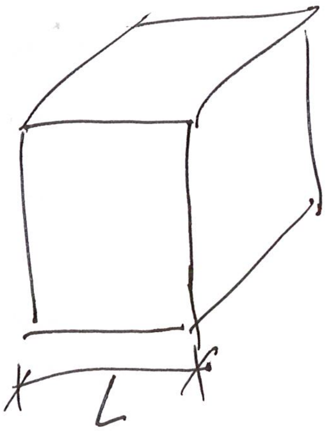


$$V = L^3$$

$$\vec{p} = \frac{2\pi}{L}(n_x, n_y, n_z) \qquad n_j \in \mathbb{Z}.$$

$$\delta^{(3)}(\vec{p}-\vec{p}\,') = \frac{1}{(2\pi)^3}\int_V d^3p\, e^{i(\vec{p}-\vec{p}\,')\vec{x}} \rightarrow \frac{V}{(2\pi)^3}\delta_{\vec{p},\vec{p}\,'}$$

$$|\psi_{in}^{(\alpha)}\rangle = |\varepsilon_1 p_1 \sigma_1; \;-\!-\!-\; \varepsilon_{N_\alpha} p_{N_\alpha} \sigma_{N_\alpha}\rangle$$

$$\langle p_1' | p_1\rangle \underset{\text{Book}}{=} 2\omega_p (2\pi)^3 \delta^{(3)}(p-p') \rightarrow 2\omega_p (2\pi)^3 \frac{V}{(2\pi)^3}\delta_{(\vec{p},\vec{p}\,')}$$

$$|p_1\rangle^{Box} = \left(\frac{\cancel{(2\pi)^3}}{V}\right)^{1/2} \frac{1}{\sqrt{2\omega_p}\,\cancel{(2\pi)^{3/2}}} |p_1\rangle$$

$$\underset{\alpha\neq\beta}{S^{Box}_{\alpha\to\beta}} = \left(\frac{\cancel{(2\pi)^3}}{V}\right)^{\frac{N_\alpha+N_\beta}{2}} \frac{1}{\prod_j \sqrt{2\omega_j}} S_{\alpha\to\beta}$$

$$\delta(E_\alpha - E_\beta) = \frac{1}{2\pi}\int_{-T/2}^{T/2} e^{i(E_\alpha-E_\beta)t}\, dt \rightarrow \delta(0) = \frac{T}{2\pi}$$

$$d^3 n_j = \frac{V}{(2\pi)^3} d^3 p_j$$

$$\prod_{j=1}^{N_\beta} d^3 n_j = \left(\frac{V}{(2\pi)^3}\right)^{N_\beta} \prod_{j=1}^{N_\beta} d^3 p_j$$

Assume prob. smooth.

$$dP(\alpha\to\beta) = P(\alpha\to\beta) \prod_{j=1}^{N_\beta} d^3 n_j$$

$$= \left(\frac{\cancel{V}}{(2\pi)^3}\right)^{N_\beta} \prod_{j=1}^{N_\beta} d^3 p_j \; \frac{1}{V^{N_\alpha \cancel{+N_\beta}}} \; \frac{|S_{\alpha\to\beta}|^2}{\prod_j 2\omega_j}$$

$$S_{\alpha\to\beta} = \mathbb{1}_{\alpha\to\beta} + i(2\pi)^4 \delta^{(4)}(P_f - P_i) \mathcal{M}_{\beta\alpha}$$

↖ Feynman diagrams.

$$dP_{\alpha\to\beta} = \frac{1}{(2\pi)^{3N_\beta}} \frac{1}{V^{N_\alpha}} \frac{1}{\prod_j 2\omega_j} (2\pi)^8 \delta(\varepsilon_f - \varepsilon_i) \frac{T}{2\pi} \delta(\vec{p}_f - \vec{p}_i) \frac{V}{(2\pi)^3} \times |\mathcal{M}_{\beta\alpha}|^2 \prod_{j=1}^{N_\beta} d^3 p_j$$

transition rate.

$$\frac{dP_{\alpha\to\beta}}{T} = d\Gamma_{\alpha\to\beta} = \frac{(2\pi)^4}{(2\pi)^{3N_\beta}} \frac{1}{V^{N_\alpha - 1}} \frac{\delta^{(4)}(P_f - P_i)}{\prod_j 2\omega_j} |\mathcal{M}_{\beta\alpha}|^2 \prod_{j=1}^{N_\beta} d^3 p_j$$

•) Decay rate $N_\alpha = 1$

$$d\Gamma_{\alpha\to\beta} = \frac{1}{(2\pi)^{3N_\beta - 4}} \frac{\delta^{(4)}(P_f - P_i)}{2\omega_i \prod_j 2\omega_j} |\mathcal{M}_{\beta\alpha}|^2 \prod_{j=1}^{N_\beta} d^3 p_j$$

Cross section

$$d\Gamma_{\alpha\to\beta} = \frac{(2\pi)^{4-3N_\beta}}{V} \frac{\delta^4(p_f - p_0)}{\prod_j 2\omega_j} |\mathcal{M}_{\beta\alpha}|^2 \prod_{j=1}^{N_\beta} d^3 p_j$$

$\frac{u_\alpha}{V}$ = flux density × relative velocity.

$$d\sigma_{\alpha\to\beta} = \frac{(2\pi)^{4-3N_\beta}}{u_\alpha} \frac{\delta^{(4)}(p_f - p_0)}{\prod_j 2\omega_j} |\mathcal{M}_{\beta\alpha}|^2 \prod_{j=1}^{N_\beta} d^3 p_j$$

2→2 scattering in CM frame

$(\mathcal{E}_1, \vec{p}_1)$ $(\mathcal{E}_2, -\vec{p}_1)$

$$\vec{v} = \vec{p}/\mathcal{E}$$

$$u_\alpha = |\vec{v}_1 - \vec{v}_2|$$

$$= \left| \frac{\vec{p}_1}{\mathcal{E}_1} + \frac{\vec{p}_1}{\mathcal{E}_2} \right| = |\vec{p}_1| \frac{\mathcal{E}_1 + \mathcal{E}_2}{\mathcal{E}_1 \mathcal{E}_2} = |\vec{p}_1| \frac{E_{cm}}{\mathcal{E}_1 \mathcal{E}_2}$$

$$d\sigma_{\alpha\to\beta} = \frac{1}{(2\pi)^2} \frac{\cancel{\mathcal{E}_1}\cancel{\mathcal{E}_2}}{|\vec{p}_1| E_{cm}} \frac{\delta(\mathcal{E}_3 + \mathcal{E}_4 - \mathcal{E}_1 - \mathcal{E}_2)\, \delta^{(3)}(\vec{p}_3 + \vec{p}_4 - \vec{p}_1 - \vec{p}_2)}{2\cancel{\mathcal{E}_1}\, 2\cancel{\mathcal{E}_2}\, 2\mathcal{E}_3\, 2\mathcal{E}_4} |\mathcal{M}_{\beta\alpha}|^2 d^3p_3\, d^3p_4$$

