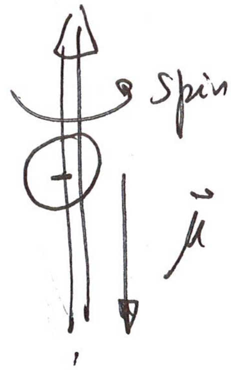
Magnetic moment of the electron



$$\vec{\mu} = g\,\frac{e}{2m}\,\vec{S}$$

(B in Tesla)

$$\vec{\mu} = g\,\frac{e}{2mc}\,\vec{S} \quad \text{if} \quad B \to \frac{\sqrt{MeV}}{fm^{3/2}}$$

$$H = -\vec{\mu}\cdot\vec{B}$$

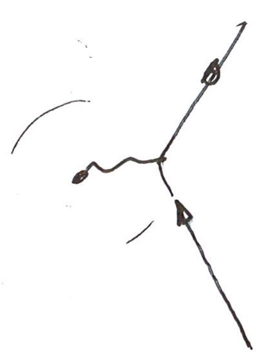
Electron → elementary particle. e, m should determine $\vec{\mu}$.

From QED theory is renormalizable, only e, m are couplings.

$$\mathcal{L} = \bar{\psi}(i\not{\partial} - m)\psi - \underbrace{e\bar{\psi}\not{A}\psi}_{\text{coupling}}$$

$$= \bar{\psi}(i\not{D} - m)\psi \qquad D_\mu = \partial_\mu + ieA_\mu$$

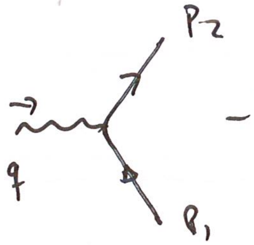
$$V = \int e\bar{\psi}\gamma^\mu\psi \, A_\mu^{cl}(x)\, d^3x$$



scattering by a field.

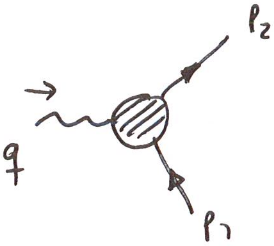
effective vertex

$$\simeq \int -\vec{\mu}\cdot\vec{B}\, d^3x = -\vec{\mu}\cdot\vec{B}$$



$-ie\gamma^\mu$

$\rightarrow$



$-ie\Gamma^\mu(p_1,p_2)$

$e\bar\psi\gamma^\mu\psi A_\mu$

$P_2 = P_1 + q$

$$\mathcal{M}_{fi} = -ie\,\bar u^{s_2}_{p_2}\gamma^\mu u^{s_1}_{p_1} A^{d}_\mu(q)$$

$$\mathcal{M}_{fi} = -ie\,\bar u^{s_2}_{p_2}\Gamma^\mu(p_1,p_2)\, u^{s_1}_{p_1} A^{d}_\mu(q)$$

$\Gamma^\mu(p_1,p_2)$ ← 4×4 matrix in Dirac indices.

$p_1^2 = m^2 \quad p_2^2 = m^2 \qquad (p_1-p_2)^2 = 2m^2 - 2\underbrace{p_1\cdot p_2} = q^2$

only invariant q^2

$$\Gamma^\mu(p_1,p_2) = \alpha p_1^\mu + \beta p_2^\mu + \delta\gamma^\mu$$

α,β,γ functions of q^2

$$A_\mu \to A_\mu + \partial_\mu\alpha \quad \to \quad A_\mu(q) \to A_\mu(q) + A(q)\,q_\mu$$

$\underline{\Gamma^\mu q_\mu = 0} \qquad \bar u^{s_2}_{p_2}\Gamma^\mu q_\mu u^{s_1}_{p_1} = 0$

$$\alpha\,\bar u_{p_2}\, p_1(p_2-p_1)u_1 + \beta\,\bar u_2\, p_2(p_2-p_1)u_2 + \delta\,\underbrace{\bar u_2(\not p_2-\not p_1)u_1}_{0}$$

