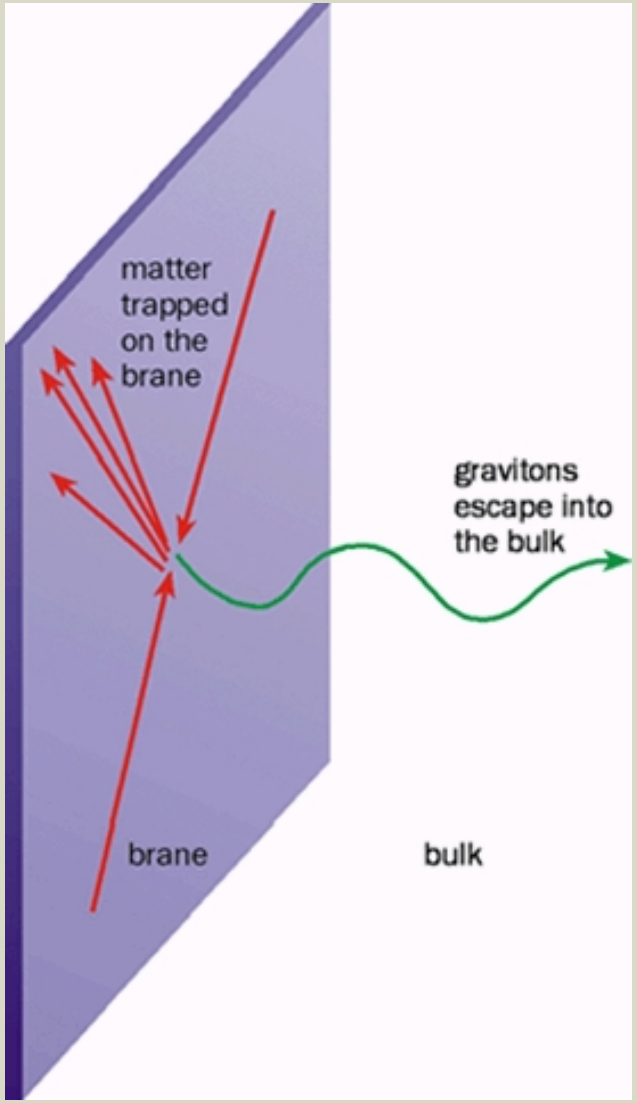


Flat extra dimension S^1 (Arkani-Hamed et.al.)

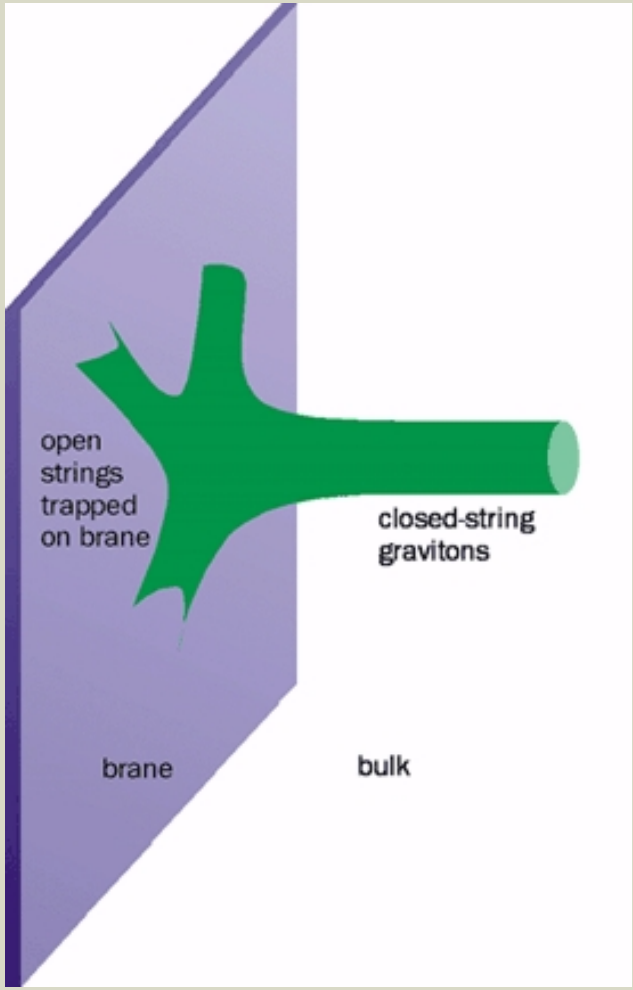


RS1 Warped extra dimension $S^1/\mathbb{Z}_2$

All SM filed should be confined on the 3+1D brane except gravity



This can be related with the string theory with brane solutions



As you know, string need to be compactified.

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Existence of massive Kaluza-Kelin(KK) graviton is cosmologically dangerous.

Randall-Sundrum model (RS1)

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RS1 is model with one warped extra-dimension.

$$ds^2 = e^{-2\sigma(y)}(dt^2 - dx^2) - dy^2,$$

$\sigma(y) = k|y|$ is the warp factor

$k \sim M_{pl}$ is the curvature of warped space.

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$k \sim M_{pl}$ is the curvature of warped space.

The 5th direction y is bounded and the bulk is AdS_5

All SM fields are confined on TeV brane (IR boundary)

Gravity resides in the bulk.

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But, how can we develop bulk SM?

The conformal coordinate of $z \equiv e^{\sigma}/k$ is useful.

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z is also bounded in $1/k \leq z \leq 1/T$.

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With $kL \approx 35$, the warp factor $\epsilon \equiv e^{-kL}$ reduces T at TeV scale from k at Planck scale:

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With this scaling the gauge hierarchy problem is answered.

Bulk fields in RS1

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- And bulk fermion was followed soon after.

  SC, Hisano, Nakano, Okada and Yamaguchi, (1999),

  Grossman and Neubert (1999).

Bulk gauge bosons

The action for a 5D $U(1)$ gauge field

$$S_{\rm gauge} = \int d^4x dz \sqrt{G} \left[-\frac{1}{4} g^{MP} g^{NQ} F_{MN} F_{PQ} + \frac{1}{2} M^2 g^{MN} A_M A_N \right],$$

where $F_{MN} = \partial_M A_N - \partial_N A_M$.

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where $F_{MN} = \partial_M A_N - \partial_N A_M$.

The general mass term $M^2(z)$,

$$M^2(z) = {a_{\rm UV}}^2 k\, \delta(z - z_{\rm UV}) + {a_{\rm IR}}^2 k\, \delta(z - z_{\rm IR}) + b^2 k^2,$$

the dimensionless b and $a_{\rm UV}(a_{\rm IR})$ are bulk mass and localized mass on the UV (IR) brane.

The KK expansion of the dimension 3/2 field $A^M(x,z)$ is

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$$-z\partial_z\left(\frac{1}{z}\partial_z f_A^{(n)}(z)\right) + \frac{M^2(z)}{k^2z^2} f_A^{(n)}(z) = m_A^{(n)^2} f_A^{(n)}(z),$$

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The mode function can be obtained

$$f_A^{(n)}(z) = \frac{z}{N_A^{(n)}}\left[J_\nu(m_A^{(n)}z) + \beta_A^{(n)} Y_\nu(m_A^{(n)}z)\right],$$

where $\nu = \sqrt{1+b^2}$, $b = 0$ if the gauge symmetry is conserved in the bulk.