$$\int d^3 p_4 \to \delta^{(3)} \ldots$$

$$d\sigma_{\alpha\to\beta} = \frac{1}{64\pi^2} \frac{\delta(\varepsilon_3+\varepsilon_4-E_{cm})}{|\vec{p}_1|\, E_{cm}\, \varepsilon_3\varepsilon_4} |\mathcal{M}_{\beta\alpha}|^2 \; p_3^2\, dp_3\, d\Omega$$

$$\varepsilon_3^2 = p_3^2 + m_3^2 \qquad \int d\varepsilon_3 \to \delta \, .$$

$$\int \delta(\varepsilon_3+\varepsilon_4-E_{cm})\, dp_3 = \frac{1}{\frac{\partial\varepsilon_3}{\partial p_3} + \frac{\partial\varepsilon_4}{\partial p_3}} = \frac{1}{p_3/\varepsilon_3 + p_3/\varepsilon_4}$$

$$= \frac{1}{p_3}\frac{\varepsilon_3\varepsilon_4}{\varepsilon_3+\varepsilon_4}$$

$$= \frac{1}{p_3}\frac{\varepsilon_3\varepsilon_4}{E_{cm}}$$

$$\varepsilon_3 d\varepsilon_3 = p_3 dp_3$$

$$\varepsilon_4^2 = p_3^2 + m_4^2 \qquad \varepsilon_4 d\varepsilon_4 = p_3 dp_3$$

$$|\vec{p}_3| = |\vec{p}_4| \qquad \vec{p}_3 = -\vec{p}_4$$

$$\frac{\partial\varepsilon_3}{\partial p_3} = p_3/\varepsilon_3 \qquad \frac{\partial\varepsilon_4}{\partial p_3} = p_3/\varepsilon_4$$

$$d\sigma_{\alpha\to\beta} = \frac{1}{64\pi^2}\frac{1}{p_1 E_{cm} \cancel{\varepsilon_3}\cancel{\varepsilon_4}}\frac{1}{\cancel{p_3}}\frac{\cancel{\varepsilon_3}\cancel{\varepsilon_4}}{E_{cm}}\, p_3^{\cancel{2}}\, |\mathcal{M}_{\beta\alpha}|^2 d\Omega$$

$$\frac{d\sigma_{\alpha\to\beta}}{d\Omega} = \frac{1}{64\pi^2}\frac{|\vec{p}_3|}{|\vec{p}_1|}\frac{1}{E_{cm}^2} |\mathcal{M}_{\beta\alpha}|^2 = \frac{1}{64\pi^2 s}\frac{|\vec{p}_3|}{|\vec{p}_1|} |\mathcal{M}_{\beta\alpha}|^2$$

$$\varepsilon_1^2 = p_1^2 + m_1^2 \qquad \varepsilon_2^2 = p_1^2 + m_2^2$$

$$E_{cm} = \sqrt{p_1^2+m_1^2} + \sqrt{p_1^2+m_2^2}$$

$$s = p_1^2+m_1^2 + p_1^2+m_2^2 + 2\sqrt{p_1^2+m_1^2}\sqrt{p_1^2+m_2^2}$$

$$\Rightarrow (s - 2p_1^2 - m_1^2 - m_2^2) = 2\sqrt{p_1^2+m_1^2}\sqrt{p_1^2+m_2^2}$$

$$s^2 + \cancel{4p_1^4} + (m_1^2+m_2^2)^2 - 4sp_1^2 - 2s(m_1^2+m_2^2) +$$

$$+ \cancel{4(m_1^2+m_2^2)p_1^2} = \cancel{4p_1^4} + \cancel{4p_1^2(m_1^2+m_2^2)} + 4m_1^2m_2^2$$

$$4sp_1^2 = s^2 + (m_1^2-m_2^2)^2 - 2s(m_1^2+m_2^2)$$

$$\boxed{p_1^2 = \frac{s^2+(m_1^2-m_2^2)^2 - 2s(m_1^2+m_2^2)}{4s}}$$

$$\boxed{p_3^2 = \frac{s^2+(m_3^2-m_4^2)^2 - 2s(m_3^2+m_4^2)}{4s}}$$

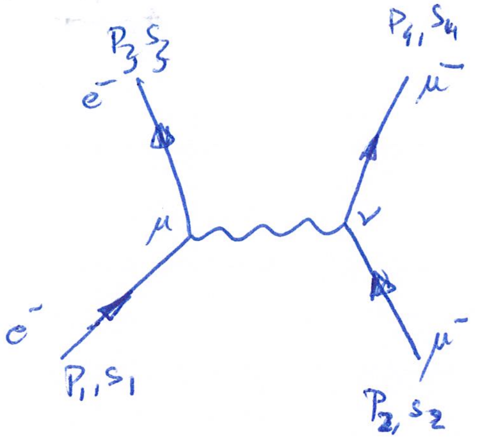
Elastic: $m_1 = m_2 = m_3 = m_4$

Same particles

$$p_1^2 = \frac{\cancel{s^2} - 4sm^2}{4\cancel{s}} = \frac{s-4m^2}{4}$$

$$p_3^2 = \frac{s-4m^2}{4}$$

$$\boxed{\frac{p_1}{p_3} = 1}$$



$$\mathcal{M}_{fi} = (-ie)^2\, \bar u^{s_3}_{p_3} \gamma^\mu u^{s_1}_{p_1}\ \bar u^{s_4}_{p_4} \gamma^\nu u^{s_2}_{p_2} \left(\frac{-i\eta_{\mu\nu}}{(p_1-p_3)^2}\right)$$

Non-relativistic

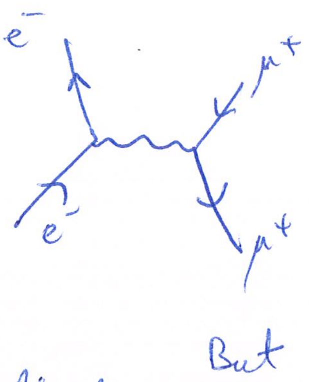
$$u = \sqrt{2m}\begin{pmatrix}\xi\\ \xi\end{pmatrix} \qquad \bar u^{s_3}_{p_3}\gamma^\mu u^{s_1}_{p_1} = 2m\left(\xi^{\dagger(s_3)}\ \ \xi^{\dagger(s_3)}\right)\begin{pmatrix}0&1\\1&0\end{pmatrix}\begin{pmatrix}0&\sigma^\mu\\ \bar\sigma^\mu&0\end{pmatrix}\begin{pmatrix}\xi^{s_1}\\ \xi^{s_1}\end{pmatrix}$$

$$= 2m\left(\xi^{\dagger(s_3)}\bar\sigma^\mu\xi^{s_1} + \xi^{\dagger(s_3)}\sigma^\mu\xi^{s_1}\right)$$

$$= 2m\,\xi^{\dagger(s_3)}\left(\bar\sigma^\mu + \sigma^\mu\right)\xi^{s_1} = 2m\,\delta^{s_3 s_1}\,\delta^{\mu 0}.$$

$$\mathcal{M}_{fi} \underset{\text{non-rel}}{\simeq} (-ie)^2\, 4 m_e m_\mu\, \delta^{s_3 s_1}\,\delta^{s_4 s_2}\, \frac{(+i)}{+(\vec p_1 - \vec p_3)^2}$$

$$V = \frac{e^2}{r}$$



$$\bar v^{s_4}_{p_4}\gamma^\nu v^{s_2}_{p_2} = 2m\left(\xi^{\dagger(s_4)}\ \ -\xi^{\dagger(s_4)}\right)\begin{pmatrix}\bar\sigma^\mu&0\\0&\sigma^\mu\end{pmatrix}\begin{pmatrix}\xi^{s_2}\\ -\xi^{s_2}\end{pmatrix}$$

= same sign.