$\not p_1 u_1 = m u_1 \qquad \bar u_2\not p_2 = m p_2$

$\alpha(p_1p_1 - p_1^2) + \beta(p_2^2 - p_1p_2)$

$\alpha = \beta$

$\underline{\alpha\,\bar u_{p_2}(p_1p_2 - p_1^2 + p_2^2 - p_1p_2)u_2}$

$$\Gamma^\mu = \cancel{\alpha(q)}\ \cancel{F_1(q^2)}\ \alpha(q^2)\,(p_1^\mu + p_2^\mu) + \delta(q^2)\gamma^\mu$$

$$\bar{u}_2 \gamma^\mu u_1 = \bar{u}_2 \left[\frac{p_1^\mu + p_2^\mu}{2m} + \frac{i\sigma^{\mu\nu} q_\nu}{2m} \right] u_1$$

$$\sigma^{\mu\nu} = \frac{i}{2}[\gamma^\mu, \gamma^\nu]$$

$$\bar{u}_2 \sigma^{\mu\nu} q_\nu u_1 = \frac{i}{2} \bar{u}_2 (\gamma^\mu\gamma^\nu - \gamma^\nu\gamma^\mu)(p_{2\nu} - p_{1\nu}) u_1$$

$$= \frac{i}{2} \bar{u}_2 \left(\gamma^\mu (\not{p}_2 - \not{p}_1) - (\not{p}_2 - \not{p}_1)\gamma^\mu\right) u_1$$

$$= \frac{i}{2} \bar{u}_2 \left(-\not{p}_2\gamma^\mu + 2p_2^\mu - \gamma^\mu \not{p}_1 - \not{p}_2\gamma^\mu + \gamma^\mu\not{p}_1 + 2p_1^\mu\right) u_1$$

$$\left(\gamma^\mu \not{a} = \gamma^\mu a_\nu \gamma^\nu = -\gamma^\nu\gamma^\mu a_\nu + 2g^{\mu\nu} a_\nu = -\not{a}\gamma^\mu + 2a^\mu\right)$$

$$= \frac{i}{2} \bar{u}_2 \left(\underline{-m\gamma^\mu} + 2p_2^\mu \underline{- m\gamma^\mu} \underline{- m\gamma^\mu} \underline{- m\gamma^\mu} + 2p_1^\mu\right) u_1$$

$$= \frac{i}{2} \bar{u}_2 \left(-4m\gamma^\mu + 2(p_1^\mu + p_2^\mu)\right) u_1$$

$$= -2im\, \bar{u}_2 \gamma^\mu u_1 + i\, \bar{u}_2 (p_1^\mu + p_2^\mu) u_1$$

$$\bar{u}_2 u_1 (p_1^\mu + p_2^\mu) = 2m \bar{u}_2 \gamma^\mu u_1 - i\, \bar{u}_2 \sigma^{\mu\nu} q_\nu u_1$$

$$\bar{u}_2 \Gamma^\mu u_1 = F_1(q^2)\gamma^\mu + F_2(q^2) \frac{i\sigma^{\mu\nu} q_\nu}{2m}$$

Effective Vertex

$$\psi_\alpha(x) = \sum_{\sigma=1,2} \int \frac{d^3k}{(2\pi)^3} \frac{1}{\sqrt{2\omega_k}} \left(e^{-ikx} u_\alpha^{(\sigma)}(k)\, c_{\sigma,\vec{k}} + e^{ikx} v_\alpha^{(\sigma)}(k)\, d^\dagger_{\sigma,\vec{k}} \right)$$

$$\bar{\psi}_\alpha = \psi^\dagger \gamma_0 = \sum_{\sigma=1,2} \int \frac{d^3k}{(2\pi)^3} \frac{1}{\sqrt{2\omega_k}} \left(e^{ikx} \bar{u}_\alpha^{(\sigma)}(k)\, c^\dagger_{\sigma,\vec{k}} + e^{-ikx} \bar{v}_\alpha^{(\sigma)}(k)\, d_{\sigma,\vec{k}} \right)$$

$$\{ c_{\sigma,\vec{k}}, c^\dagger_{\sigma',\vec{k}'} \} = (2\pi)^3\, \delta^{(3)}(\vec{k}-\vec{k}')\, \delta_{\sigma\sigma'}$$

$$|1_{k,\sigma}\rangle = \sqrt{2\omega_k}\; c^\dagger_{k,\sigma} |0\rangle \Rightarrow c_{k',\sigma'} |1_{k,\sigma}\rangle = (2\pi)^3 \sqrt{2\omega_k}\, \delta(\vec{k}-\vec{k}')\, \delta^{ss'} |0\rangle$$