Fermions in RS1 Bulk

$$S_{\text{fermion}} = \int d^4x dy \left[\overline{\hat{\Psi}} e^{\sigma} i\gamma^{\mu}\partial_{\mu}\hat{\Psi} - \frac{1}{2}\overline{\hat{\Psi}}\gamma_5\partial_y\hat{\Psi} + \frac{1}{2}(\partial_y\overline{\hat{\Psi}})\gamma_5\hat{\Psi} + m_D\overline{\hat{\Psi}}\hat{\Psi} \right]$$

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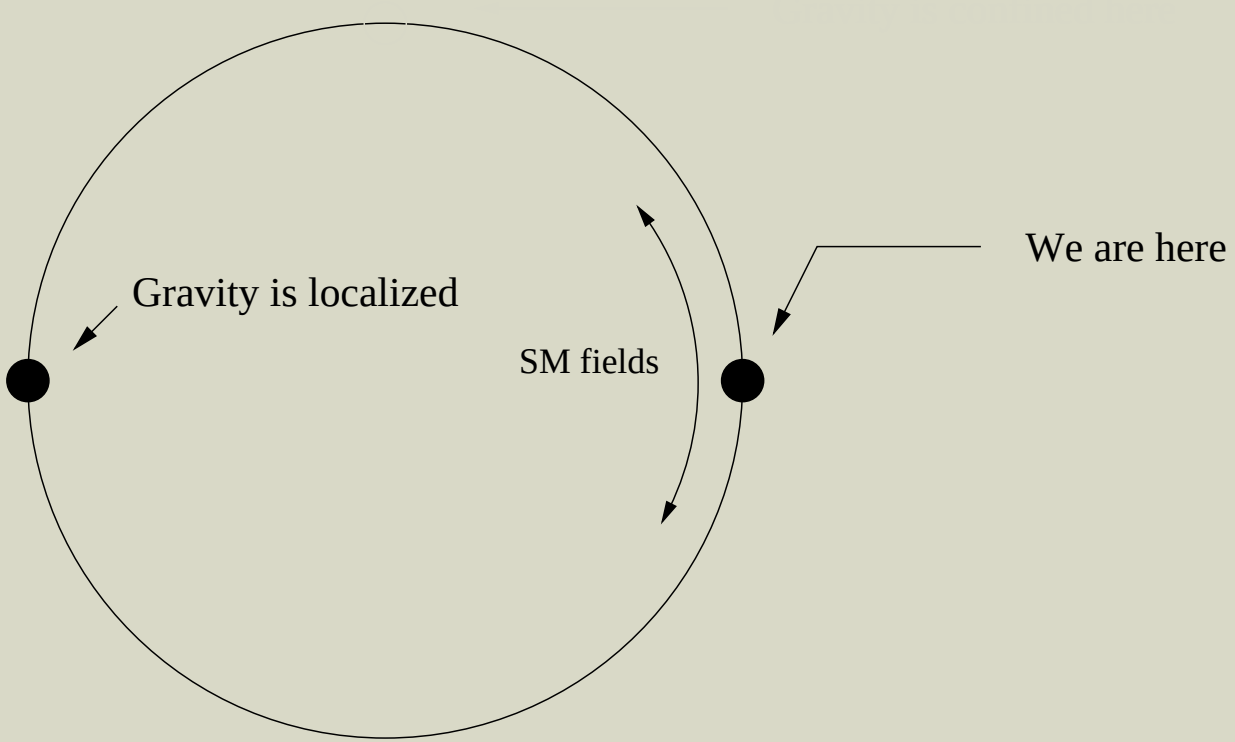
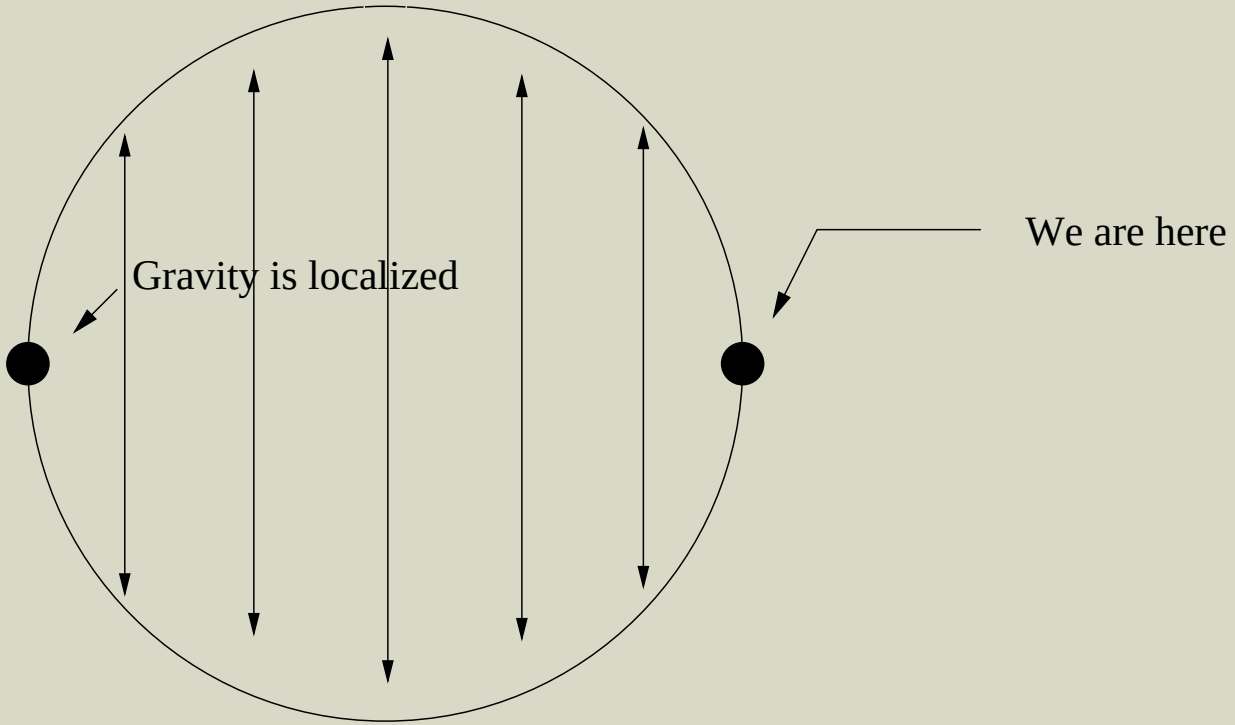
which can be expanded to KK modes

$$\hat{\Psi}(x,y)_{L(R)} = \sqrt{k}\sum_n \psi^{(n)}_{L(R)}(x) f^{(n)}_{L(R)}(y).$$



Parity of RS1 Warped extra dimension $S^1/\mathbb{Z}_2$



Parity of RS1 Warped extra dimension $S^1/\mathbb{Z}_2$

- The 5-th D parity should be conserved.
- Bulk fermion with odd parity cannot have a zero mode.
- The zero mode is chiral (thus massless).
- Chiral fermion is automatically localized on IR boundary.
- All bulk gauge boson have (massless) zero mode.
- (Massless) SM fields can be induced from bulk fields (except the Higgs).

Will KK fields survive experimental bounds?

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Experimental signals

Energy scale is warped down at IR boundary ($y = y_0$).

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The QG scale is about TeV.

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Quark KK mode contribution to M_W/M_Z conflicts with experimental data

Custodial isospin

In 2003, Agashe et.al. suggested that if there is

$SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ gauge symmetry on $S^1/\mathbb{Z}_2 \times \mathbb{Z}_2'$.

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There exist Ads5/CFT conformal dual global $SU(2)$ custodial isospin.

Which protect M_W/M_Z ratio from KK mode contributions.

Agashe, Delgado, May and Sundrum, (2003)

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Two independent parities on each boundary (UV,IR).

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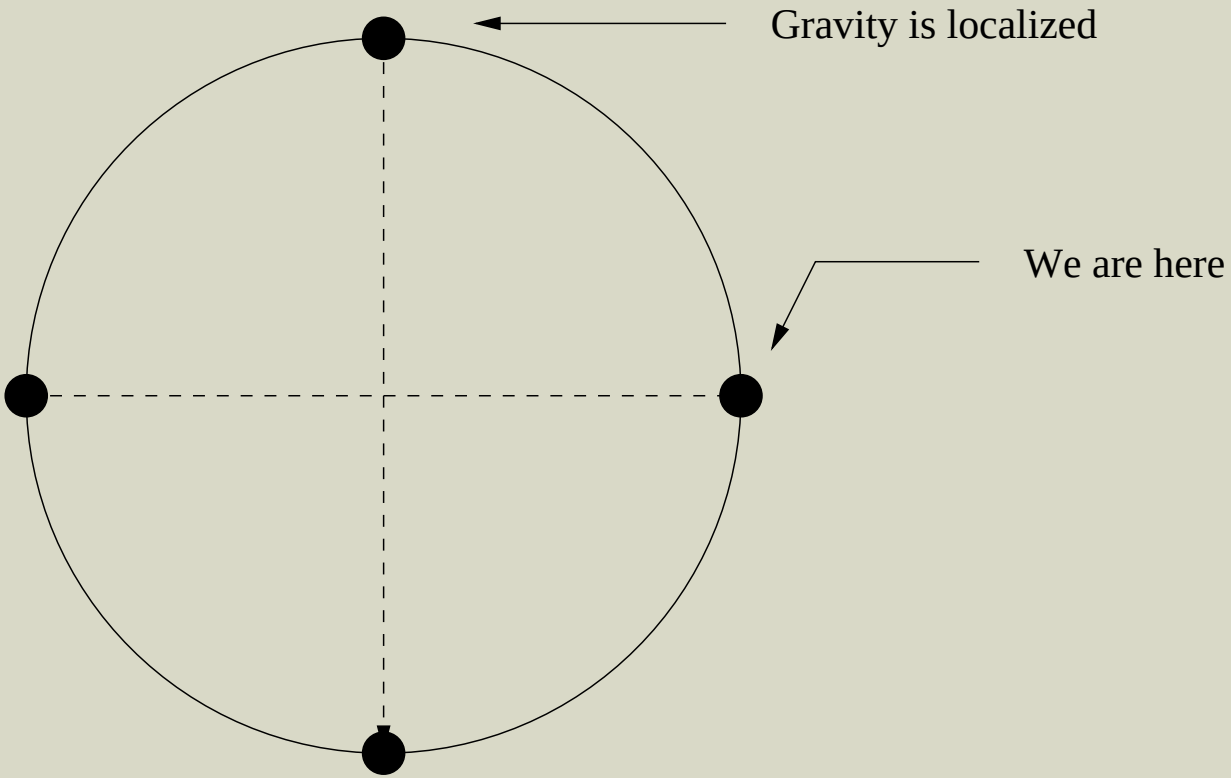
Two independent parities on each boundary (UV,IR).