But $\langle p_3\, p_4 |\, \bar\psi_\mu\gamma^\nu\psi_\mu\ \bar\psi_e\gamma^\mu\psi_e\, | p_1\, p_2\rangle$ ← gives (−) sign.

attractive.

Ultra-relativistic.

$\mathbb{R}$ $\quad \varepsilon_1 \to \infty \qquad (s \to \infty)$

$$u_{p_1}^{s_1} = \begin{pmatrix} e^{\beta\sigma/2} & 0 \\ 0 & e^{-\beta\sigma/2} \end{pmatrix} \begin{pmatrix} \xi^{(s)} \\ \xi^{(s)} \end{pmatrix} \sqrt{2m}$$

$$e^{\beta\sigma/2} = ch\beta/2 + sh\beta/2 \; \hat{\beta}.\vec{\sigma} \underset{\beta\to\infty}{\simeq} \frac{1}{2} e^{\beta/2} \left(1 + \hat{\beta}.\vec{\sigma}\right)$$

$$\cancel{u_{p_1}^{s_1}} = e^{-\beta\sigma/2} = ch\beta/2 - sh\beta/2 \; \hat{\beta}.\vec{\sigma} \simeq \frac{1}{2} e^{\beta/2} \left(1 - \hat{\beta}.\vec{\sigma}\right)$$

$$u_{p_1}^{s_1} \simeq \frac{1}{2} e^{\beta/2} \begin{pmatrix} 1 + \hat{\beta}.\vec{\sigma} & 0 \\ 0 & 1 - \hat{\beta}\vec{\sigma} \end{pmatrix} \begin{pmatrix} \xi^{(s)} \\ \xi^{(s)} \end{pmatrix}$$

take $(\hat{\beta}\vec{\sigma})\,\xi^{(1)} = 1 \qquad (\hat{\beta}\vec{\sigma})\,\xi^{(2)} = -1$

$$u_{p_1}^{(1)} \simeq \text{[redacted]}\, e^{\beta/2} \begin{pmatrix} \xi^{(1)} \\ 0 \end{pmatrix} \qquad u_{p_1}^{(2)} \simeq e^{\beta/2} \begin{pmatrix} 0 \\ \xi^{(2)} \end{pmatrix}$$

up and down in dir. of mom.

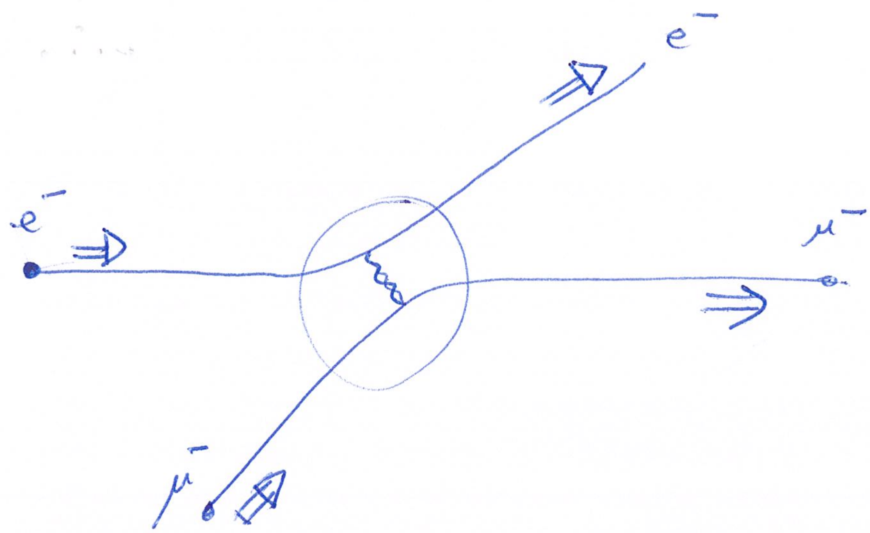
rot: $\begin{pmatrix} e^{i\frac{\hat{\theta}}{2}\vec{\sigma}} & 0 \\ 0 & e^{+i\frac{\hat{\theta}}{2}\vec{\sigma}} \end{pmatrix}$ $\quad |\mathcal{M}|^2 \sim \dfrac{(e^{\beta}.e^{\beta})^2}{(p_1-p_2)^4} = \dfrac{e^{4\beta}}{(p_1-p_3)^4} \sim \dfrac{E^4}{} = s^2/(p_1-p_3)^4$

cancels with $(p_1 - p_3)^4$ $\quad \dfrac{1}{(p_1-p_3)^4}$ denominator

$$\bar{u}_{p_3}^{s_3} \gamma^\mu u_{p_1}^{s_1} = 2m \begin{pmatrix} \xi^{\dagger(1)} & 0 \end{pmatrix} \begin{pmatrix} \bar{\sigma}^\mu & 0 \\ 0 & \sigma^\mu \end{pmatrix} \begin{pmatrix} \xi^{(1)} \\ 0 \end{pmatrix} = 2m \; \xi^{\dagger(1)} \bar{\sigma}^\mu \xi^{(1)}$$

but $2m \begin{pmatrix} 0 & \xi^{\dagger(2)} \end{pmatrix} \begin{pmatrix} \bar{\sigma}^\mu & 0 \\ 0 & \sigma^\mu \end{pmatrix} \begin{pmatrix} \xi^{(1)} \\ 0 \end{pmatrix} = 0$

helicity is conserved!!



at high energies helicity is conserved. (or polarization)

at low " spin " "

— · —

ν

CM

April 2nd

$$\frac{d\sigma_{\alpha\to\beta}}{d\Omega} = \frac{1}{64\pi^2}\frac{|\vec{p}_3|}{|\vec{p}_1|}\frac{1}{s}\left|\mathcal{M}^{\text{book}}_{\beta\alpha}\right|^2$$

$2\to 2$

$\Omega = (\theta, \varphi)$

renormalized.

from Feynmann diagrams.

$$\sigma_T = \int d\Omega \, \frac{d\sigma_{\alpha\to\beta}}{d\Omega}$$

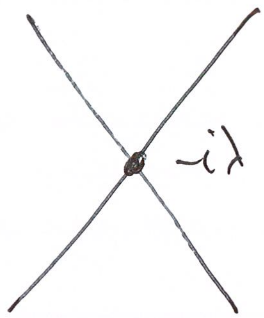
partial waves.