$$\boxed{\langle 0|0\rangle = 1} \qquad \langle 1_{k,\sigma}| c^\dagger_{k',\sigma'} = (2\pi)^3 \sqrt{2\omega_k}\, \delta(\vec{k}-\vec{k}')\, \delta^{ss'} \langle 0|$$

$$\langle 1_{k,\sigma} | 1_{k',\sigma'} \rangle = 2\omega_k \langle 0| c_{k,\sigma} c^\dagger_{k',\sigma'} |0\rangle = (2\pi)^3\, 2\omega_k\, \delta^{(3)}(k-k')$$

$$V = \lambda_1 \int d^3x\; i(\bar{\psi} \partial^\mu \psi - \partial^\mu \bar{\psi} \psi) A_\mu$$

$$e^{-i\int dt V} \xrightarrow{1^{st}\text{ order}} -i\lambda_1 \int d^4x\; i\,(\bar{\psi}\partial^\mu\psi - \partial^\mu\bar{\psi}\psi) A_\mu$$

$$\lambda_1 \int \langle 1_{p_2\sigma_2} | \bar{\psi} \partial^\mu \psi - \partial^\mu \bar{\psi} \psi\, | 1_{p_1\sigma_1} \rangle \langle A_\mu \rangle\, d^4x$$

$$\bar{u}_{p_2}^{\sigma_2} u_{p_1}^{\sigma_1} (-ip_1^\mu - ip_2^\mu)\; e^{-ip_1x + ip_2x}$$

$$= -i\lambda_1\, \bar{u}_2 u_1 \int d^4x\; (p_1^\mu + p_2^\mu) \langle A_\mu(x) \rangle\, e^{-i(p_1-p_2)x}$$

$$= -i\lambda_1\, \bar{u}_2 u_1\, (p_1^\mu + p_2^\mu)\, A_\mu^{(cl)}(q)$$

$$V = \lambda_2 \int d^3x \; \bar{\psi}\, \sigma^{\mu\nu} \psi \; F_{\mu\nu}$$

$$-i \lambda_2 \int d^4x \; F^{(cl)}_{\mu\nu} \quad \langle 1_{p_2 \sigma_2} | \; \bar{\psi}\, \sigma^{\mu\nu} \psi \; | 1_{p_1 \sigma_1} \rangle$$

$$\bar{u}^{\sigma_2}_{p_2}\, \sigma^{\mu\nu} u^{\sigma_1}_{p_1} \; \underbrace{e^{ip_2 x - ip_1 x}}_{e^{iqx}}$$

$$-i\lambda_2 \; \bar{u}_2 \sigma^{\mu\nu} u_1 \quad \int d^4x \; (\partial_\mu A_\nu - \partial_\nu A_\mu)\, e^{iqx}$$

$$-iq_\mu A_\nu + iq_\nu A_\mu$$

$$-2\lambda_2 \; \bar{u}_2 \sigma^{\mu\nu} u_1 \; q_\mu A^{cl}_\nu \; = \; 2\lambda_2 \; \bar{u}_2 \sigma^{\mu\nu} u_1 \; q_\nu A^{cl}_\mu$$

$$-ie\, \bar{u}_2 \Gamma^\mu u_1 \underset{q^2 \to 0}{\simeq} - ie \; F_1(0) \; \bar{u}_2 u_1 \; \frac{p_1^\mu + p_2^\mu}{2m} \; + e \; (F_1 + F_2) \; \frac{\bar{u}_2 \sigma^{\mu\nu} q_\nu u_1}{2m}$$

$$\lambda_2 = \frac{e}{4m}\, (F_1 + F_2) \qquad\qquad \lambda_1 = \frac{e}{2m} F_1$$

$$V = \frac{e}{2m} F_1(0) \int d^3x \;\; i\,(\bar{\psi}\, \partial^\mu \psi - \partial^\mu \bar{\psi}\, \psi)\, A_\mu$$

$$+ \frac{e}{4m}\, (F_1 + F_2) \int d^3x \;\; \bar{\psi}\, \sigma^{\mu\nu} \psi \; F_{\mu\nu}$$

