Superconformal symmetry.

a) Supersymmetry

Supersymmetric algebra.

Includes fermionic operators.

$\boxed{4d}$

$$M_{\mu\nu},\quad P_\mu,\quad Q_\alpha{}^A \underset{\dagger}{\longleftrightarrow} \bar{Q}_{\dot\alpha A}$$

$$\{Q_\alpha{}^A, \bar{Q}_{\dot\beta B}\} = 2\sigma^\mu_{\alpha\dot\beta} P_\mu \delta^A{}_B$$

$$\{Q_\alpha{}^A, Q_\beta{}^B\} = \{\bar{Q}_{\dot\alpha A}, \bar{Q}_{\dot\beta B}\} = 0$$

$$[P_\mu, Q_\beta{}^A] = [P_\mu, \bar{Q}_{\dot\alpha A}] = 0$$

$$[P_\mu, P_\nu] = 0$$

<u>Lorentz symmetry.</u>

$\sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ $\sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$ $\sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ ②

$\sigma^0 = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$

$$\Delta s^2 = -t^2 + x^2 + y^2 + z^2$$

$$= -x_0^2 + x_1^2 + x_2^2 + x_3^2$$: preserves interval.

$$\mathbb{X} = \begin{pmatrix} x_0 + x_3 & x_1 - i x_2 \\ x_1 + i x_2 & x_0 - x_3 \end{pmatrix} = x_0 \sigma^0 + x_j \sigma^j = x_\mu \sigma^\mu$$

$$\det \mathbb{X} = x_0^2 - x_1^2 - x_2^2 - x_3^2 = -\Delta s^2$$

$\mathbb{X}^+ = \mathbb{X} \leftarrow \mathbb{X}$: most general

$$\underbrace{\tilde{\mathbb{X}} = A}_{\text{linear.}} \mathbb{X} A^+ \quad ; \quad \tilde{\mathbb{X}}^+ = A \mathbb{X}^+ A^+ = A \mathbb{X}^+ A = \tilde{\mathbb{X}}$$

$$\det \tilde{\mathbb{X}} = |\det A|^2 \det \mathbb{X} \qquad |\det A| = 1.$$

$$\det A = e^{i\alpha} \qquad \tilde{A} = e^{-i\alpha/2} A \qquad \det \tilde{A} = 1$$

↑ generates the same transf.

$A \in SL(2, \mathbb{C})$ 2x2 complex matrices. with det 1.

$SO(3,1) \equiv SL(2, \mathbb{C})$ up to double cover.

$A = \mathbb{1}, -\mathbb{1}$ same.

↑ 2π rotation.

if $A \in SU(2)$ $\quad \tilde{X} = x_0 \underbrace{A \sigma^0 A^+}_{\sigma^0} + x_j A \sigma^j A^+$

$$= x_0 \sigma^0 + x_j R_{jk} \sigma^k \quad \text{rotation.}$$

Spinors : 2 kinds.

$$\xi \to A\xi \quad \text{left}$$

$$\eta \to A^*\eta \quad \text{right.}$$

if ξ left ξ^* right.

for $SU(2)$ they rotate as usual.

Indeed.

$$U = \begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix} \qquad \alpha\delta - \beta\gamma = 1 \qquad U^\dagger = \begin{pmatrix} \alpha^* & \gamma^* \\ \beta^* & \delta^* \end{pmatrix}$$

$$U^{-1} = \begin{pmatrix} \delta & -\beta \\ -\gamma & \alpha \end{pmatrix} \qquad =$$

$$U = \begin{pmatrix} \alpha & \beta \\ -\beta^* & \alpha^* \end{pmatrix} \qquad |\alpha|^2 + |\beta|^2 = 1.$$

$$U^* = \begin{pmatrix} \alpha^* & \beta^* \\ -\beta & \alpha \end{pmatrix}$$

$$\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} \alpha & \beta \\ -\beta^* & \alpha^* \end{pmatrix}\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} = \begin{pmatrix} -\beta^* & \alpha^* \\ -\alpha & -\beta \end{pmatrix}\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} = \begin{pmatrix} -\alpha^* & -\beta^* \\ \beta & -\alpha \end{pmatrix} = -\begin{pmatrix} \alpha^* & \beta^* \\ -\beta & \alpha \end{pmatrix}$$

$$\boxed{U^* = \sigma_2 U \sigma_2}$$ same rep. for $SU(2)$ but different for $SL(2,\mathbb{C})$.

