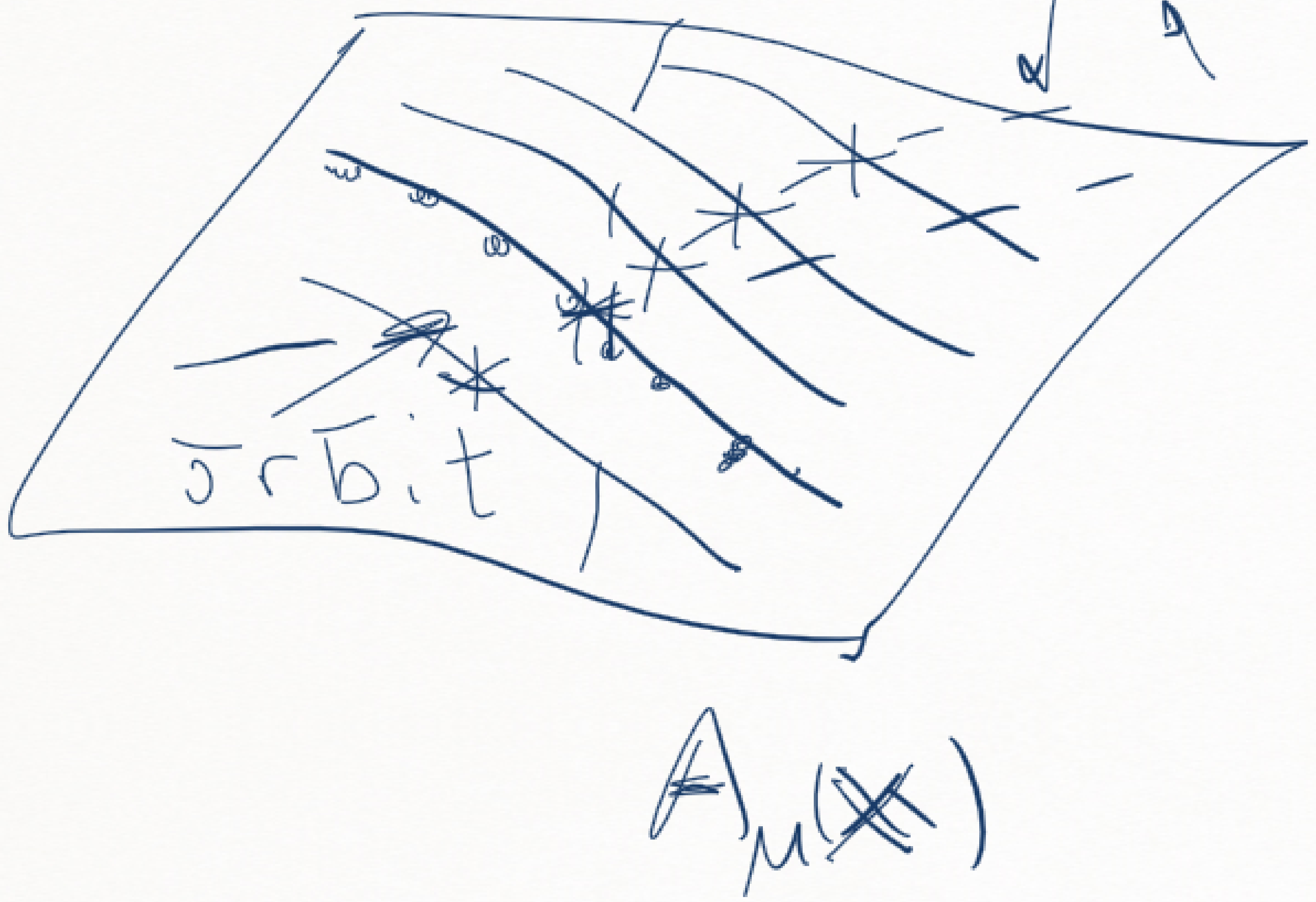
QFT II (2/7/2023)



$$\mathcal{F}_a(A^b_\mu(x)) = 0$$

$$\boxed{\theta_a(x)}$$

$$A_0 + A_3 \equiv A_+ = 0$$

$$\underbrace{\int \mathcal{D}A_\mu(x)}\ \Longrightarrow\ \boxed{\int \mathcal{D}\theta_a(x)}\ \int \mathcal{D}A_\mu(x)$$