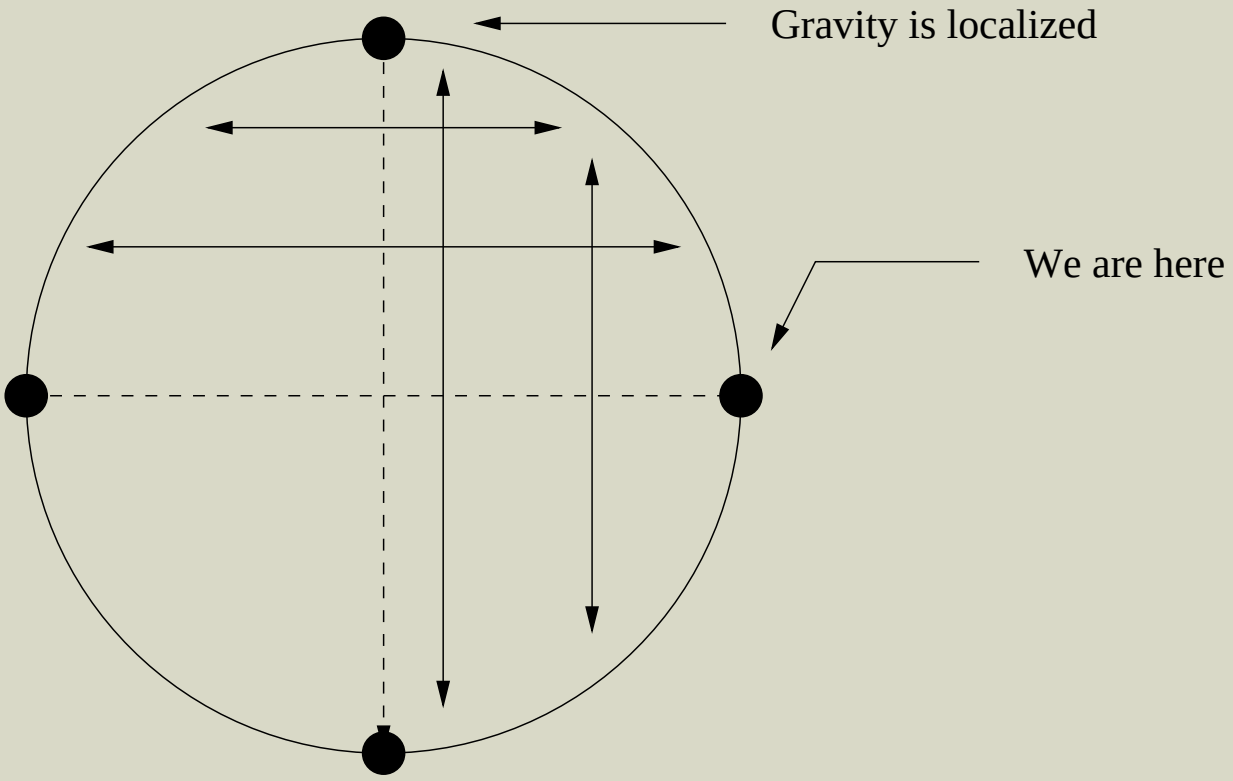
$(\pm, \pm)$ and $(\pm, \mp)$

For bulk fermions, this gives symmetry

$$\gamma_5 \Psi(x, -y) = \pm \Psi(x, y), \qquad \gamma_5 \Psi(x, L-y) = \pm \Psi(x, L+y)$$

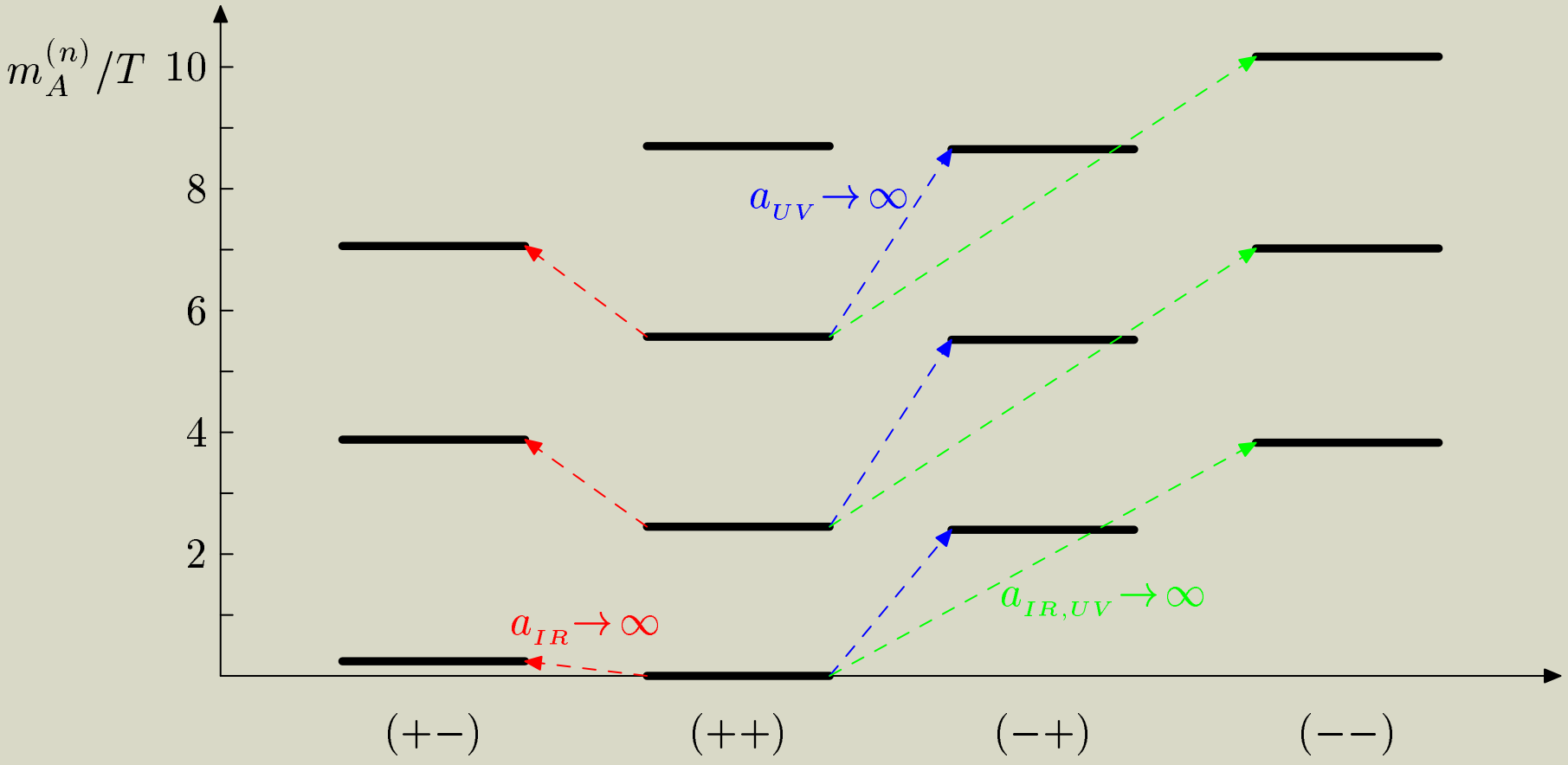


Parities of Agashe et.al. Warped extra dimension $S^1/\mathbb{Z}_2 \times \mathbb{Z}_2'$



Parities of Agashe et.al. Warped extra dimension $S^1/\mathbb{Z}_2 \times \mathbb{Z}_2'$

Gauge boson KK mode mass on $S^1/\mathbb{Z}_2 \times \mathbb{Z}_2'$



Only $(++)$ parity bulk field can have zero mode.