$$B = B_z \hat{z} \qquad B_z = \partial_x A_y - \partial_y A_x = F_{xy}$$

$$\scriptstyle {}^{12}{}_{12} + {}^{21}{}_{21}$$

$$V = -\frac{eF_2}{4m} \; 2 \int d^3x \, \langle \bar{\psi} \sigma^{12} \psi \rangle B_z$$

$$\sigma^{12} = \frac{i}{2}[\gamma^1, \gamma^2] = \frac{i}{2}\left[\begin{pmatrix} 0 & \sigma_1 \\ -\sigma_1 & 0 \end{pmatrix}\begin{pmatrix} 0 & \sigma_2 \\ -\sigma_2 & 0 \end{pmatrix} - \begin{pmatrix} 0 & \sigma_2 \\ -\sigma_2 & 0 \end{pmatrix}\begin{pmatrix} 0 & \sigma_1 \\ \sigma_1 & 0 \end{pmatrix}\right]$$

$$= \frac{i}{2}\begin{pmatrix} -[\sigma_1,\sigma_2] & 0 \\ 0 & -\underbrace{[\sigma_1,\sigma_2]}_{2i\sigma_3} \end{pmatrix} = -\frac{i}{2} 2i \begin{pmatrix} \sigma_3 & 0 \\ 0 & \sigma_3 \end{pmatrix} = \begin{pmatrix} \sigma_3 & 0 \\ 0 & \sigma_3 \end{pmatrix}$$

$$V = -\frac{eF_2(0)}{2m} B_3 \int d^3x \, \bar{u}^\sigma \sigma^{12} u^\sigma \, e^{i(k'-k)x}$$

$\sqrt{m}\begin{pmatrix} \xi \\ \xi \end{pmatrix}$ normalization

$$= -\frac{eF_2(0)}{2m} B_3 \, (2\pi)^3 \delta^{(3)}(\vec{k}'-\vec{k}) \; m \underbrace{(\xi^\dagger \xi^\dagger)\begin{pmatrix} \sigma_3 & 0 \\ 0 & \sigma_3 \end{pmatrix}\begin{pmatrix} \xi \\ \xi \end{pmatrix}}_{2\xi^\dagger \sigma_3 \xi = 4 S_3}$$

$(\sigma_3/2 = S_3)$

$$= -\frac{eF_2(0)}{m} S_3 B_3 \underbrace{(2\pi)^3 2m \, \delta^{(3)}(k'-k)}$$

$$\langle 1_{p'\sigma} | 1_{p\sigma} \rangle = (2\pi)^3 2\omega_k \, \delta^{(3)}(k-k')$$

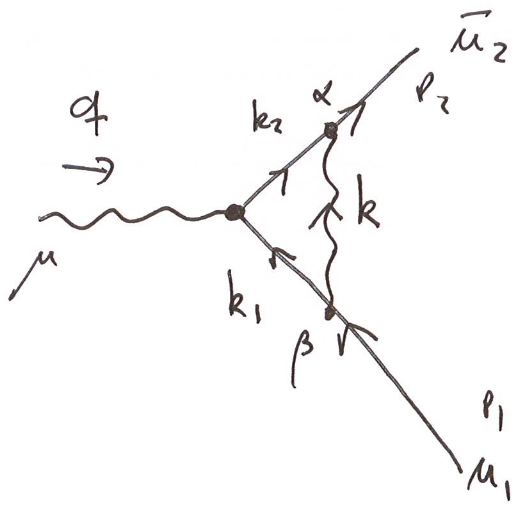
$\omega_k \to m$ at rest.

$$= -\frac{e}{m} F_2(0) \, B_3 \cdot S_3$$

$$\mu = \frac{e}{m}(F_1(0) + F_2(0)) \vec{S} \qquad \Rightarrow g = 2(F_1 + F_2)$$