$$\mathcal{M}_{\beta\alpha} = \frac{4E}{\sqrt{|\vec{p}_1||\vec{p}_3|}} \sum_{\substack{j\sigma l s m\mu \\ l's'm'\mu'}} \langle s_3\sigma_3 s_4\sigma_4 | s'\mu'\rangle \langle l'm's'\mu' | j\sigma\rangle Y_{l'm'}(\hat{p}_3) \times$$
$$\times \langle s\mu | s_1\sigma_1 s_2\sigma_2\rangle \langle j\sigma | lms\mu\rangle Y_{lm}(\hat{p}_1).$$

$\mathcal{M}^j_{\underbrace{l's'n'}_{p}\,\underbrace{lsn}_{p}}$

$$\sigma_T = \frac{\pi}{(2s_1+1)(2s_2+1)}\frac{1}{|\vec{p}_1|}\sum_{\substack{jls \\ jl's'}}(2j+1)\Big|\underbrace{\delta_{ll'}\delta_{ss'}\delta_{nn'} - S^j_{\substack{lsn \\ l's'n'}}}_{4\pi^2|\mathcal{M}_j|^2}\Big|^2$$

$S^j \cancel{\not=}$

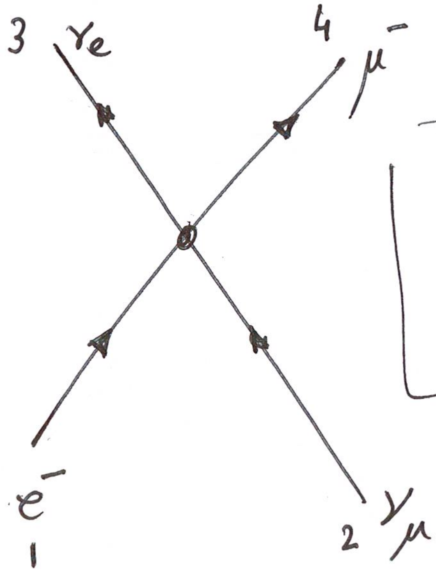


$$\frac{d\sigma_{\alpha\to\beta}}{d\Omega} = \frac{\lambda^2}{64\pi^2 E_{cm}^2} = \frac{\lambda^2}{64\pi^2 s}$$

$$\sigma = \frac{\lambda^2}{32\pi s} \underset{s\to\infty}{\sim} 1/s$$

$$\mathcal{M}_0 = -\frac{i\lambda}{8\pi^2}\frac{|\vec{p}_1|}{E} \qquad \text{s-wave}$$

$\boxed{j=0}$



$$\nu_\mu + e^- \to \mu^- + \nu_e$$

$$\boxed{-\frac{iG_F}{\sqrt{2}}\left(\underset{\nu_e}{\overline{u}_3^{(s_3)}}\,\gamma^\alpha \underset{\substack{\text{projector over}\\ \text{left}}}{(1-\gamma_5)}\, \underset{e^-}{u_1^{(s_1)}}\right)\left(\underset{\mu^-}{\overline{u}_4^{(s_4)}}\,(1+\gamma_5)\gamma_\alpha\, \underset{\nu_\mu}{u_2^{(s_2)}}\right)}$$

$$\underset{L}{\left(\tfrac{1}{2},0\right)} \oplus \underset{R}{\left(0,\tfrac{1}{2}\right)}$$

$$\frac{d\sigma_{\alpha\to\beta}}{d\Omega} = \frac{G_F^2}{8\pi^2}\frac{(s-m_\mu^2)^2}{s} \underset{s\to\infty}{\sim} s$$

(s, t, u)

$$\mathcal{M}_{\beta\alpha} = -\frac{iG_F}{\sqrt{2}} \left(\bar{u}_3^{(\sigma_3)} \gamma^\alpha (1-\gamma_5) u_1^{(\sigma_1)}\right)\left(\bar{u}_4^{(\sigma_4)} (1+\gamma_5)\gamma_\alpha u_2^{(\sigma_2)}\right)$$

Fierz identity.

$$P_L = \frac{1-\gamma_5}{2} \qquad P_L u_1 = u_{1L} \qquad P_L \begin{pmatrix} u_{1L} \\ u_{1R} \end{pmatrix} = \begin{pmatrix} u_{1L} \\ 0 \end{pmatrix}$$

$$(\not{p} - m_e) u_1 = 0 \qquad (u_{1L}^\dagger \; 0)\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = (0, u_{1L}^\dagger)$$

$$P_L^2 = P_L \qquad \bar{u}_3 \gamma^\alpha \frac{(1-\gamma_5)}{2} \frac{(1-\gamma_5)}{2} u_1$$

$$\underbrace{\bar{u}_3 \left(\frac{1+\gamma_5}{2}\right)}_{\bar{u}_{3L}} \gamma^\alpha \underbrace{\frac{(1-\gamma_5)}{2} u_1}_{u_{1L}}$$

$$\boxed{\mathcal{M}_{\beta\alpha} = -\frac{iG_F}{\sqrt{2}} \left(\bar{u}_{3L} \gamma^\alpha u_{1L}\right)\left(\bar{u}_{4L} \gamma_\alpha u_{2L}\right)}$$

only left part ~~enter~~ couple for the weak interaction.

$$\gamma^\alpha = \begin{pmatrix} 0 & \sigma^\alpha \\ \bar{\sigma}^\alpha & 0 \end{pmatrix} \qquad \sigma^\alpha = (\mathbb{1}, \vec{\sigma}) \qquad \bar{\sigma}^\alpha = (\mathbb{1}, -\vec{\sigma})$$

$$\begin{pmatrix} 0 & \sigma^\alpha \\ \bar{\sigma}^\alpha & 0 \end{pmatrix}\begin{pmatrix} u_{4L} \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ \bar{\sigma}^\alpha u_{4L} \end{pmatrix}$$

$$\mathcal{M}_{\beta\alpha} = -\frac{iG_F}{\sqrt{2}} \left(\bar{u}_{3L} \bar{\sigma}^{\alpha} u_{1L} \right) \left(\bar{u}_{4L} \bar{\sigma}_{\alpha} u_{2L} \right)$$

$$\boxed{\sum_{\mu=0}^{3} (\bar{\sigma}^{\mu})_{ab} (\bar{\sigma}_{\mu})_{cd} = 2 \epsilon_{ac} \epsilon_{bd}}$$

$\epsilon_{12} = 1 = -\epsilon_{21}$

$1,0$

11   22   1 1   22

                 1 2   12

$\mu = 1,2$

$\mu = 0,3$

~~11 22~~

~~$1.1 + (-1)(+1) = 0$~~

~~11 22~~

$$\sum_{\mu=0}^{3} (\bar{\sigma}^{\mu})_{ab} (\bar{\sigma}_{\mu})_{cd} = 2 \epsilon_{ac} \epsilon_{bd}$$

12   12   11   22

$$(\bar{\sigma}^{1})_{12} (\bar{\sigma}_{1})_{12} + (\bar{\sigma}^{2})_{12} (\bar{\sigma}_{2})_{12}$$