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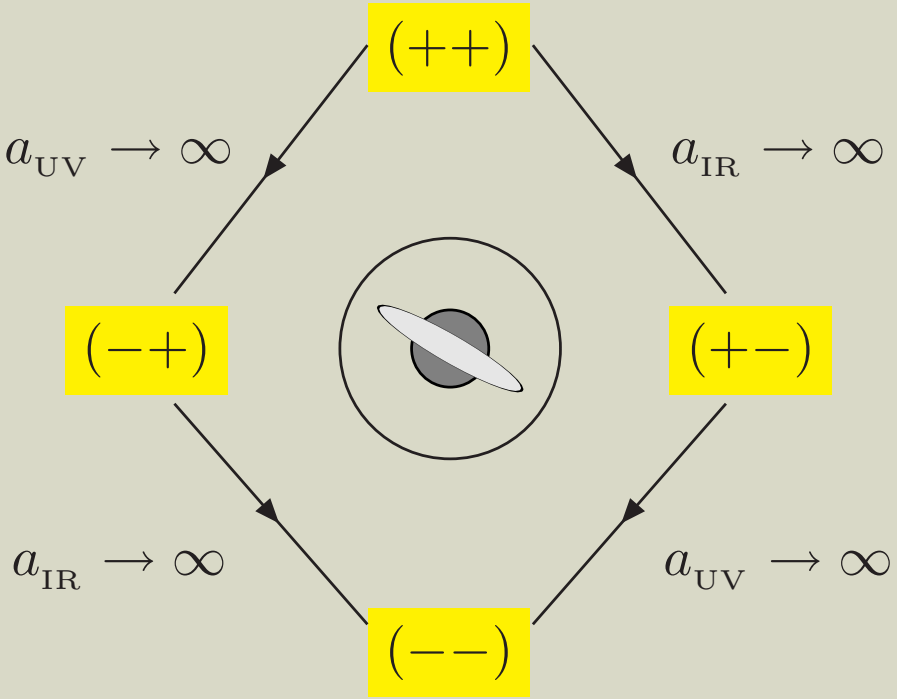
Thus $\mathbf{W_R}$ fields are massive without bulk mass term.

The $(+-)$ field contains a very light KK mode.

If the boundary mass term turns on and start to increase,

each mode mass transform to the other parity

The boundary mass modifies the KK modes masses



KK mass of bulk gauge boson

$$
\begin{aligned}
m^{(1)}_{A(++)} &\approx 2.45\,T, & m^{(2)}_{A(++)} &\approx 5.57\,T, & m^{(3)}_{A(++)} &\approx 8.70\,T,\\
m^{(1)}_{A(+-)} &\approx 0.24\,T, & m^{(2)}_{A(+-)} &\approx 3.88\,T, & m^{(3)}_{A(+-)} &\approx 7.06\,T,\\
m^{(1)}_{A(-+)} &\approx 2.40\,T, & m^{(2)}_{A(-+)} &\approx 5.52\,T, & m^{(3)}_{A(-+)} &\approx 8.65\,T,\\
m^{(1)}_{A(--)} &\approx 3.83\,T, & m^{(2)}_{A(--)} &\approx 7.02\,T, & m^{(3)}_{A(--)} &\approx 10.17\,T.
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If $T \sim$ TeV, $m^{(1)}_{A(+-)}$ can be about 100 GeV. (Without Higgs)

Bulk fermion can have four different $\mathbb{Z}_2 \times \mathbb{Z}_2'$ parity sets,

$$\hat{\Psi}_i(x,y) = \sqrt{k}\sum_n [\psi_{iL}^{(n)}(x) f_{iL}^{(n)}(y) + \psi_{iR}^{(n)}(x) f_{iR}^{(n)}(y)]$$

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$i = 3, 4$ represent the crossed conditions, where $(\pm\mp)$

f_{iL} has $(\pm\mp)$ parity and f_{iR} has $(\mp\pm)$.

$$f_{iL}^{(n)}(z) = \frac{\sqrt{z}}{N_i^{(n)}}\left[J_{c_i+\frac{1}{2}}(m_i^{(n)}z) + \beta_i^{(n)} Y_{c_i+\frac{1}{2}}(m_i^{(n)}z)\right],$$
$$f_{iR}^{(n)}(z) = \frac{\sqrt{z}}{N_i^{(n)}}\left[J_{c_i-\frac{1}{2}}(m_i^{(n)}z) + \beta_i^{(n)} Y_{c_i-\frac{1}{2}}(m_i^{(n)}z)\right].$$

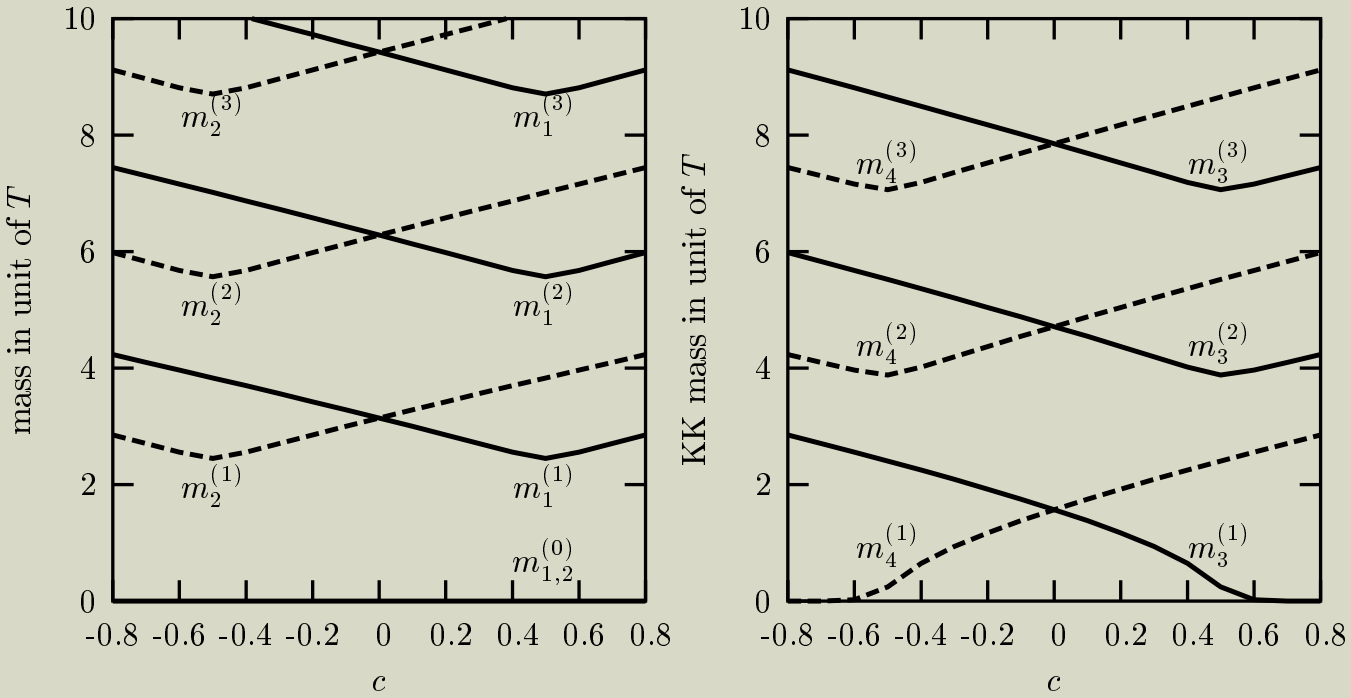

Figure: KK mass spectra of bulk fermion without Higgs in unit of T

Only $(++)$ parity bulk mode function can be a zero mode.

The gauge field with other parity is massive in 4D.

$(+-)$ parity mode has a light gauge boson without Higgs.

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The fermions with other than $(++)$ parity become massive in 4D.

$(+-)$ and $(-+)$ mode can have very light KK modes.

Higgs should be confined on IR boundary

Bulk Higgs scalar brings back hierarchy problem again.

Higgsless Standard Model

If the VEV of Higgs become very large, SM fields

$((++)$ mode) simulate the $(+-)$ KK modes without Higgs field.

