P2 (1)

$$\Delta_E = \int \frac{d^d p}{(2\pi)^d} \frac{e^{ipx}}{p^2+m^2} = \int_0^\infty d\alpha \int \frac{d^d p}{(2\pi)^d} e^{ipx-\alpha p^2-\alpha m^2}$$

$$= \int_0^\infty d\alpha \frac{\pi^{d/2}}{\alpha^{d/2}} \frac{1}{(2\pi)^d} e^{-\frac{x^2}{4\alpha} - \alpha m^2} \quad ; \quad \alpha \to \alpha/m^2$$

$$= \frac{(m^2)^{-1+\frac{d}{2}}}{(4\pi)^{d/2}} \int_0^\infty d\alpha \, \alpha^{-d/2} e^{-\alpha - \frac{m^2x^2}{4\alpha}} = \frac{m^{d-2}}{(4\pi)^{d/2}} 2\left(\frac{mx}{2}\right)^{1-\frac{d}{2}} K_{\frac{d}{2}-1}(mx)$$

$$m^{1-d/2+d-2} = m^{d/2-1}$$

$$K_\nu(z) = \frac{1}{2}\left(\frac{z}{2}\right)^\nu \int_0^\infty \frac{e^{-t-\frac{z^2}{4t}}}{t^{\nu+1}} dt \quad ; \quad \nu = \frac{d}{2}-1 \quad ; \quad z^2 = x^2m^2$$

$|\arg z| < \frac{\pi}{2}$ ✓ $\quad$ $\mathrm{Re}\, z^2 > 0$ ✓

$$\Delta_E(x) = \frac{2^{\cancel{1-1}+d/2-d}}{\pi^{d/2}} m^{d/2-1} x^{1-d/2} K_{\frac{d}{2}-1}(mx)$$

$$\boxed{\Delta_E(x) = \frac{1}{(2\pi)^{d/2}} \left(\frac{m}{x}\right)^{\frac{d}{2}-1} K_{\frac{d}{2}-1}(mx)}$$

$$\boxed{\begin{array}{ll} K_\nu(z) \sim \sqrt{\frac{\pi}{2z}}\, e^{-z} & |z| \to \infty \\ K_n(z) \sim \frac{1}{z^n} & z \to 0. \end{array}}$$

$$\Delta_E(x\to\infty) \simeq \frac{1}{(2\pi)^{d/2}} \left(\frac{m}{x}\right)^{\frac{d}{2}-1} \sqrt{\frac{\pi}{2mx}}\, e^{-mx} \sim m^{\frac{d}{2}-\frac{3}{2}} \frac{e^{-mx}}{x^{\frac{d-1}{2}}}$$

$$\Delta_E(x\to 0) \approx m^{d/2-1} x^{-d/2+1} (mx)^{-\frac{d}{2}+1} \sim x^{-d+2}$$

$$\Delta_E(m\to 0) \sim x^{-d+2}$$

$$\Delta_F = \int \frac{d^d p}{(2\pi)^d} \frac{i e^{ipx}}{p^2 - m^2 + i\epsilon}$$

$$= \int \frac{d^d p}{(2\pi)^d} \int_0^\infty d\alpha \; e^{-(\epsilon + ip^2 + im^2)\alpha + ipx}$$

$e^{-i\pi/2}$ $e^{i\pi/2}$

$-(-ip_0^2 + ip^2)\alpha$

$$= \int_0^\infty d\alpha \frac{e^{-\epsilon\alpha - im^2\alpha}}{(2\pi)^d} \frac{\pi^{d/2}}{\alpha^{d/2}} e^{-\frac{i\pi}{4}(d-1) + i\frac{\pi}{4}} e^{+\frac{t^2}{+4i\alpha} - \frac{x^2}{4i\alpha}}$$

$$= \frac{1}{(4\pi)^{d/2}} e^{-\frac{i\pi}{4}(d-2)} \int_0^\infty d\alpha \; \alpha^{-d/2} e^{-\epsilon\alpha - im^2\alpha - \frac{i}{4\alpha}(t^2 - x^2)}$$

$$H_\nu^{(2)}(xz) = \frac{i}{\pi} e^{\frac{i}{2}\nu\pi} z^\nu \int_0^\infty e^{-\frac{ix}{2}\left(t + \frac{z^2}{t}\right)} t^{-\nu-1} dt$$

$x>0$, real z real $\mathrm{Re}\,\nu > -1$, ν real.