gaugefixed

Jacobian

$$\frac{\delta G_a(A^\theta_b(x))}{\delta \theta_b(y)} = M_{ab}(x,y)$$

$(x,a) \quad (b,y)$

$$\det M_{ab}(x,y) = \int \mathcal{D}\bar{c}_a \, \mathcal{D}c_b \, e^{i\int \bar{c}^a(x) M_{ab}(x,y) c_b(y)}$$

$$\int d\bar{z}\, dz \; e^{-a\bar{z}z} \qquad \bar{c}_a \neq c_b^+ \qquad \boxed{c_a^+ = c_a} \quad \text{o} \quad \boxed{\bar{c}_a^+ = \bar{c}_a}$$

$$\int d\bar{c}\, dc \; e^{-a\bar{c}c} = \underbrace{\int d\bar{c}\, dc}\left(1 - a\bar{c}c + \cancel{\tfrac{1}{2}(a\bar{c}c)^2} + \dots\right) = a$$

$\bar{c} \neq c^+$

$$\int \mathcal{D}A_\mu \to \int \mathcal{D}\theta_a \int \mathcal{D}\bar{c}_a \mathcal{D}c_b \underbrace{\int \mathcal{D}A_\mu\, \delta(\mathcal{F}_a(A))}_{\text{over gauge fixed}} e^{\int \mathcal{L}}$$

$$\mathcal{F}_a = B_a(x)$$

$$\int \mathcal{D}B_a(x)\, e^{-\frac{i}{2\xi}\int B_a B_a}\, \delta(\mathcal{F}_a - B_a)$$

$$= e^{-\frac{i}{2\xi}\mathcal{F}_a \mathcal{F}_a} \qquad \mathcal{F}_a = \partial_\mu A^{\mu a}$$

$$Z = \int \mathcal{D}A^a_\mu \int \mathcal{D}\bar{c}_a \mathcal{D}c_a \, e^{i\int d^4x \, \mathcal{L}}$$

$$\frac{\partial}{\partial x^\mu} \qquad \partial_\mu - g A_\mu \qquad F = \partial A \quad [A, A]$$

$$\boxed{\begin{aligned} \mathcal{L} = -\frac{1}{2} \mathrm{Tr}\, \underbrace{F_{\mu\nu} F^{\mu\nu}}_{4} &+ \overset{3/2}{\bar{\psi}} \overset{1}{(} i \overset{3}{\not{D}} - m) \overset{3/2}{\psi} + |D_\mu \phi|^2 - m^2 |\phi|^2 \\ &+ \underbrace{\partial_\mu \bar{c}^a D^\mu c^a}_{4} - \frac{1}{2\xi} \boxed{(\partial_\mu A^{a\mu})^2} + \cancel{\frac{1}{2} M^2 A_\mu A^\mu} \end{aligned}}$$