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((++) mode) simulate the (+−) KK modes without Higgs field.

Since (+−) has light gauge boson, and can have light fermions

There is a possible Higgsless SM (Higgsless gauge symmetry breaking).

Unfortunately, due to the parity conservation,

one of the fermion doublet component should have zero mass.

Basic assumptions

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All 5D SM parameters are considered $\mathcal{O}(1)$.

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All 5D SM parameters are considered $\mathcal{O}(1)$.

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With this two simple assumptions, we construct a bulk SM in warped RS1.

Universal Yukawa coupling

The SM fermion mass and mixing is decided by

both bulk fermion masses and Yukawa coupling with Higgs.

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There is no hierarchy within Yukawa coupling matrix.

$$y_f \simeq \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}.$$

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c_i is mass of bulk fermion f_i.

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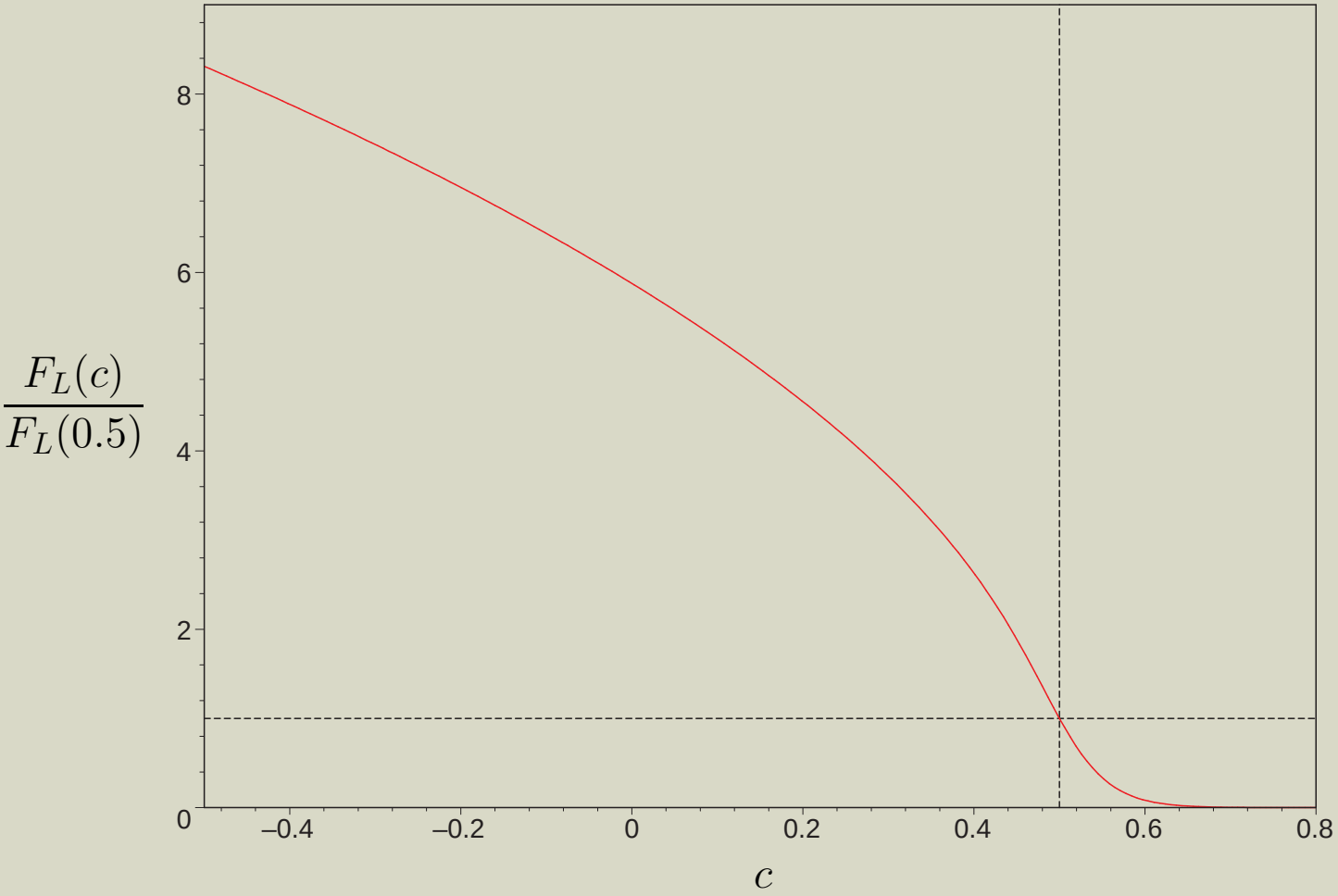
c_i is mass of bulk fermion f_i.

Left-handed and right handed SM fermion comes from different bulk fermions.

$F_L(c)$ as a function of bulk mass c.



If we increase $(-)c_i$, $F_{L(R)}(c_i)$ decrease slowly until $(-)c_i = 1/2$.

For $(-)c_i > 1/2$, it decrease fast in power of $\epsilon^{(-)c_i}$.

Without a large hierarchy in bulk fermion mass c_i,

All SM fermion masses can be generated.

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Without a large hierarchy in bulk fermion mass c_i,

All SM fermion masses can be generated.

If $(-)c_i < 0$, the 1st KK mode can be very light. $m^{(1)} \ll$ TeV.

Model with larger symmetry might contains light neutral and stable KK mode, i.e. WIMPS.

WIMPS are ...

Loading..

Bulk fermions

Q_i contains left handed quark

U_i, D_i contains right handed quark

L_i contains lepton doublet

E_i contains lepton singlet

N_i contains right handed neutrino

Quark mass and mixing

$$(M_a)_{ij} \simeq v_W F_L(c_{Qi}) F_R(c_{Aj}),$$

where $a = u, d$ and $A = U, D$.

$$U_{qL}^T M_q U_{qR} = M_q^{\rm diag} \qquad \text{for } q = u, d.$$

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The CKM matrix is defined as $K = U_{uL}^T U_{dL}$.

$$K \simeq \begin{pmatrix} 1 & \lambda & \lambda^3 \\ \lambda & 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix},$$

For simplified Wolfenstein parameterization and $\lambda \simeq 0.22$.

We also assume that there is no order changing cancellation in matrix diagonalization or other algebras.

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i.e. 1 is a approximation of a number between $\sqrt{\lambda}$ and $1/\sqrt{\lambda}$.

Thus the order λ in matrix will not change after the algebra.

This simple assumption leads to a unique bulk mass structure for SM fermions.

Since u_L and d_L has almost the same matrix form

$$U_{u_L} \simeq U_{d_L} \simeq \begin{pmatrix} 1 & \lambda & \lambda^3 \\ \lambda & 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix}.$$

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Write the quark masses with λ

$$M_u^{\rm diag} = diag(m_u, m_c, m_t) \simeq v_W \; diag(\lambda^8, \lambda^{3.5}, 1),$$
$$M_d^{\rm diag} = diag(m_d, m_s, m_b) \simeq v_W \; diag(\lambda^7, \lambda^5, \lambda^{2.5}).$$

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With known masses of SM fermions,

Following hierarchical relations are obtained:

$$F_L(c_{Q_1}) : F_L(c_{Q_2}) : F_L(c_{Q_3}) \simeq \lambda^3 : \lambda^2 : 1,$$
$$F_R(c_{A_1}) : F_R(c_{A_2}) : F_R(c_{A_3}) \simeq \lambda^3 : \lambda^2 : 1,$$
$$F_R(c_{D_1})^2 + F_R(c_{D_2})^2 + F_R(c_{D_2})^2 \simeq$$
$$\lambda^5 \left[F_R(c_{U_1})^2 + F_R(c_{U_2})^2 + F_R(c_{u_2})^2\right] .$$

The SM quark mixing matrices can be approximated as

$$(U_{qL})_{ij(i\leq j)} \approx \frac{F_L(c_{Q_i})}{F_L(c_{Q_j})}, \quad (U_{qR})_{ij(i\leq j)} \approx \frac{F_R(c_{A_i})}{F_R(c_{A_j})},$$

where $A = U, D$.

From various experimental constraints, (e.g. $Z \to b\bar{b}$)

The bulk quark masses which satisfy all SM parameters can be determined for $T \sim$ TeV.

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The bulk quark masses which satisfy all SM parameters can be determined for $T \sim$ TeV.