$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

$\bar{\sigma}^{\mu} = (1, -\sigma)$

$\sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$

$$(-\sigma_1)_{12} (\sigma_1)_{12} + (-\sigma_2)_{12} (\sigma_2)_{12}$$

$$-1 \times 1 + i \cdot (-i) = -1 + 1 = 0$$

$$\sum_{\mu=0}^{3} (\bar{\sigma}^{\mu})_{ab} (\bar{\sigma}_{\mu})_{cd} = 2\epsilon_{ac}\epsilon_{bd}$$

$$\begin{array}{ccc} 11 & 22 & \underbrace{12 \quad 12}_{2} \\ 21 & 12 & \end{array}$$

$$(\bar{\sigma}^0)_{11} (\bar{\sigma}_0)_{22} + (\bar{\sigma}^3)_{11} (\bar{\sigma}_3)_{22}$$

$$\sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$1 \cdot 1 + (-\sigma_3)_{11} (\sigma_3)_{22}$$

$$1 + (-1).(-1) = 2$$

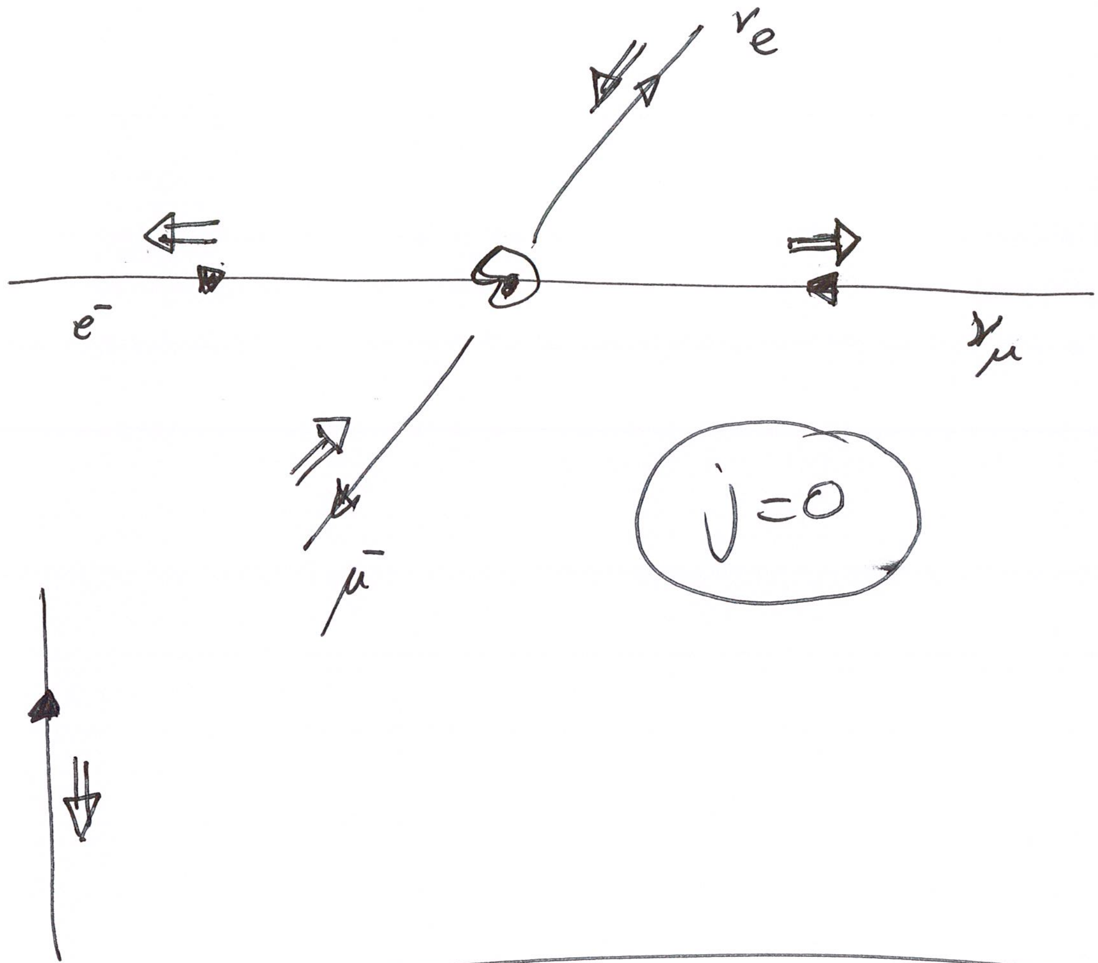
$$\mathcal{M}_{\beta\alpha} = -\frac{iG_F}{\sqrt{2}} \bar{u}^{(\sigma_3)}_{3La} \bar{\sigma}^{\alpha}_{ab} u^{(\sigma_1)}_{1Lb} \bar{u}^{(\sigma_4)}_{4Lc} \bar{\sigma}_{\alpha cd} u^{(\sigma_2)}_{2Ld}$$

$$2\epsilon_{ac}\epsilon_{bd}$$

$$= -\frac{2iG_F}{\sqrt{2}} \underbrace{(\bar{u}_{3a} \epsilon_{ac} \bar{u}_{4c})}_{\text{final}} \underbrace{(u_{1Lb} \epsilon_{bd} u_{2Ld})}_{\text{initial}}$$

$$u_1^{(\uparrow)} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad u_2^{(\downarrow)} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \qquad (u^{\uparrow} \; u^{\downarrow}) \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} u^{\uparrow} \\ u^{\downarrow} \end{pmatrix} = -u_1^{\downarrow} u_2^{\uparrow} + u_1^{\uparrow} u_2^{\downarrow}$$

S-wave $\quad l=0 \quad s=0 \quad j=0$



$$(\bar{u}_{3L}\bar{\sigma}^{\alpha}u_{1L})(\bar{u}_{4L}\bar{\sigma}_{\alpha}u_{2L}) = 2(\bar{u}_3\epsilon\bar{u}_4)(u_1\epsilon u_2)$$

$$(\bar{u}_{3L}\bar{\sigma}^{\alpha}u_{2L})(\bar{u}_{4L}\bar{\sigma}_{\alpha}u_{1L}) = 2(\bar{u}_3\epsilon\bar{u}_4)(u_2\epsilon u_1)$$

$$u_{2c}\epsilon_{cd}u_{1d}$$

$$\boxed{(\bar{u}_{3L}\bar{\sigma}^{\alpha}u_{1L})(\bar{u}_{4L}\bar{\sigma}_{\alpha}u_{2L}) = -(\bar{u}_{3L}\bar{\sigma}^{\alpha}u_{2L})(\bar{u}_{4L}\bar{\sigma}_{\alpha}u_{1L})}$$

Fierz identity.