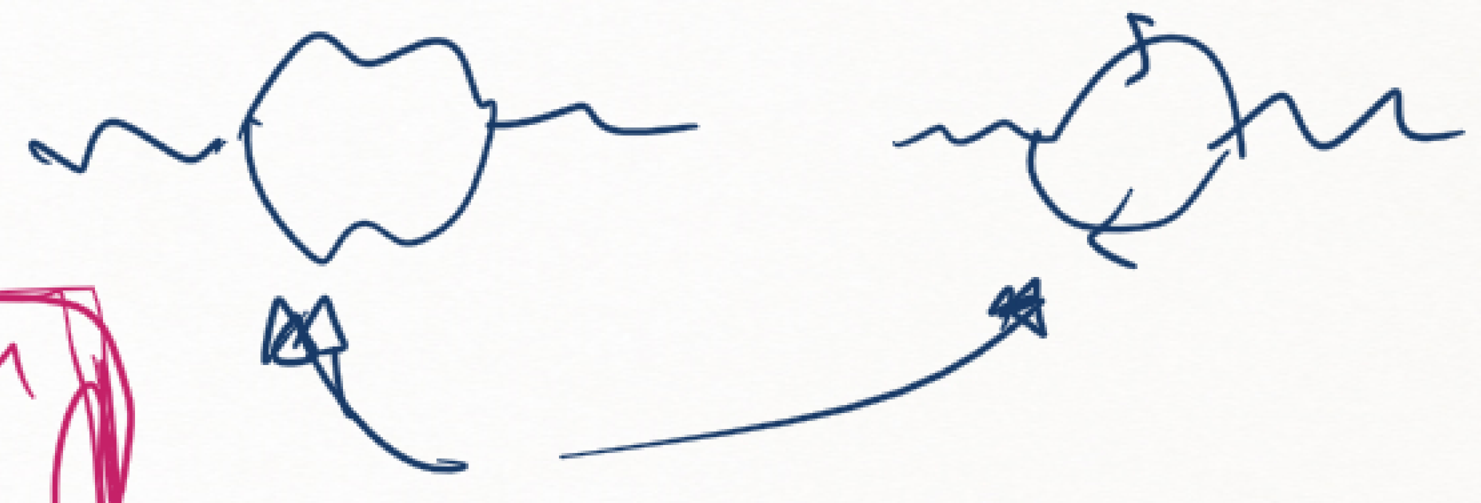
BRST

$a = 1 \ldots N^2 - 1$

$$\partial^\mu c^a - g f^{abc} A^b_\mu c^c$$

ghosts

$$\boxed{A_\mu A^\mu}$$



$$\boxed{\begin{aligned} c^a &\to \lambda c^a \\ \bar{c}^a &\to \frac{1}{\lambda} \bar{c}^a \end{aligned}}$$

$\bar{c}^a \to \bar{c}^a + \ldots$

$$S^{(2)} = \int d^4x \left(-\frac{1}{4} \left(\partial_\mu A^a_\nu - \partial_\nu A^a_\mu \right)^2 - \frac{1}{2\xi} \left(\partial^\mu A^a_\mu \right)^2 \right)$$

$$= \int d^4x \left(-\frac{1}{2} \partial_\mu A^a_\nu \, \partial^\mu A^{a\nu} + \frac{1}{2} \partial_\mu A^a_\nu \, \partial^\nu A^{a\mu} - \frac{1}{2\xi} \left(\partial^\mu A^a_\mu \right)^2 \right)$$

$$= \int d^4x \, \frac{1}{2} \left(A^a_\nu \, \partial^2 A^{a\nu} - A^a_\nu \, \partial^\mu \partial^\nu A^{a\mu} + \frac{1}{\xi} A^a_\mu \, \partial^{\mu\nu} A^a_\nu \right)$$

$$= \int d^4x \, \frac{1}{2} A^a_\nu \left(\delta^{ab} \eta^{\mu\nu} \partial^2 - \left(1 - \frac{1}{\xi}\right) \delta^{ab} \partial^\mu \partial^\nu \right) A^b_\mu$$

$$-\left(k^2 \eta^{\mu\nu} - \left(1 - \frac{1}{\xi}\right) k^\mu k^\nu \right) k_\nu = \cancel{k^2 k^\mu} - \left(\cancel{1} - \frac{1}{\xi}\right) k^2 k^\mu$$

$$= \frac{1}{\xi} k^2 k^\mu$$

$$-\left(k^2\eta^{\mu\nu}-\left(1-\frac{1}{\xi}\right)k^\mu k^\nu\right)\boxed{\Delta_{\nu\alpha}(k)}=i\delta^\mu_\alpha \qquad \frac{i}{k^2-m^2}$$

$$\begin{pmatrix} k^2 h_4 & & & \\ & -k^2 & & \\ & & -k^2 & \\ & & & -k^2 \end{pmatrix}^{-1}$$

$$\Delta_{\nu\alpha}=\underbrace{a k_\nu k_\alpha+b\eta_{\nu\alpha}}$$

$$-k^2 a k^\mu k_\alpha \underbrace{-bk^2\delta^\mu_\alpha}+\left(1-\frac{1}{\xi}\right)ak^2k^\mu k_\alpha+\left(1-\frac{1}{\xi}\right)bk^\mu k_\alpha=$$

$$\delta^\alpha_\mu \quad -k^4 a k_\alpha - bk^2 k_\alpha + \left(1-\frac{1}{\xi}\right)ak^4 k_\alpha+\left(1-\frac{1}{\xi}\right)bk^2k_\alpha \underline{=i\delta^\mu_\alpha}$$

$$\cancel{-ak^4}\cancel{-bk^2}+\left(\cancel{1}-\frac{1}{\xi}\right)ak^4+(\cancel{1}-1/\xi)\,bk^2=i \qquad =ik_\alpha$$