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$$c_{Q1} \simeq 0.61, \qquad c_{Q2} \simeq 0.56, \qquad c_{Q3} \simeq 0.3 \pm 0.03.$$

$$c_{U1} \simeq -0.70, \qquad c_{U2} \simeq -0.52, \qquad 0 \lesssim c_{U3} \lesssim 0.2,$$

$$c_{D1} \simeq -0.66, \qquad c_{D2} \simeq -0.61, \qquad c_{D3} \simeq -0.56.$$

Lepton mass matrix

$$U_{MNS} = U_e^\dagger U_\nu,$$

where

$$M_\nu^\dagger M_\nu = U_\nu (M_\nu^{\rm diag})^2 U_\nu^\dagger, \qquad M_e^\dagger M_e = U_e (M_e^{\rm diag})^2 U_e^\dagger.$$

MNS matrix can be approximated as

$$|U_{\rm MNS}| \sim \begin{pmatrix} 1 & 1 & \lambda^m \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

where the experimental constraint on U_{e3} gives $m > 1.3$.

Universal lepton mass matrices are

$$(M_\nu)_{ij} \simeq v_W F_L(c_{Li}) F_R(c_{Nj}),$$
$$(M_e)_{ij} \simeq v_W F_L(c_{Li}) F_R(c_{Ej}).$$

Individual neutrino masses are not yet measured.

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But from the neutrino oscillation

$$\Delta m^2_{sol} = m_2^2 - m_1^2 \simeq 7.5 \times 10^{-5}\ \mathrm{eV}^2,$$
$$\Delta m^2_{atm} = |m_3^2 - m_2^2| \simeq 2.5 \times 10^{-3}\ \mathrm{eV}^2.$$

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Neutrino mass can be either Normal Hierarchy(NH) ($m_1 \simeq 0$)

Or Inverse Hierarchy(IH) ($m_3 = 0$)

Assumption of no order changing cancellation in matrix product allows only NH for neutrino mass.

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$$M_\nu^T M_\nu \propto \begin{pmatrix} \lambda^{2n} & \lambda^n & \lambda^n \\ \lambda^n & 1 & 1 \\ \lambda^n & 1 & 1 \end{pmatrix} \quad \text{(NH)}.$$

Also we parameterize lepton masses

$$M_\nu^{\rm diag} = diag(m_1, m_2, m_3) = v_W \; diag(< \lambda^{20.5}, \lambda^{20.5}, \lambda^{19}),$$
$$M_e^{\rm diag} = diag(m_e, m_\mu, m_\tau) = v_W \; diag(\lambda^{8.5}, \lambda^5, \lambda^3).$$

Maximal mixing between 2 and 3 flavors suggests the mixing

$$U_f \simeq \begin{pmatrix} 1 & \lambda^{a_f} & \lambda^n \\ \lambda^{b_f} & 1 & 1 \\ \lambda^{c_f} & 1 & 1 \end{pmatrix},$$

with $f = e, \nu$.

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with $f = e, \nu$.

To have a desired mass hierarchy,

$$1.5 + a_\nu \gtrsim n, 2 + a_e \gtrsim n.$$

Unlike quark mass, lepton mass has some ambiguity.

With MNS matrix $U_{\rm MNS} = U_e^T U_\nu$

$$\lambda^{a_\nu} \sim \lambda^{b_\nu} + \lambda^{c_\nu} \sim 1, \qquad \lambda^{b_e} + \lambda^{c_e} \lesssim \lambda^m, \qquad \lambda^n \lesssim \lambda^m.$$

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The allowed values of n and m in a narrow range

$$1.3 \lesssim m \lesssim n \lesssim 1.5.$$

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Thus, as a representative value, we expect

$$U_{e3} \simeq \lambda^m \simeq 0.10 - 0.14,$$

which is almost marginal to experimental bound.

Lepton flavor violation

If we focus the KK mass within detectable range $2-3$ TeV,

The Lepton flavor violation(LFV) processes gives a strong bound.

$$\frac{\Gamma(\tau \to 3e)}{\Gamma(\mu \to e\nu_\mu \bar{\nu}_e)} \simeq (K^{(1)}_{L11} K^{(1)}_{L12})^2 \left(\frac{m_Z}{M_A^{(1)}}\right)^4 \lesssim 1.0 \times 10^{-12},$$

$$\frac{\Gamma(\tau \to 3\mu)}{\Gamma(\tau \to \mu\nu_\tau \bar{\nu}_\mu)} \simeq (K^{(1)}_{L22} K^{(1)}_{L23})^2 \left(\frac{m_Z}{M_A^{(1)}}\right)^4 \lesssim 10^{-6},$$

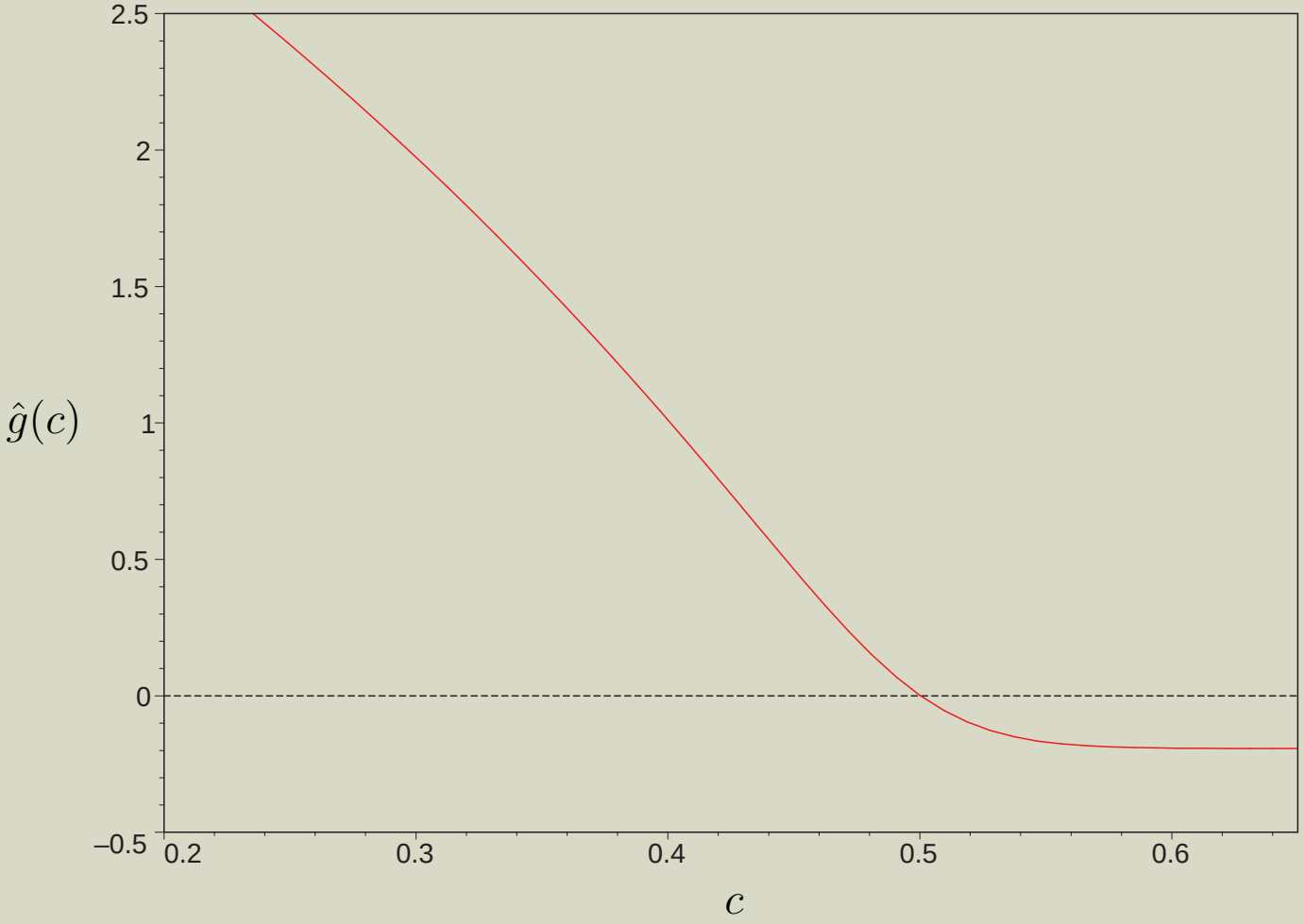
The $K^{(1)}_{Lij}$ factor represents l_i-l_j-$A^{(1)}$ vertex,

$$K^{(n)}_{Lij} = \sum_{k=1}^{3} \left(U_{eL}\right)_{ik} \hat{g}^{(n)}(c_{L_k}) \left(U^{\dagger}_{eL}\right)_{kj} ,$$

where effective coupling of KK gauge mode is,

$$\hat{g}^{(n)}_{L}(c_{f_i}) = \sqrt{kL} \int \mathrm{d}zk \left[f^{(0)}_{L}(z, c_{f_i})\right]^2 f^{(n)}_{A}(z) \equiv \hat{g}^{(n)}(c_{f_i}).$$