$-\nu - 1 = -d/2$ $\boxed{\nu = -1 + d/2 > -1}$ ✓ $x = 2m^2$ $z^2 = \frac{t^2 - x^2}{4m^2} = \frac{s^2}{4m^2}$

$t^2 > x^2$ $z = s/2m$

$$\underset{t^2 > x^2}{\Delta_F =} \frac{1}{(4\pi)^{d/2}} e^{-\frac{i\pi}{4}(d-2)} (-i\pi) e^{-\frac{i}{2}\pi(-1 + d/2)} \left(\frac{s}{2m}\right)^{1-d/2} H^{(2)}_{-1+\frac{d}{2}}(ms)$$

$-\frac{i\pi}{4}(-2+d)$

$-\frac{i\pi}{4}(d-2) - \frac{i\pi}{4}(d-2) = -\frac{i\pi}{2}(d-2)$

$$\Delta_F = -\frac{i\pi}{(4\pi)^{d/2}} e^{-\frac{i\pi}{2}(d-2)} \cancel{2}^{d/2 - 1} \left(\frac{m}{s}\right)^{\frac{d}{2}-1} H^{(2)}_{\frac{d}{2}-1}(ms)$$

$$= \cancel{-} \frac{i\pi}{2\pi^{d/2}} e^{-\frac{i\pi d}{2}} \cancel{e^{i\pi}} \left(\frac{m}{s}\right)^{\frac{d}{2}-1} H^{(2)}_{\frac{d}{2}-1}(ms) = \frac{i}{2} e^{-\frac{i\pi d}{2}} \left(\frac{m}{\pi s}\right)^{\frac{d}{2}-1} H^{(2)}_{\frac{d}{2}-1}(ms)$$

③

$$\Delta_F \underset{t^2 > x^2}{=} \frac{i}{2} e^{-\frac{i\pi d}{2}} \left(\frac{m}{\pi s}\right)^{\frac{d}{2}-1} H^{(2)}_{\frac{d}{2}-1}(ms)$$

$s \to \infty$ e.g. ($t \to \infty$ $|\vec{x}|$ fixed).

up to constant

$$\Delta_F \sim \frac{m^{d/2-1}}{s^{d/2-1}} \frac{1}{\sqrt{ms}} e^{-ims} = \frac{m^{d/2-\frac{3}{2}}}{s^{\frac{d-2}{2}}} e^{-ims}$$

$s \to 0$ ~~[illegible]~~ (near light-cone).

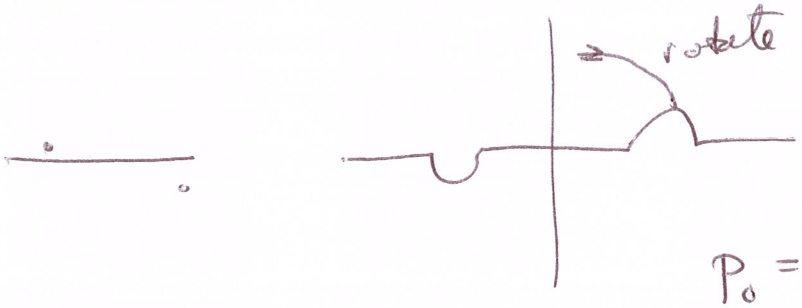
$$\Delta_F \sim \frac{m^{d/2-1}}{s^{d/2-1}} (ms)^{-\frac{d}{2}+1} \sim s^{-d+2}$$

same as $m \to 0$.

$\Delta_F \quad (t^2 < x^2)$

$$\Delta_F = \int \frac{d^dp}{(2\pi)^d} \, \frac{i\, e^{ip_0 t - i\vec{p}\vec{x}}}{p_0^2 - \vec{p}^2 - m^2 + i\epsilon}$$

$$p_0 = \pm\sqrt{\omega_p^2 - i\epsilon} = \omega_p - i\epsilon, \quad -\omega_p + i\epsilon$$



$p_0 = i p_4 \qquad t = i x_4$

$$\int i\, dp_4 \, \frac{i\, e^{i\, i p_4 t - i\vec{p}\vec{x}}}{-p_4^2 - \vec{p}^2 - m^2} = \int dp_4 \, \frac{e^{-ip_4 x_4 - i\vec{p}\vec{x}}}{p_4^2 + \vec{p}^2 + m^2}$$

$s^2 = t^2 - \vec{x}^2 = -x_4^2 - \vec{x}^2 < 0$ ← outside light cone.

