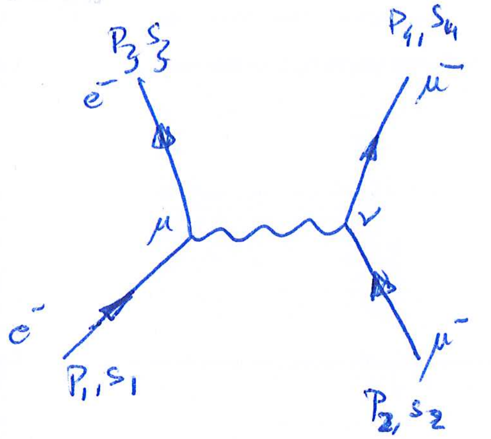


$$\mathcal{M}_{fi} = (-ie)^2\, \bar{u}^{s_3}_{p_3} \gamma^\mu u^{s_1}_{p_1}\ \bar{u}^{s_4}_{p_4} \gamma^\nu u^{s_2}_{p_2} \left(\frac{-i\eta_{\mu\nu}}{(p_1-p_3)^2} \right)$$

Non-relativistic

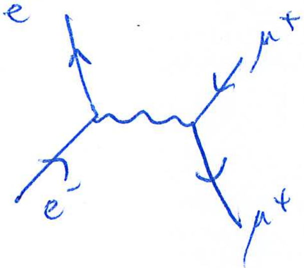
$$u = \sqrt{2m} \begin{pmatrix} \xi \\ \xi \end{pmatrix} \qquad \bar{u}^{s_3}_{p_3} \gamma^\mu u^{s_1}_{p_1} = 2m \begin{pmatrix} \xi^{+(s_3)} & \xi^{+(s_3)} \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & \sigma^\mu \\ \bar{\sigma}^\mu & 0 \end{pmatrix} \begin{pmatrix} \xi^{s_1} \\ \xi^{s_1} \end{pmatrix}$$

$$= 2m \left(\xi^{+(s_3)} \bar{\sigma}^\mu \xi^{s_1} + \xi^{+(s_3)} \sigma^\mu \xi^{s_1} \right)$$

$$= 2m\, \xi^{+(s_3)} \left(\bar{\sigma}^\mu + \sigma^\mu \right) \xi^{s_1} = 2m\, \delta^{s_3 s_1} \delta^{\mu 0}$$

$$\mathcal{M}_{fi} \underset{\substack{p \\ \text{non-rel}}}{\simeq} (-ie)^2\, 4 m_e m_\mu\, \delta^{s_3 s_1} \delta^{s_4 s_2} \frac{(+i)}{+(\vec{p}_1 - \vec{p}_3)^2}$$

$$V = \frac{e^2}{r}$$



$$\bar{v}^{s_4}_{p_4} \gamma^\nu v^{s_2}_{p_2} = 2m \begin{pmatrix} \xi^{+(s_4)} & -\xi^{+(s_4)} \end{pmatrix} \begin{pmatrix} \bar{\sigma}^\mu & 0 \\ 0 & \sigma^\mu \end{pmatrix} \begin{pmatrix} \xi^{s_2} \\ -\xi^{s_2} \end{pmatrix}$$

= same sign.

But $\langle p_3\ p_4 |\ \bar{\psi}_\mu \gamma^\nu \psi_\mu\ \bar{\psi}_e \gamma^\mu \psi_e | p_1\ p_2 \rangle$ ← gives (−) sign.

attractive,

Ultra-relativistic.

R $\varepsilon_1 \to \infty \qquad (s \to \infty)$

$$u_{p_1}^{s_1} = \begin{pmatrix} e^{\beta\sigma/2} & 0 \\ 0 & e^{-\beta\sigma/2} \end{pmatrix} \begin{pmatrix} \xi^{(s)} \\ \xi^{(s)} \end{pmatrix} \sqrt{2m}$$

$$e^{\beta\sigma/2} = \operatorname{ch}\beta/2 + \operatorname{sh}\beta/2 \;\hat{\beta}.\vec{\sigma} \underset{\beta\to\infty}{\simeq} \frac{1}{2} e^{\beta/2} \,(1+\hat{\beta}.\vec{\sigma})$$