$$\tilde{\xi} = U\xi$$

$$\tilde{\eta} = U^*\eta = \sigma_2 U \sigma_2 \eta \qquad \underbrace{\sigma_2\tilde{\eta}}_{\tilde{\xi}} = U\underbrace{(\sigma_2\eta)}_{\xi}$$

$$A_{\alpha\beta} A_{\gamma\delta} \varepsilon_{\beta\delta} = A_{\alpha 1} A_{\gamma 2} - A_{\alpha 2} A_{\gamma 1} = \varepsilon_{\alpha\gamma}$$

$\varepsilon_{\alpha\beta} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$ if $\alpha = \gamma \to 0$

$\alpha = 1 \quad \gamma = 2 \qquad A_{11} A_{22} - A_{12} A_{21} = \det A = 1$

$\alpha = 2 \quad \gamma = 1 \qquad A_{12} A_{22} - A_{22} A_{11} = -1$

$\varepsilon_{\alpha\beta}$: invariant symbol (instead of $\delta_{\alpha\beta}$).

$$\tilde{X}_{\alpha\dot\beta} = A_{\alpha\gamma} X_{\gamma\dot\delta} A^{\dagger}_{\dot\delta\dot\beta} = A_{\alpha\gamma} A^{*}_{\dot\beta\dot\delta} X_{\gamma\dot\delta}$$

$\boxed{\sigma^{\mu}_{\alpha\dot\beta}}$ with a left & right spinor we can get a <u>vector</u>.

<u>Representations</u> $A = 1 \ldots N$

Consider massive state at rest $P_\mu = (-M, 0, 0, 0)$

$$\{Q^A_\alpha, \bar{Q}_{\dot\beta B}\} = 2M (+\mathbb{1})_{\alpha\dot\beta} \, \delta^A{}_B$$

$P^\mu = (\varepsilon, \vec{p})$

$P_\mu = (-\varepsilon, \vec{p})$

$$a^A{}_\alpha = \frac{1}{\sqrt{2M}} Q^A{}_\alpha$$

$$(a^A{}_\alpha)^\dagger = \frac{1}{\sqrt{2M}} \bar{Q}_{\dot\alpha A}$$

$$\{a^A{}_\alpha, (a^B{}_\beta)^\dagger\} = \delta^{AB} \delta_{\alpha\beta}$$

↑

$2N$ fermions.

<u>2^{2N} states</u> $\qquad a^A_\alpha |\Omega\rangle = 0$.

5

$\boxed{M=0}$ $\qquad P_\mu = (-k,0,0,k)$

$$\{Q^A_\alpha, \bar{Q}_{\dot\beta B}\} = k(\sigma_0+\sigma_3)\delta^A_B = \begin{pmatrix} 2k & 0 \\ 0 & 0 \end{pmatrix}_{\alpha\dot\beta} \delta^A{}_B$$

$$a^A_1 = \frac{1}{\sqrt{2k}} Q^A_\alpha \qquad \{(a^A{}_1), (a^B{}_1)^\dagger\} = \delta^{AB}$$

$$\{a^A_2, (a^B_2)^\dagger\} = 0$$

$$cc^\dagger + c^\dagger c = 0 \qquad \| c^\dagger|\Omega\rangle\|^2 = \langle\Omega|cc^\dagger|\Omega\rangle = -\langle\Omega|c^\dagger c|\Omega\rangle$$

$$= 0.$$

$$\underline{c^\dagger|\Omega\rangle = 0} \qquad c|\Omega\rangle = 0$$

2^N states <u>only</u>.

BPS multiplet.