7

$$\sigma_T = \frac{\pi}{2}\frac{1}{|\vec{p}_1|^2}\sum_j (2j+1)\,\Big|\underbrace{\delta_{\ell\ell'}\delta_{ss'}\delta_{nn'}}_{0} - S^j_{\substack{\ell s n\\ \ell' s' n'}}\Big|^2$$

$$= \frac{\pi}{2}\frac{1}{|\vec{p}_1|^2}\Big|\underbrace{S^0_{\substack{\ell s n\\ \ell' s' n'}}}_{\lesssim 1}\Big|^2 \lesssim \frac{\pi}{2|\vec{p}_1|^2} = \frac{4\pi s}{\cancel{2}(s-m_e^2)^2} \underset{s\to\infty}{\sim} \frac{1}{s}$$

σ_T growing as s violates unitarity..

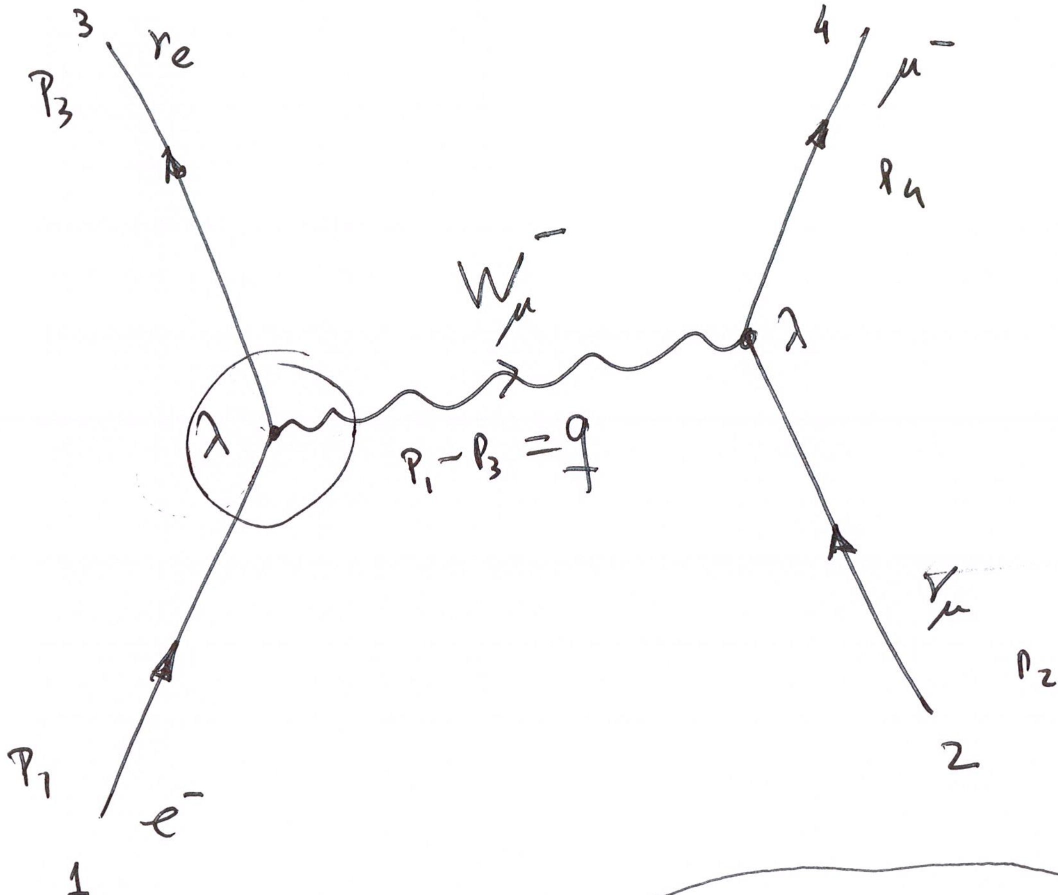
$$\frac{G_F^2}{2\pi}\frac{(s-m_\mu^2)^2}{s} \sim \frac{G_F^2}{2\pi} s \lesssim \frac{1}{s} \qquad G_F^2 s^2 \lesssim 1$$

$$s \lesssim 1/G_F \qquad G_F \sim M^{-2}$$

$$E \sim \sqrt{G_F}$$

$$G_F^{eff} = G_F E^2$$

$$E \sim 1/\sqrt{G_F}$$



$$\mathcal{M}_{\beta\alpha} = \lambda^2 \underbrace{\left(\bar{u}_3 \gamma^\alpha (1-\gamma_5) u_1\right)}\, \frac{(-i)\left(\eta_{\alpha\beta} - \dfrac{q_\alpha q_\beta}{M_W^2}\right)}{q^2 - M_W^2} \times$$

$$\times \underbrace{\left(\bar{u}_4 \gamma_\beta (1-\gamma_5) u_2\right)}$$

$$\boxed{\frac{\lambda^2}{M_W^2} = \frac{G_F}{\sqrt{2}}}$$

$$(\bar{u}_3 \not{q} (1-\gamma_5) u_1)$$

$$(\bar{u}_3 (\not{p}_1 - \not{p}_3)(1-\gamma_5) u_1)$$

$$(1+\gamma_5$$

$$\not{p}_3 u_3 = 0 \qquad \not{p}_1 u_1 = m_e u_1$$

$$\frac{d\sigma_{\alpha\to\beta}}{d\Omega} = \frac{1}{64\pi^2}\,\frac{|\vec{p}_3|}{|\vec{p}_1|}\,\frac{1}{s}\,|\mathcal{M}_{\beta\alpha}|^2$$

$$= \frac{\lambda^4}{4\pi^2\,(\underset{\downarrow\atop t}{q^2}-M_W^2)^2}\,\frac{(s-m_\mu^2)^2}{s} = \frac{\lambda^4\,(s-m_\mu^2)^2}{4\pi^2\left(4\underset{\uparrow\atop \frac{s}{2}}{\varepsilon_1}\underset{\uparrow\atop \frac{s}{2}}{\varepsilon_3}\sin^2\frac{\theta}{2}+M_W^2\right)^2 s}$$

$$t = (p_1-p_3)^2 = (\varepsilon_1-\varepsilon_3)^2 - (\vec{p}_1-\vec{p}_3)^2$$

$$\underset{e^-}{p_1} = (\varepsilon_1, \vec{p}_1) \qquad p_3 = (\underset{\uparrow\atop \nu_e}{\varepsilon_3}, \vec{p}_3)$$

$$= \frac{\lambda^4}{4\pi^2}\,\frac{(s-m_\mu^2)^2}{s\left(s\,\sin^2\frac{\theta}{2}+M_W^2\right)^2}$$

$$t = (p_1-p_3)^2 = p_1^2 + p_3^2 - 2\,p_1\cdot p_3 =$$

$$= m_e^2 - 2\left(\varepsilon_1\varepsilon_3 - \vec{p}_1\cdot\vec{p}_3\right)$$

$$= m_e^2 - 2\left(\varepsilon_1\varepsilon_3 - |\vec{p}_1||\vec{p}_3|\,c\theta\right)$$