$\hat{g}^{(n)}(c)$ saturates around -0.2 if $c > 0.55$.



if KK gauge boson is experimentally testable, i.e. $M_A^{(1)} \simeq$ 2-3 TeV, we can specify its mixing matrix

$$U_{eL} \simeq \begin{pmatrix} 1 & \delta & \delta \\ < \delta^2 & 1 & 1 \\ \delta & 1 & 1 \end{pmatrix},$$

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where $\delta = \lambda^m \simeq 0.1$, and bulk lepton masses (SC, Kim, Song)

$$c_{L1} \simeq 0.59, \qquad c_{L2} \simeq 0.5, \qquad c_{L3} \simeq 0.5,$$

$$c_{E1} \simeq -0.74, \qquad c_{E2} \simeq -0.65, \qquad c_{E3} \simeq -0.55.$$

$$c_{N2} \simeq -1.2, \qquad c_{N3} \simeq -1.1.$$

Is there experimental application of this model?

Theoretical or Experimental?

Loading..

$B_q^0 - \bar{B}_q^0$ mixing data

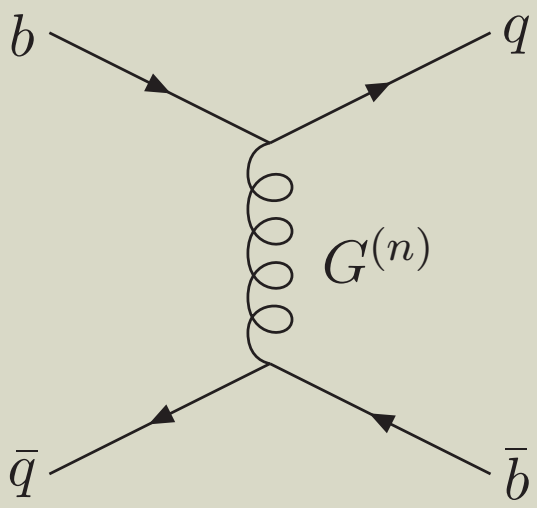
The current experimental results are well measured

$$\Delta M_d^{\rm exp} = (0.507 \pm 0.004)\,{\rm ps}^{-1}\,,$$
$$\Delta M_s^{\rm exp} = \left[17.33^{+0.42}_{-0.21}({\rm stat}) \pm 0.07({\rm syst})\right]\,{\rm ps}^{-1}\,.$$

There is no tree level contribution to $B_q^0 - \bar{B}_q^0$ in SM.

Can be a good test for the RS bulk SM.

$B_q^0 - \bar{B}_q^0$ mixing at tree level by bulk gluon



The KK gluon contribution to $B_q^0 - \bar{B}_q^0$ is tree level while SM contribution is from box diagram.

For the $B_q^0 - \bar{B}_q^0$ transition amplitude $M_{b\bar{b}}^q$ defined by

$$\langle B_q^0 | \mathcal{H}_{\rm eff}^{\Delta B=2} | \bar{B}_q^0 \rangle = 2 M_{B_q} M_{b\bar{b}}^q,$$

where

$$\Delta M_q = 2\,|M_{b\bar{b}}^q|.$$

"mixing-induced" CP violation by phase

$$\phi_q = \arg\left(M_{b\bar{b}}^q\right).$$

$B_q^0 - \bar{B}_q^0$ mixing from the SM box diagrams and the RS KK gluons:

$$M_{b\bar{b}}^q = M_{b\bar{b}}^{q,\mathrm{SM}} \left(1 + \frac{M_{b\bar{b}}^{q,\mathrm{RS}}}{M_{b\bar{b}}^{q,\mathrm{SM}}} \right).$$

We parameterize the new physics effect by

$$r_q e^{i\sigma_q} = \frac{M_{b\bar{b}}^{q,\mathrm{RS}}}{M_{b\bar{b}}^{q,\mathrm{SM}}},$$

where $r_q \geq 0$ and σ_q is real.

The r_q and σ_q are constrained by the experimental result for ΔM_q and the theoretical calculation of the $\Delta M_q^{\rm SM}$,
of which the ratio is defined by ρ_q:

$$\rho_q \equiv \left|\frac{\Delta M_q}{\Delta M_q^{\rm SM}}\right| = \left|\frac{M_{b\bar{b}}^{q,\rm SM} + M_{b\bar{b}}^{q,\rm RS}}{M_{b\bar{b}}^{q,\rm SM}}\right| = \sqrt{1 + 2r_q \cos\sigma_q + r_q^2}.$$

ϕ_d can be divided into the SM phase and that from New Physics (NP) contributions.

$$\phi_q = \phi_q^{\rm SM} + \phi_q^{\rm NP} = \phi_q^{\rm SM} + \arg(1 + r_q e^{i\sigma_q}).$$

$\phi_q^{\rm NP}$ is determined by r_q and σ_q.

$$\begin{aligned} \sin\phi_q^{\rm NP} &= \frac{r_q \sin\sigma_q}{\sqrt{1 + 2r_q\cos\sigma_q + r_q^2}}, \\ \cos\phi_q^{\rm NP} &= \frac{1 + r_q\cos\sigma_q}{\sqrt{1 + 2r_q\cos\sigma_q + r_q^2}}. \end{aligned}$$

SM amplitude is

$$M_{b\bar{b}}^{q,\mathrm{SM}} = \frac{G_F^2 m_W^2}{12\pi^2} M_{B_q} \hat{\eta}^B \hat{B}_{B_q} f_{B_q}^2 (V_{tq}^* V_{tb})^2 S_0(x_t)\,,$$

where $x_t = m_{top}^2/m_W^2$, and S_0 is an "Inami–Lim" function.

CKM components are

$$|V_{td}^* V_{tb}| = (8.6 \pm 1.3) \times 10^{-3}\,, |V_{ts}^* V_{tb}| = (41.3 \pm 0.7) \times 10^{-3}.$$

Short-distance QCD correction $\hat{\eta}^B = 0.552$

$\hat{B}_{B_{d,s}} f_{B_{d,s}}^2$ is determined from lattice simulation by JLQCD collaboration.

New physics effect becomes

$$r_q e^{i\sigma_q} \equiv \frac{M_{b\bar{b}}^{q,\mathrm{RS}}}{M_{b\bar{b}}^{q,\mathrm{SM}}} = \frac{16\pi^2}{N_C}\frac{8g_s^2}{g^4 S_0(x_t)} m_W^2 \kappa_{33}^2 \kappa_{q3}^2 \sum_{n=1} \left(\frac{\hat{g}^{(n)}(c_{Q_3})}{m_A^{(n)}} \right)^2 .$$

$\mathcal{O}(1)$ ambiguity and phases in mixings are absorbed in $\sqrt{\lambda} < \kappa_{ij} < \frac{1}{\sqrt{\lambda}}$.

$$(U_{qL})_{ij} = \kappa_{ij} V_{ij}^{\mathrm{CKM}},$$

ϕ_d associated with $B_d^0 - \bar{B}_d^0$ is well measured,

$$(\sin\phi_d)_{cc\bar{s}} = \sin(2\beta + \phi_d^{\rm NP}) = 0.687 \pm 0.032,$$

where $\phi_d^{\rm SM} = 2\beta$.

$\phi_d^{\rm NP}$ is estimated by (Ball and Fleischer, 2006)

$$\phi_d^{\rm NP}\big|_{\rm incl} = -(10.1 \pm 4.6)^\circ, \quad \phi_d^{\rm NP}\big|_{\rm excl} = -(2.5 \pm 8.0)^\circ.$$

New Physics parameters are constrained by the CP phases.