$\boxed{F_1 = 1}$ charge renormalization

$$\boxed{g = 2 + 2F_2(0)}$$



$$k_1 = p_1 - k$$

$$k_2 = k_1 + q = p_2 - k$$

$$(-ie)^2 \quad i^2 \int \frac{d^4k}{(2\pi)^4} \quad \bar{u}_2 \gamma^\alpha \frac{\not{k}_2 + m}{k_2^2 - m^2 + i\varepsilon} \gamma^\mu \frac{\not{k}_1 + m}{k_1^2 - m^2 + i\varepsilon} \gamma^\beta u_1 \quad \frac{(-i\eta_{\alpha\beta})}{k^2 + i\varepsilon}$$

$$\bar{u}_2 \gamma^\alpha (\not{k}_2 + m) \gamma^\mu (\not{k}_1 + m) \gamma^\beta u_1 \quad \eta_{\alpha\beta}$$

$$\gamma^\alpha \gamma^\mu \gamma_\alpha = -\gamma^\alpha \gamma_\alpha \gamma^\mu + 2\gamma^\mu = -4\gamma^\mu + 2\gamma^\mu = -2\gamma^\mu$$

$$\gamma^\alpha \gamma^\mu \gamma^\nu \gamma_\alpha = -\gamma^\alpha \gamma^\mu \gamma_\alpha \gamma^\nu + 2\gamma^\nu \gamma^\mu = 2(\gamma^\mu \gamma^\nu + \gamma^\nu \gamma^\mu) = 4\eta^{\mu\nu}$$

$$\gamma^\alpha \gamma^\mu \gamma^\nu \gamma^\rho \gamma_\alpha = -\gamma^\alpha \gamma^\mu \gamma^\nu \gamma_\alpha \gamma^\rho + 2\gamma^\rho \gamma^\mu \gamma^\nu = -4\eta^{\mu\nu} \gamma^\rho + 2\gamma^\rho \gamma^\mu \gamma^\nu$$

$$= -2\gamma^\rho \gamma^\nu \gamma^\mu$$

$$-ie^2 \int \frac{d^4k}{(2\pi)^4} \frac{\bar{u}_2 \left(-2 \not{k}_1 \gamma^\mu \not{k}_2 + m\, 4k_1^\mu + m^2(-2\gamma^\mu) + m\, 4k_2^\mu\right) u_1}{(k_2^2 - m^2 + i\varepsilon)(k_1^2 - m^2 + i\varepsilon)(k^2 + i\varepsilon)}$$

$$\int d\alpha\, d\beta\, d\gamma \frac{2\,\delta(\alpha + \beta + \gamma - 1)}{(\alpha k_2^2 - \alpha m^2 + \beta k_1^2 - \beta m^2 + \gamma k^2 + i\varepsilon)^3}$$

$$\alpha(p_2-k)^2+\beta(p_1-k)^2-(\alpha+\beta)m^2+\gamma k^2+i\varepsilon$$

$$\cancel{\alpha m^2}-2\alpha p_2 k+\underline{\alpha k^2}+\cancel{\beta m^2}-2\beta p_1\cdot k+\underline{\beta k^2}-\cancel{(\alpha+\beta)m^2}+\underline{\gamma k^2}+i\varepsilon$$

$$k^2-2(\alpha p_2+\beta p_1)k+i\varepsilon$$

$$\underbrace{(k-\alpha p_2-\beta p_1)^2}_{\tilde{k}}-(\alpha p_2+\beta p_1)^2+i\varepsilon$$

$$(p_2-p_1)^2=q^2=2m^2-2p_1p_2 \qquad 2p_1p_2=2m^2-q^2$$

$$\tilde{k}^2-\alpha^2m^2-\beta^2m^2-2\alpha\beta\, p_1\cdot p_2+i\varepsilon$$

$$\tilde{k}^2-(\alpha^2+\beta^2+2\alpha\beta)m^2+\alpha\beta q^2+i\varepsilon$$

$$\bar{u}_2\left(-2(\not{p}_1-\not{k})\gamma^\mu(\not{p}_2-\not{k})+4m(p_1^\mu+p_2^\mu-2k^\mu)-2m^2\gamma^\mu\right)u_1$$

$$+4m(p_1^\mu+p_2^\mu-2k^\mu-2\alpha p_2^\mu-2\beta p_1^\mu)-2m^2\gamma^\mu$$

$$-2ie^2\int d\alpha\, d\beta\, d\gamma\;\delta(\alpha+\beta+\gamma-1)\int\frac{d^4k}{(2\pi)^4}\frac{\bar{u}_2\left(-2(\not{p}_1-\not{k}-\alpha\not{p}_2-\beta\not{p}_1)\gamma^\mu(\not{p}_2-\not{k}-\alpha\not{p}_2-\beta\not{p}_1)\right.}{(k^2-(\alpha+\beta)^2m^2+\alpha\beta q^2+i\varepsilon)^3}$$

$$\bar{u}_2\left(-2((1-\beta)\not{p}_1-\alpha\not{p}_2-\not{k})\gamma^\mu((1-\alpha)\not{p}_2-\beta\not{p}_1-\not{k})+4m((1-2\beta)p_1^\mu+(1-2\alpha)p_2^\mu-2k^\mu)-2m^2\gamma^\mu\right)u_1$$