$\Delta_F(t, x) = \Delta_E(i x_0, \vec{x})$ outside light-cone

$x^2 \to -s^2$

$$\Delta_F(t, x) = \frac{1}{(2\pi)^{d/2}} \left(\frac{m}{\sqrt{-s^2}}\right)^{\frac{d}{2}-1} K_{\frac{d}{2}-1}\left(\sqrt{-m^2 s^2}\right)$$

outside light cone.

Limits are the same as for Euclidean.

P3 (1)

$$Z = \int \mathcal{D}\sigma \mathcal{D}\rho \mathcal{D}\lambda \, e^{-N\int \frac{1}{2}\partial_\mu \sigma \partial^\mu \sigma + \frac{1}{2} r\rho + \frac{\bar{u}}{4!}\rho^2 + \frac{1}{2}\lambda\sigma^2 - \frac{1}{2}\lambda\rho}$$

$$e^{-\frac{N-1}{2} \mathrm{Tr}\ln(-\partial^2+\lambda) \cancel{+} \int H\sigma}$$

e.o.m.)

$$\partial^2\sigma = \lambda\sigma \cancel{+} H$$

$$\frac{1}{2}r + \frac{\bar{u}}{12}\rho - \frac{1}{2}\lambda = 0$$

$$\frac{1}{2}\sigma^2 - \frac{1}{2}\rho + \frac{N-1}{2}\frac{\delta}{\delta\lambda}\mathrm{Tr}\,\ln(-\partial^2+\lambda) = 0$$

If we look for constant solutions

$$\lambda\sigma = +H \quad ; \quad \rho = \frac{6}{\bar{u}}(\lambda - r)$$

$$\sigma^2 - \rho + \cancel{N} \int \frac{d^dk}{(2\pi)^d} \frac{1}{k^2+\lambda} = 0$$

$$\sigma^2 - \frac{6}{\bar{u}}(\lambda - r) + \cancel{N} \int \frac{d^dk}{(2\pi)^d} \frac{1}{k^2+\lambda} = 0$$

Before $\lambda\sigma = 0 \rightarrow \lambda = 0$ or $\sigma = 0$

now $\lambda \neq 0$ & $\sigma \neq 0$.

$\sigma = +H/\lambda$

$$\frac{H^2}{\lambda^2} - \frac{6}{\bar{u}}(\lambda - r) + \cancel{N}\int \frac{d^dk}{(2\pi)^d}\frac{1}{k^2+\lambda} = 0.$$

from notes

$\lambda = m^2$ (definition)

$$\frac{H^2}{m^4} - \frac{6}{\bar{u}}(m^2 - r) + \cancel{N}\left(-\frac{6 r_c}{\bar{u}} - \frac{2\Gamma(\frac{4-d}{2})}{(4\pi)^{d/2}(d-2)} m^{d-2}\right) = 0$$

$$\frac{H^2}{m^4} - \frac{6m^2}{\bar{u}} + \frac{6}{\bar{u}}(r - r_c) - \frac{2\Gamma(\frac{4-d}{2})}{(4\pi)^{d/2}(d-2)} m^{d-2} = 0.$$

$\boxed{\sigma = H/m^2} \rightarrow \chi = \frac{1}{m^2} \sim t^{-\frac{2}{d-2}} \Rightarrow \gamma = \frac{2}{d-2}$; $\gamma = 2\nu = \frac{2}{d-2}$ ✓

Suppose $\boxed{r = r_c}$

$$H^2 - \frac{6m^6}{\bar{u}} - \frac{2\Gamma(\frac{4-d}{2})}{(4\pi)^{d/2}(d-2)} m^{2+d} = 0$$

$H \to 0$ $m \to 0$ but $m^6 \ll m^{2+d}$ if $d < 4$

$$\Rightarrow H^2 \sim m^{2+d} \sim m \sim H^{\frac{2}{2+d}} \rightarrow \lambda \sim H^{\frac{4}{2+d}}$$

$$\sigma \sim H^{1-\frac{4}{d+2}} = H^{\frac{d-2}{d+2}} \rightarrow \boxed{\delta = \frac{d+2}{d-2}} = \frac{d+2-\eta}{d-2+\eta}$$

critical exponent ✓ $(\eta = 0)$