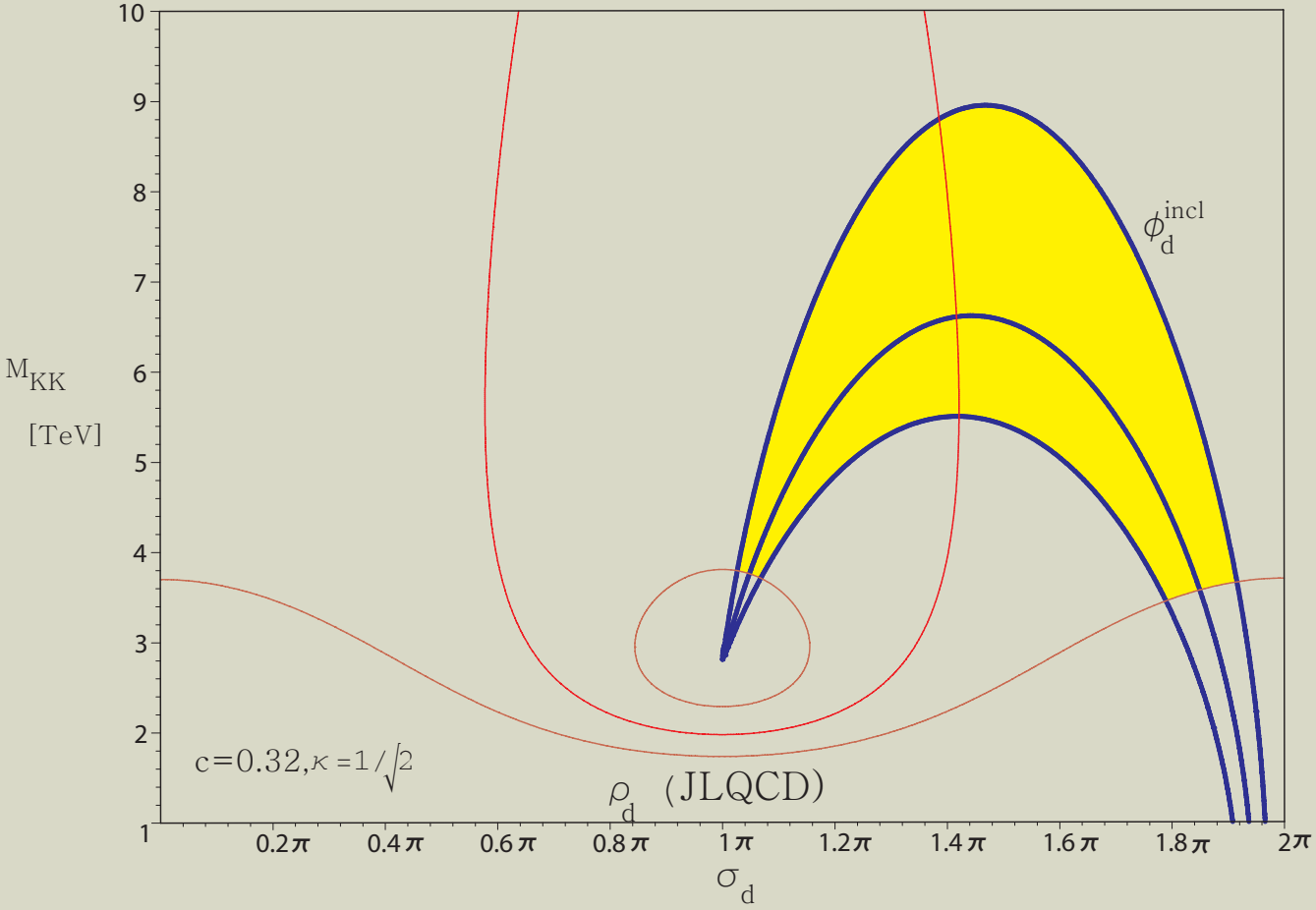


Figure: Allowed parameter space of $(\sigma_d,\ M_{KK})$. Red lines satisfy the observed ρ_d and blue lines for $\phi_d^{\rm NP}|_{\rm incl}$, with 1σ uncertainty.

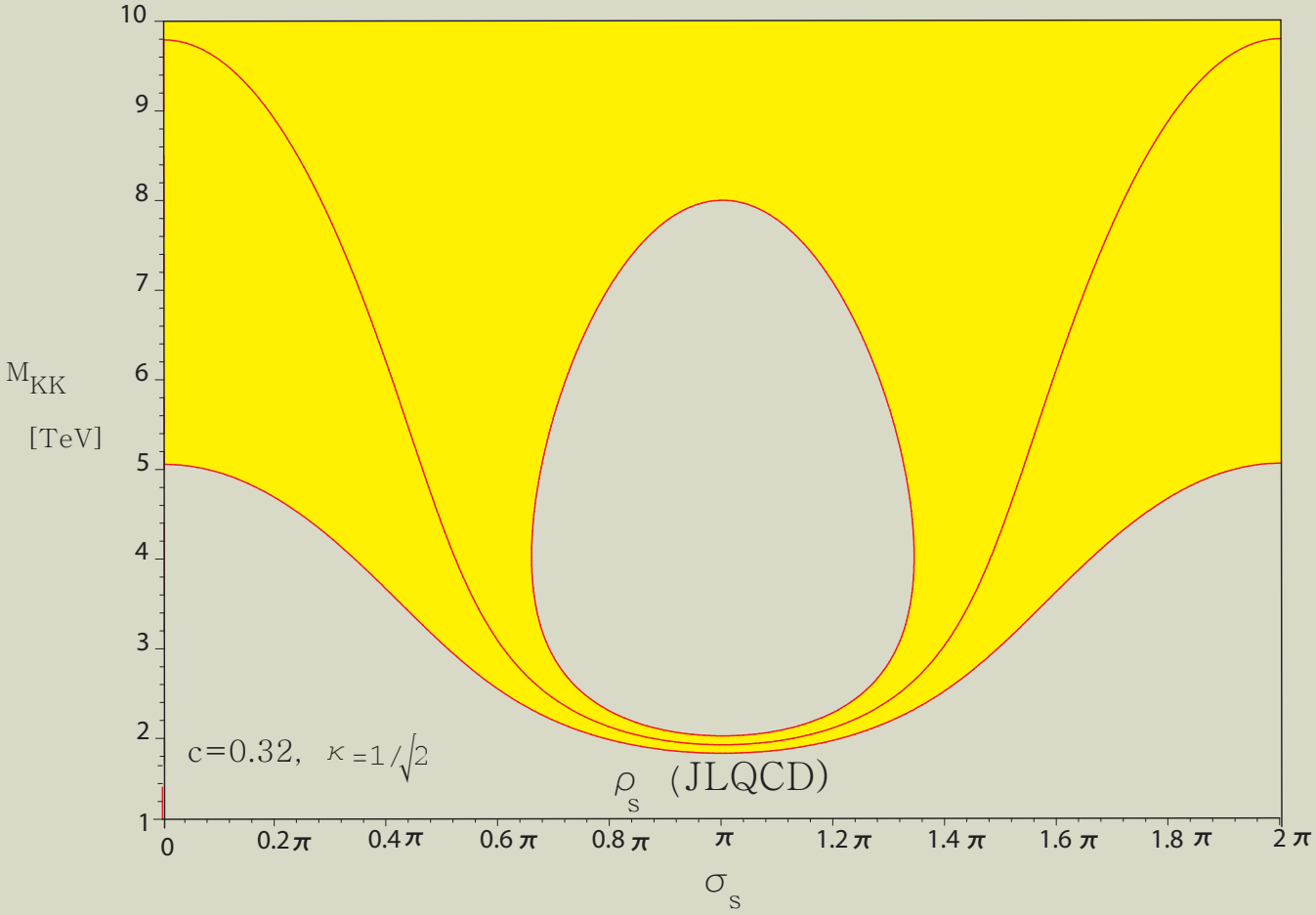


Figure: Allowed parameter space of $(\sigma_s,\ M_{KK})$ from the observed ΔM_s.

Results

For the choice of optimistic parameters for bulk SM,

$$c_{Q3} = 0.32, \quad \kappa = 1/\sqrt{2}$$

$B_d^0 - \bar{B}_d^0$ bounds allow KK mode mass as low as 3 TeV.

B_s mixing gives a weaker bound than B_d mixng.

Summary

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Summary

1. SM field can be resides on $S^1/\mathbb{Z}_2 \times \mathbb{Z}_2'$ bulk space.
2. Fermion bulk masses Can be estimate from SM parameters.
3. In universal Yukawa coupling model, $U_{e3} \sim 0.1$ in the neutrino mixing matrix.
4. $B_d^0 - \bar{B}_d^0$ constrains 1st KK mode $M_A^{(1)} \gtrsim 3$ TeV.

The last one is rather disappointing, since it will be impossible to detect for LC and will be hard for LHC.

Credit for the movie files

- NBC TV series *Law & Order*

  Episode : Big Bang

- WB TV series *Angel*

  Episode : Supersymmetry