$k\to 0$.

$$\bar{u}_2\left(-2((1-\beta)\not{p}_1-\alpha\not{p}_2)\gamma^\mu((1-\alpha)\not{p}_2-\beta\not{p}_1)-2\not{k}\gamma^\mu\not{k}+4m((1-2\beta)p_1^\mu+(1-2\alpha)p_2^\mu)-2m^2\gamma^\mu\right)u_1$$

$$\not{k}\gamma^\mu\not{k}=k_\alpha k_\beta\,\gamma^\alpha\gamma^\mu\gamma^\beta=-k_\alpha k_\beta\,\gamma^\alpha\gamma^\beta\gamma^\mu+2k_\alpha k_\mu\gamma^\alpha$$

$$=-k^2\gamma^\mu+2k_\mu\not{k}$$

$$-2\left((1-\beta)\not{p}_1-\alpha\not{p}_2\right)\gamma^\mu\left((1-\alpha)\not{p}_2-\beta\not{p}_1\right)-2\not{k}\gamma^\mu\not{k}+$$

$$+4m\left((1-2\beta)p_1^\mu+(1-2\alpha)p_2^\mu\right)-2m^2\gamma^\mu$$

$$\not{k}\gamma^\mu\not{k}=-\not{k}\not{k}\gamma^\mu+2k^\mu\not{k}=-k^2\gamma^\mu+2k^\mu k_\nu\gamma^\nu\ \underbrace{}_{\delta^\mu_\nu}\rightarrow\gamma^\mu\ \underline{\text{ }}.$$

$$-2\left((1-\beta)\not{p}_1-\alpha\not{p}_2\right)\gamma^\mu\left((1-\alpha)\not{p}_2-\beta\not{p}_1\right)+4m(1-\alpha-\beta)(p_1^\mu+p_2^\mu)$$

$$-2\left((1-\beta)\not{p}_1-\alpha m\right)\gamma^\mu\left((1-\alpha)\not{p}_2-\beta m\right)+4m(1-\alpha-\beta)(p_1^\mu+p_2^\mu)$$

$$-2(1-\alpha)(1-\beta)\not{p}_1\gamma^\mu\not{p}_2+2\alpha m(1-\alpha)\underbrace{\gamma^\mu\not{p}_2}_{\cancel{-\not{p}_2\gamma^\mu}+2p_2^\mu}+2\beta m(1-\beta)\underbrace{\not{p}_1\gamma^\mu}_{2p_1^\mu}+4m(1-\alpha-\beta)(p_1^\mu+p_2^\mu)$$

$$-2(1-\alpha)(1-\beta)\left(-\gamma^\mu\underbrace{\not{p}_1\not{p}_2}_{-\not{p}_2\not{p}_1+\underbrace{2p_1\cdot p_2}_{\gamma}}+2p_1^\mu\not{p}_2\right)+4\alpha(1-\alpha)m\,p_2^\mu+4\beta(1-\beta)m\,p_1^\mu+$$
$$+4m(1-\alpha-\beta)(p_1^\mu+p_2^\mu)$$

$$-2(1-\alpha)(1-\beta)\left(m\underbrace{\gamma^\mu\not{p}_2}_{-\not{p}_2\gamma^\mu+2mp_2^\mu}+2mp_1^\mu\right)+4\alpha(1-\alpha)m(p_1^\mu+p_2^\mu)+$$
$$+4m(1-\alpha-\beta)(p_1^\mu+p_2^\mu)$$

$$\left(-4m(1-\alpha)(1-\beta)+2m\alpha(1-\alpha)+2m\beta(1-\beta)+4m(1-\alpha-\beta)\right)(p_1^\mu+p_2^\mu)$$