$$u_{p_1}^{s_1} = e^{-\beta\sigma/2} = \operatorname{ch}\beta/2 - \operatorname{sh}\beta/2 \;\hat{\beta}.\vec{\sigma} \simeq \frac{1}{2} e^{\beta/2} (1-\hat{\beta}.\vec{\sigma})$$

$$u_{p_1}^{s_1} \simeq \frac{1}{2} e^{\beta/2} \begin{pmatrix} 1+\hat{\beta}.\vec{\sigma} & 0 \\ 0 & 1-\hat{\beta}\vec{\sigma} \end{pmatrix} \begin{pmatrix} \xi^{(s)} \\ \xi^{(s)} \end{pmatrix}$$

take $(\hat{\beta}\vec{\sigma})\,\xi^{(1)} = 1 \qquad (\hat{\beta}\vec{\sigma})\,\xi^{(2)} = -1$

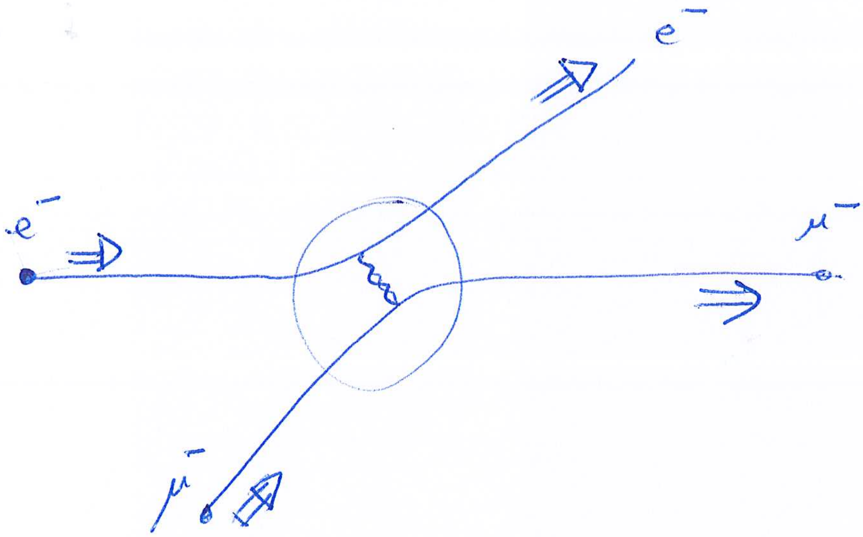
$$u_{p_1}^{(1)} \simeq [redacted]\, e^{\beta/2} \begin{pmatrix} \xi^{(1)} \\ 0 \end{pmatrix} \qquad u_{p_1}^{(2)} \simeq e^{\beta/2} \begin{pmatrix} 0 \\ \xi^{(2)} \end{pmatrix}$$

up and down in dir. of mom.

rot: $\begin{pmatrix} e^{i\frac{\hat{\theta}}{2}\vec{\sigma}} & 0 \\ 0 & e^{+i\frac{\hat{\theta}}{2}\vec{\sigma}} \end{pmatrix}$

$$\bar{u}_{p_3}^{s_3} \gamma^\mu u_{p_1}^{s_1} = 2m \begin{pmatrix} \xi^{\dagger(1)} & 0 \end{pmatrix} \begin{pmatrix} \bar{\sigma}^\mu & 0 \\ 0 & \sigma^\mu \end{pmatrix} \begin{pmatrix} \xi^{(1)} \\ 0 \end{pmatrix} = 2m\, \xi^{\dagger(1)} \bar{\sigma}^\mu \xi^{(1)}$$

but $2m \begin{pmatrix} 0 & \xi^{\dagger(2)} \end{pmatrix} \begin{pmatrix} \bar{\sigma}^\mu & 0 \\ 0 & \sigma^\mu \end{pmatrix} \begin{pmatrix} \xi^{(1)} \\ 0 \end{pmatrix} = 0$ helicity is conserved!!



at high energies helicity is conserved. (or polarization)

at low " spin " "

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