$$-\frac{1}{\xi}\left(ak^4 + bk^2\right) = i \qquad b = -i/k^2$$

$$\boxed{\xi = 1} \text{ Feymann}$$

$$ak^4 - i = -i\xi \qquad ak^4 = -i\xi + i$$

$$\Delta^{ab}_{\mu\nu} = -\frac{i}{k^2+i\varepsilon}\eta_{\mu\nu}\delta^{ab}$$

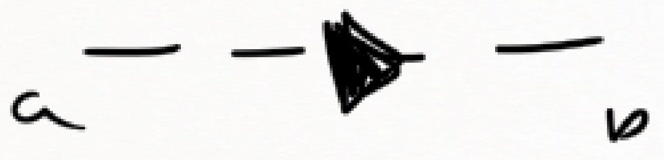
$$\Delta_{\mu\nu} = -\frac{i}{k^2}\eta_{\mu\nu} + \left(-i\xi + i\right)\frac{1}{k^4}k_\mu k_\nu$$

$$\boxed{\Delta^{ab}_{\mu\nu} = -\frac{i}{k^2+i\varepsilon}\left(\eta_{\mu\nu} - (1-\xi)\frac{k_\mu k_\nu}{k^2}\right)\delta^{ab}}$$

$$\Delta_{\mu\nu}k^\nu = -\frac{i}{k^2}\xi k_\mu \qquad \boxed{\xi = 0} \text{ Landau gauge}$$

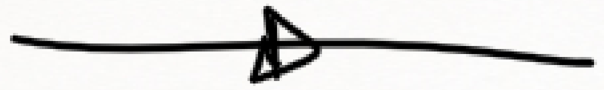
$$g f^{abc} \bar{c}^a A^b_\mu c^c$$

$$\mathcal{L}^{(4)} = \partial^\mu \bar{c}^a D_\mu c^a \to \partial^\mu \bar{c}^a \partial_\mu c^a \qquad \int d^4k \frac{\qquad}{k^2}$$