$$2m(p_1^\mu+p_2^\mu)\left(-2(1-\alpha)(1-\beta)+\alpha(1-\alpha)+\beta(1-\beta)+2(1-\alpha-\beta)\right)$$

$$-ie^2 \int \frac{d^4k}{(2\pi)^4} \int d\alpha\, d\beta\, d\gamma \; \frac{2\,\delta(\alpha+\beta+\gamma-1)\; 2m\, \bar{u}_2 u_1\, (p_1^\mu + p_2^\mu) \times}{(k^2 - (\alpha+\beta)^2 m^2 + \alpha\beta q^2 + i\varepsilon)^3}$$

$$\times\left(\cancel{-2} + \cancel{2\alpha} + \cancel{2\beta} - 2\alpha\beta + \alpha - \alpha^2 + \beta - \beta^2 \cancel{+2} - \cancel{2\alpha} - \cancel{2\beta}\right)$$

$$\alpha+\beta - (\alpha+\beta)^2 = (\alpha+\beta)(\alpha+\beta-1)$$

$$-ie^2 \int_0^1 d\alpha \int_0^{1-\alpha} d\beta \; 4m\, \bar{u}_2 u_1\, (p_1^\mu + p_2^\mu) \int \frac{d^4k}{(2\pi)^4} \frac{(\alpha+\beta)(\alpha+\beta-1)}{(k^2 - (\alpha+\beta)^2 m^2 + \alpha\beta q^2 + i\varepsilon)^3}$$

$$\frac{(-)^3\, i\, \Gamma(1)}{(4\pi)^2\, \Gamma(3)} \; \frac{1}{((\alpha+\beta)^2 m^2 - \alpha\beta q^2)}$$

$$= -\frac{e^2}{32\pi^2}\, 4m\, \bar{u}_2 u_1\, (p_1^\mu + p_2^\mu) \int_0 d\alpha\, d\beta\, d\gamma\; \delta(\alpha+\beta+\gamma-1)\, \frac{(\alpha+\beta)(\alpha+\beta-1)}{((\alpha+\beta)^2 m^2 - \alpha\beta q^2)^3}$$

$$\bar{u}_2 \Gamma^\mu u_1 \sim \frac{-e^2}{32\pi^2}\, 4m\, \underbrace{\bar{u}_2 u_1\, (p_1^\mu + p_2^\mu)}_{-i\, \bar{u}_2 \sigma^{\mu\nu} q_\nu u_1} \int - -$$

$$-\frac{e^2}{8\pi^2}\, 2m^2\, \frac{\bar{u}_2 \sigma^{\mu\nu} q_\nu u_1}{2m} \int - -$$

(11)

$$F_2(q^2) = -\frac{e^2 m^2}{4\pi^2}\int d\alpha\, d\beta\, d\gamma\ \delta(\alpha+\beta+\gamma-1)\frac{(\alpha+\beta)(\alpha+\beta-1)}{((\alpha+\beta)^2 m^2 - \alpha\beta q^2)}$$

$$F_2(0) = -\frac{e^2 \cancel{m^2}}{4\pi^2}\frac{1}{\cancel{m^6}}\int d\alpha\, d\beta\, d\gamma\ \delta(\alpha+\beta+\gamma-1)\frac{(1-\gamma)(-\gamma)}{(1-\gamma)^2}$$

$$= +\frac{e^2}{4\pi^2}\int_0^1 d\gamma \int_0^{1-\gamma} d\beta\ \frac{\gamma}{(1-\gamma)}$$

$$= \frac{e^2}{4\pi^2}\int_0^1 d\gamma\ \gamma = \frac{e^2}{8\pi^2} = \frac{\alpha}{2\pi}$$

$e^2/4\pi = \alpha \quad \Rightarrow \quad g = 2 + \alpha/\pi$

$\alpha^5 \sim 10^{-13}$

3 e difference

$g_e = 2.00\underline{23}\,\underline{19}\,304\,362\,56(35)$

$g_\mu = 2.00\underline{23}\,\underline{3}18418(13)$

8 difference

2.0023193044

~~2.002331~~

2.002331836 1

$g_{\text{hadronic}} \sim 10^{-12}$

$\alpha = 0.007\,297\,352\,5693(1)$

$\frac{\alpha}{\pi} = 0.0023228$