$m_e \approx 0$

$$= -2\,\varepsilon_1\varepsilon_3\,(1-c\theta) = \underline{-4\,\varepsilon_1\varepsilon_3\,\sin^2\frac{\theta}{2}}$$

for $s \gg M_\mu^2 > m_e^2$

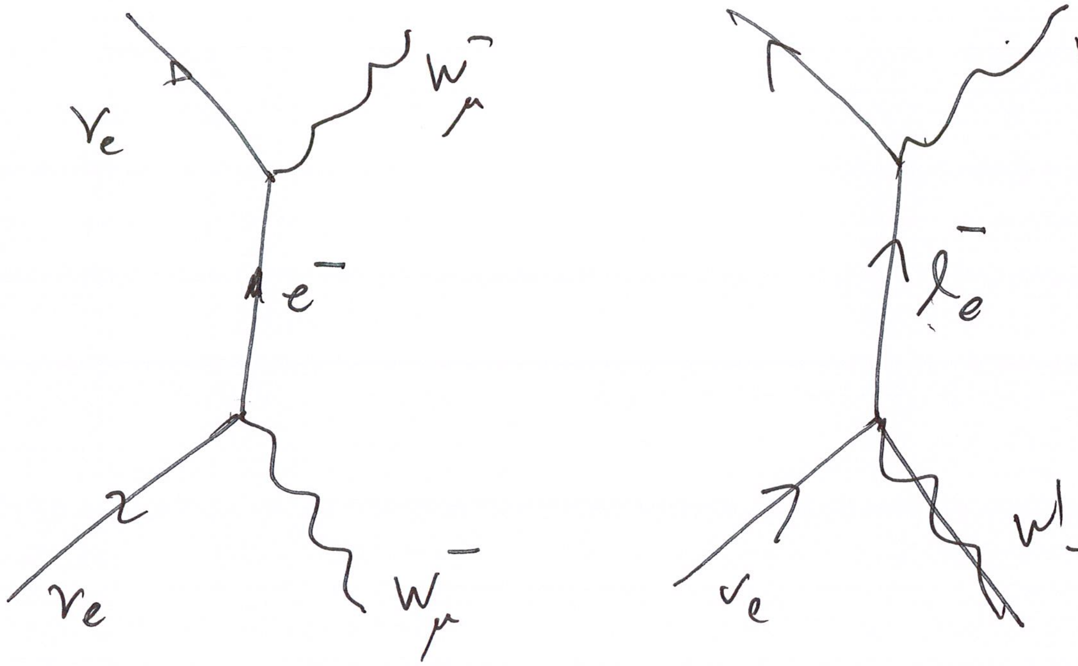
$$\frac{d\sigma_{\alpha\to\beta}}{d\Omega} = \frac{\lambda^4}{4\pi^2} \frac{(s-\cancel{m_\mu^2})^2}{s\left(s_* \, s^2\frac{\theta}{2} + M_W^2\right)^2}$$

$$= \frac{\lambda^4}{4\pi^2} \frac{s}{\left(M_W^2 + s_* \sin^2\frac{\theta}{2}\right)^2}$$

$$\int s\theta d\theta d\varphi \frac{d\sigma_{\alpha\to\beta}}{d\Omega} = \frac{\lambda^4}{4\pi^2} \frac{4s}{M_W^2(s+M_W^2)}$$

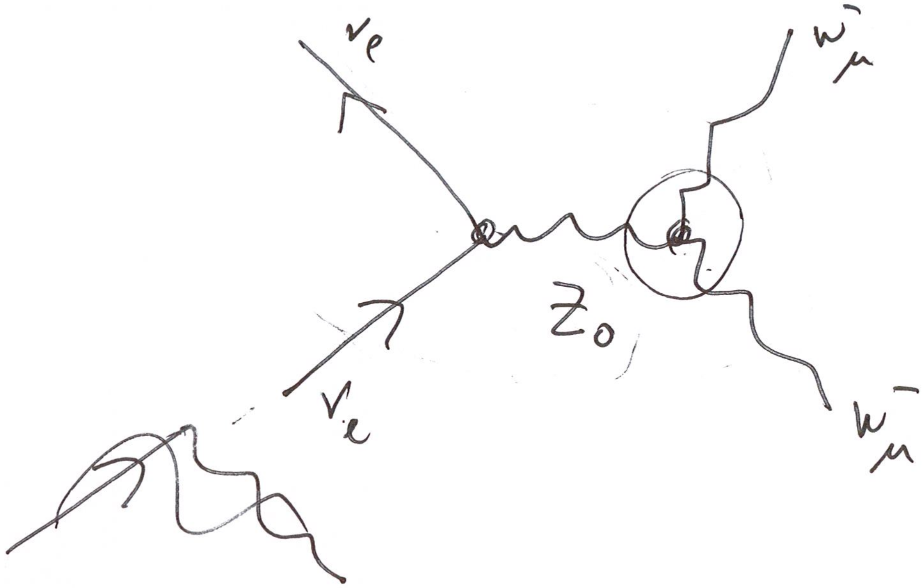
$$\sigma_T = \frac{\lambda^4}{\pi^2} \frac{s}{M_W^2(s+M_W^2)} \rightarrow \frac{\lambda^4 \cancel{s}}{\pi^2 M_W^2 \cancel{s}}$$

$$\frac{G_F^2 M_W^2}{2\pi}$$



$\nu_e + W_\mu^- \longrightarrow \nu_e + W_\mu^-$

$\underline{\sigma \sim s}$



$$\left| \left(\bar{u}_3 \gamma^\alpha (1-\gamma_5) u_1 \right) \left(\bar{u}_4 (1+\gamma_5) \gamma_\alpha u_2 \right) \right|^2$$

(1)

$$\left(\bar{u}_3 \gamma^\alpha (1-\gamma_5) u_1 \right)^+ = u_1^+ (1-\gamma_5) (\gamma^\alpha)^+ \gamma^0 u_3$$

$$= \bar{u}_1 \gamma_0 (1-\gamma_5) \gamma_0 \gamma^\alpha u_3$$

$$\sum_{\sigma=1,2} u^\sigma \bar{u}^\sigma = \not{p} + m$$

$$= \bar{u}_1 (1+\gamma_5) \gamma^\alpha u_3$$

$$\bar{u}_3 \gamma^\alpha (1-\gamma_5) u_1 \bar{u}_1 (1+\gamma_5) \gamma^\beta u_3 \quad \bar{u}_4 (1+\gamma_5) \gamma_\alpha u_2 \bar{u}_2 \gamma^\beta (1-\gamma_5) u_4$$

$$\mathrm{Tr}(\underbrace{u_3 \bar{u}_3} \gamma^\alpha (1-\gamma_5) u_1 \bar{u}_1 (1+\gamma_5) \gamma^\beta) \; \mathrm{Tr}(u_4 \bar{u}_4 (1+\gamma_5) \gamma_\alpha u_2 \bar{u}_2 \gamma^\beta (1-\gamma_5))$$

$$\mathrm{Tr}(\not{p}_3 \gamma^\alpha (1-\gamma_5)(\not{p}_1 + m_e)(1+\gamma_5)\gamma^\beta) \; \mathrm{Tr}((\not{p}_4 + m_\mu)(1+\gamma_5)\gamma_\alpha \not{p}_2 \gamma^\beta (1-\gamma_5))$$