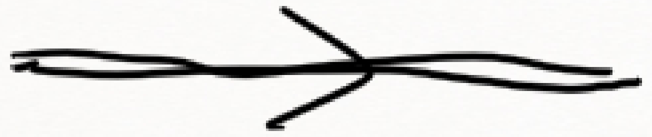


$$\boxed{\frac{i}{k^2+i\epsilon}\delta^{ab}}$$

$$-\frac{i}{k^2+i\epsilon}\eta_{\mu\nu}\delta^{ab}$$

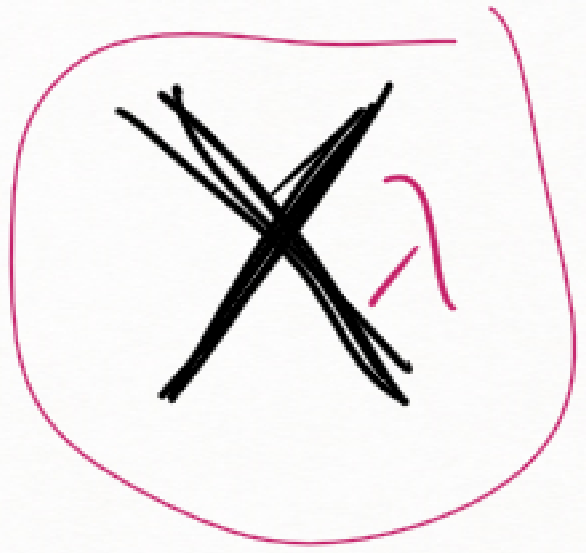


$$\frac{i}{\not{k}-m+i\epsilon}$$



$$\frac{i}{k^2-M^2+i\epsilon}$$

$$|D_\mu \phi|^2$$

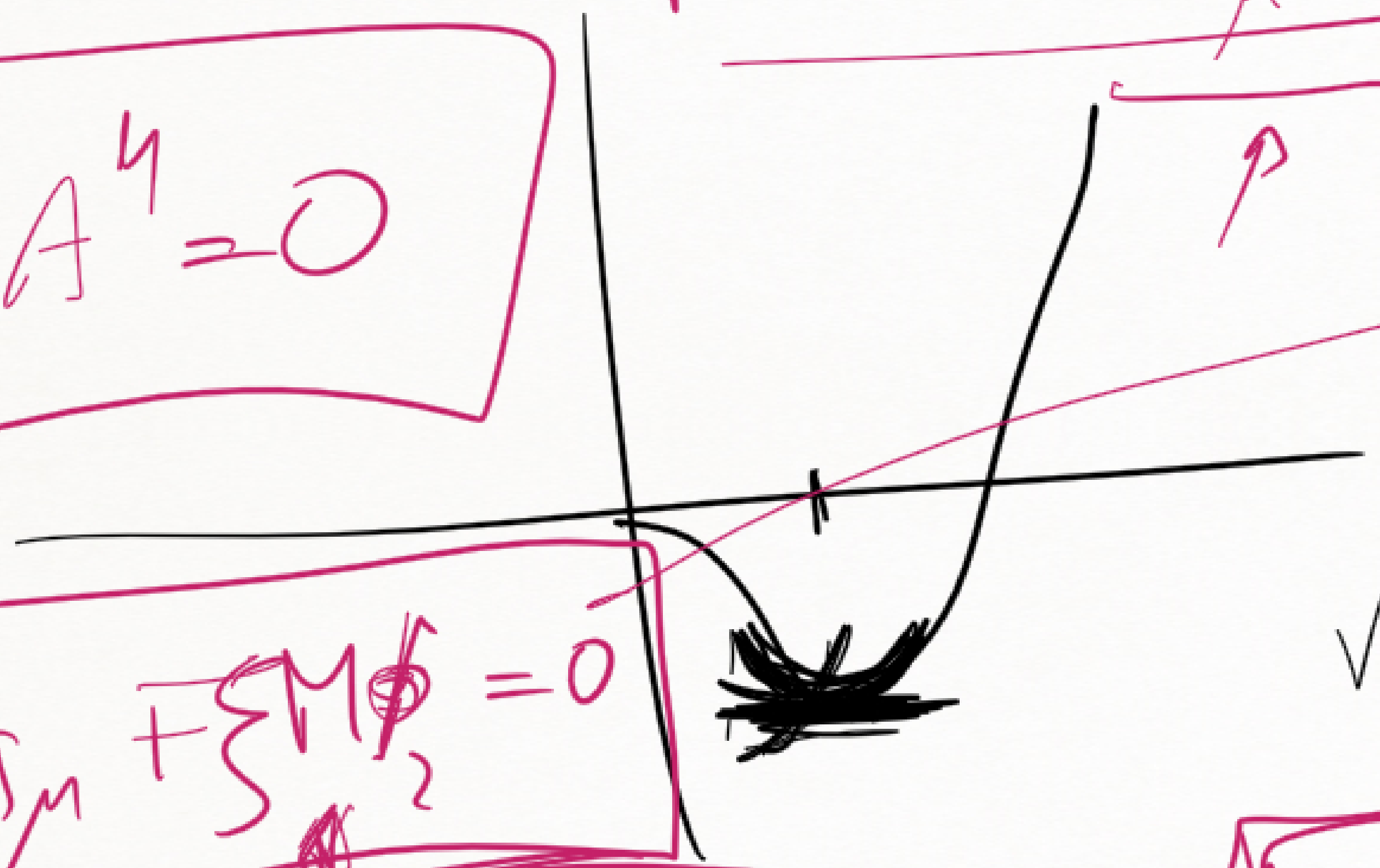


$$g + \delta g$$

$$\mathcal{L} = (D_\mu \phi)^\dagger (D^\mu \phi) - \mu^2 (\phi^\dagger \phi) - \lambda (\phi^\dagger \phi)^2 - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

$$+ \partial^\mu \bar{c}^a \partial_\mu c^a \cancel{+} \frac{1}{2\xi} (\partial_\mu A^\mu)^2$$

$$\boxed{\partial_\mu A^\mu = 0}$$



$\mu^2 < 0$

$$v = \left(-\frac{\mu^2}{\lambda} \right)^{1/2}$$

$M \phi_2 \partial^\mu A_\mu$

$\sqrt{\phi^\dagger \phi}$ $\quad M A_\mu \partial^\mu \phi_2$ $\quad \sqrt{\phi^\dagger \phi} = v/\sqrt{2}$

$$\boxed{\partial^\mu A_\mu + \xi M \phi_2 = 0}$$

$\partial^2 \theta$

$\phi_0 = v/\sqrt{2}$

$e^{i\theta_0}$

$$\boxed{\phi = \frac{v}{\sqrt{2}} + \frac{1}{\sqrt{2}} \tilde{\phi}_1 + \frac{i}{\sqrt{2}} \phi_2}$$

$\phi_1 = v + \tilde{\phi}_1$

$$\phi = e^{i \frac{\xi(x)}{v}} \frac{(v + \eta(x))}{\sqrt{2}}$$

$A_\mu \to$ massive