663, Homework II, (2 problems)

Problem 1

Consider an $SU(N)$ gauge theory and show the validity of the Bianchi identity

$$D_\mu F_{\nu\rho} + D_\nu F_{\rho\mu} + D_\rho F_{\mu\nu} = 0 \tag{0.1}$$

Problem 2

Consider an $SU(N)$ gauge theory with a fermion in the fundamental. In the BRST approach one introduces extra fields c, $\bar{c}$ and B. We say, by definition, that c has ghost number 1 and $\bar{c}$ has ghost number -1.

a) Use the gauge fixing function $\partial^\mu A_\mu$ (Lorentz gauge) and compute the dimensions of the different fields (including B).

b) Consider the Lagrangian density to be a BRST, Lorentz, and global $SU(N)$ invariant polynomial in the fields. Show that a BRST variation preserves the number $N_B + N_{\bar{c}}$, where N_B is the number of fields B in a given term, and the same for $N_{\bar{c}}$. Notice that the operator that counts this number can be written as

$$N_B + N_{\bar{c}} = B\frac{\delta}{\delta B} + \bar{c}\frac{\delta}{\delta \bar{c}} \tag{0.2}$$

c) Show that, if a function $\mathcal{L}$ of the fields is BRST invariant then

$$\delta_{BRST}\left[\left(\bar{c}\frac{\delta}{\delta B}\right)\mathcal{L}\right] = (N_B + N_{\bar{c}})\mathcal{L} \tag{0.3}$$

Hint: Use that the BRST variation can be written as $\delta_{BRST} = B\frac{\delta}{\delta \bar{c}} + \tilde{\delta}$ where $\tilde{\delta}$ does not involve B or $\bar{c}$.

d) Conclude that any BRST invariant Lagrangian of ghost number zero can be written as the usual Yang-Mills Lagrangian plus the BRST variation of another function Ψ

e) Using that the dimension of all the vertices in the Lagrangian is 4 or less, write the most general function Ψ and the corresponding Lagrangian.