$$\gamma^\beta (1-\gamma_5)$$

$$1-\gamma_5$$

$$\mathrm{Tr}((\not{p}_1 + \cancel{m_e}) \gamma^\beta \not{p}_3 \gamma^\alpha (1-\gamma_5)) \; \mathrm{Tr}((\cancel{\not{p}_4 + m_\mu}) \gamma_\alpha \not{p}_2 \gamma^\beta (1-\gamma_5))$$

$$\left(4 \left(p_1^\beta p_3^\alpha - (p_1 p_3) \eta^{\alpha\beta} + p_1^\alpha p_3^\beta \right) + 4i \, \varepsilon^{\mu\beta\nu\alpha} p_{1\mu} p_{3\nu} \right) \cdot$$

$$\cdot \left(4 \left(p_{4\alpha} p_{2\beta} - (p_2 p_4) \eta_{\alpha\beta} + p_{4\beta} p_{2\alpha} \right) + 4i \, \varepsilon^{\mu\alpha\nu\beta} p_{4\mu} p_{2\nu} \right)$$

$$16 \{ (p_1 p_2)(p_3 p_4) - \cancel{(p_1 p_3)(p_2 p_4)} + (p_1 p_4)(p_2 p_3) - \cancel{(p_1 p_3)(p_2 p_4)} + \cancel{4 (p_1 p_3)(p_2 p_4)}$$

$$- \cancel{(p_1 p_3)(p_2 p_4)} + (p_1 p_4)(p_2 p_3) - \cancel{(p_1 p_3)(p_2 p_4)} + (p_1 p_2)(p_3 p_4) -$$

$$+ \; \varepsilon^{\mu'\nu'\beta\alpha} \varepsilon^{\mu\nu\beta\alpha} \; p_{1\mu'} p_{3\nu'} \; p_{4\mu} p_{2\nu} \Big)$$

$$16 \Big\{ 2(p_1 p_2)(p_3 p_4) + \cancel{2(p_1 p_4)(p_2 p_3)} -$$

$$\cancel{-2 (p_1 p_4)(p_2 p_3)} + 2 \; (p_1 p_2)(p_3 p_4) \Big\}$$

$$64 \; (p_1 p_2)(p_3 p_4)$$

$$(p_1 + p_2)^2 = p_1^2 + p_2^2 + 2 p_1 p_2 \; = m_\mu^2 + 2 p_1 p_2$$

$$p_1 p_2 = \frac{s - m_\mu^2}{2}$$

$$(p_3 + p_4)^2 = p_3^2 + p_4^2 + 2 p_3 p_4 = m_e^2 + 2 p_3 p_3$$

$$p_3 p_4 = \frac{s - m_e^2}{2}$$

$$\frac{64}{4} \; (s - m_\mu^2)(s - m_e^2) \; = 16 \; (s - m_\mu^2)(s - m_e^2)$$

$$\frac{d\sigma_{\alpha\to\beta}}{d\Omega} = \frac{1}{64\pi^2 s}\frac{|\vec{p}_3|}{|\vec{p}_1|}|\mathcal{M}_{\beta\alpha}|^2$$

$$\mathcal{M}_{\beta\alpha} = -\frac{iG_F}{\sqrt{2}}\left(\bar{u}_3^{s_3}\gamma^\alpha(1-\gamma_5)u_1^{s_1}\right)\left(\bar{u}_4^{s_4}(1+\gamma_5)\gamma_\alpha u_2^{s_2}\right)$$

ν_e $\quad e^-$ $\quad \mu^-$ $\quad \nu_\mu$

$$= -\frac{2iG_F}{\sqrt{2}}\left(\bar{u}_{3La}\epsilon_{ac}\bar{u}_{4Lc}\right)\left(u_{1Lb}\epsilon_{bd}u_{2Ld}\right)$$

$$|\mathcal{M}_{\beta\alpha}|^2$$

$$\frac{G_F^2}{2}\left(\bar{u}_3^{s_3}\gamma^\alpha(1-\gamma_5)u_1^{s_1}\right)^*\left(\bar{u}_3^{s_3}\gamma^\alpha(1-\gamma_5)u_1^{s_1}\right).$$

$$\cdot\left(\bar{u}_4^{s_4}(1+\gamma_5)\gamma_\alpha u_2^{s_2}\right)^*\left(\bar{u}_4^{s_4}(1+\gamma_5)\gamma_\alpha u_2^{s_2}\right)$$

$u_3^+\gamma_0$

$\cancel{\bar{u}_1\gamma^0\gamma^\alpha(1-\gamma_5)}$

$$\bar{u}_1^{s_1}(1-\gamma_5)\gamma^\alpha u_3^{s_3}\,\bar{u}_3^{s_3}\gamma^\alpha(1-\gamma_5)u_1^{s_1}$$

$\bar{u}_1(1+\gamma_5)\gamma^\alpha u_3 \quad \bar{u}_1\gamma^\alpha(1-\gamma_5)u_3$

$$u_1^+(1-\gamma_5)(\gamma^\alpha)^{+}\gamma_0 u_3 = \bar{u}_1\gamma_0(1-\gamma_5)\gamma_0\gamma^\alpha u_3$$

$$\frac{d\sigma_{\alpha\to\beta}}{d\Omega} = \frac{1}{64\pi^2 s}\,\frac{|\vec{p}_3|}{|\vec{p}_1|}\,|\mathcal{M}_{\beta\alpha}|^2$$

$$= \frac{1}{64\pi^2 s}\,\frac{|\vec{p}_3|}{|\vec{p}_1|}\,\frac{G_F^2}{2}\,16\,(s-m_\mu^2)(s-m_e^2)$$

$s = (p_1+p_2)^2$

(ε_1, p_1) $\quad (\varepsilon_2, -p_1)$

$\varepsilon_1^2 = p_1^2 + m_e^2$

$\varepsilon_2^2 = p_1^2$.

$\sqrt{s} = \sqrt{p_1^2+m_e^2} + p_1$

$(\sqrt{s}-p_1)^2 = p_1^2 + m_e^2 \rightarrow \quad s - 2\sqrt{s}\,p_1 + \cancel{p_1^2} = \cancel{p_1^2} + m_e^2$

$s - m_e^2 = 2\sqrt{s}\,p_1$

$p_1 = \frac{s-m_e^2}{2\sqrt{s}} \qquad p_3 = \frac{s-m_\mu^2}{2\sqrt{s}}$

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2 s}\,\frac{(s-m_\mu^2)}{\cancel{2\sqrt{s}}}\,\frac{\cancel{2\sqrt{s}}}{\cancel{s-m_e^2}}\,\frac{G_F^2}{2}\,16\,(s-m_\mu^2)\cancel{(s-m_e^2)}$$

$$= \frac{G_F^2}{8\pi^2 s}(s-m_\mu^2)^2 \quad \frac{1}{4}$$ unpolarized cross section

average.