Central charge.

$$\{Q^L_\alpha, (Q^M_\beta)^\dagger\} = 2\sigma^\mu_{\alpha\dot\beta} P_\mu \delta^L{}_M$$

$$\{Q^A_\alpha, Q^B_\beta\} = \varepsilon_{\alpha\beta} Z^{[AB]}$$

$$\{\bar{Q}_{\dot\alpha A}, \bar{Q}_{\dot\beta B}\} = \varepsilon_{\dot\alpha\dot\beta} Z^*_{[AB]}$$

$$[P_\mu, P_\nu] = 0 \qquad [P_\mu, Q^A_\beta] = [P_\mu, Q^\dagger_{\dot\alpha A}] = 0$$

$$Z^{AB} = Z\,\epsilon^{AB} \quad \text{example} \quad \underline{\mathcal{N}=2}$$

$$\{Q^L_\alpha, (Q^M_\beta)^\dagger\} = 2\sigma^\mu_{\alpha\dot\beta} P_\mu \delta^L_M$$

$$\{Q^A_\alpha, Q^B_\beta\} = \epsilon_{\alpha\beta}\,\epsilon^{AB}\, Z$$

$$\{\bar Q_{\dot\alpha A}, \bar Q_{\dot\beta B}\} = \epsilon_{\dot\alpha\dot\beta}\,\epsilon_{AB}\, Z$$

$$P_\mu = (-M, 0, 0, 0)$$

$$\{Q^L_\alpha, (Q^M_\beta)^\dagger\} = 2M\,\delta_{\alpha\dot\beta}\,\delta_{LM}$$

$$\{Q^A_\alpha, Q^B_\beta\} = \epsilon_{\alpha\beta}\,\epsilon^{AB}\, Z$$

$$\{(Q^A_\alpha)^\dagger, (Q^B_\beta)^\dagger\} = \epsilon_{\alpha\beta}\,\epsilon_{AB}\, Z$$

$$Q^L_\alpha : \quad Q^1_1 \quad Q^1_2 \quad Q^2_1 \quad Q^2_2$$

<u>non-zero</u>

$$\{Q^1_1, (Q^1_1)^\dagger\} = 2M \qquad \{Q^1_2, (Q^1_2)^\dagger\} = 2M$$

$$\{Q^2_1, (Q^2_1)^\dagger\} = 2M \qquad \{Q^2_2, (Q^2_2)^\dagger\} = 2M$$

$$\{Q^1_1, Q^2_2\} = Z \qquad \{Q^2_1, Q^1_2\} = -Z \qquad \{(Q^1_1)^\dagger, (Q^2_2)^\dagger\} = Z$$

$$\{(Q^2_1)^\dagger, (Q^1_2)^\dagger\} = -Z$$

$$a_1 = \frac{1}{\sqrt{2}}\left(Q^1_1 + (Q^2_2)^\dagger\right) \qquad b_1 = \frac{1}{\sqrt{2}}\left(Q^1_1 - Q^{\dagger 2}_2\right)$$

$$a_2 = \frac{1}{\sqrt{2}}\left(Q^1_2 - (Q^2_1)^\dagger\right) \qquad b_2 = \frac{1}{\sqrt{2}}\left(Q^1_2 + Q^{\dagger 2}_1\right)$$

$$\{a_1, a_1^\dagger\} = \frac{1}{2}\left\{Q^1_1 + (Q^2_2)^\dagger, (Q^1_1)^\dagger + Q^2_2\right\}$$

$$= \frac{1}{2}\left(2M + Z + Z + 2M\right) = 2M + Z$$

$$\{b_1, b_1^\dagger\} = \frac{1}{2}\left\{Q^1_1 - Q^{\dagger 2}_2, (Q^1_1)^\dagger - Q^2_2\right\}$$

$$= \frac{1}{2}\left(2M - Z - Z + 2M\right) = 2M - Z$$

$$Z \leq 2M$$

if $\underline{Z = 2M}$ ~~here~~ b vanishes.

2^N states short multiplet.

mass can be predicted by charge Z.

Superconformal algebra: $\mathcal{N}=1$

$$Q_\alpha\,,\quad \bar{Q}_{\dot\alpha}\,,\qquad S_\alpha\,,\quad \bar{S}_{\dot\alpha}$$

$$[D, Q_\alpha] = -\frac{i}{2} Q_\alpha \qquad\qquad [D, S_\alpha] = \frac{i}{2} S_\alpha$$

$$[D, \bar{Q}^{\dot\alpha}] = -\frac{i}{2} \bar{Q}^{\dot\alpha} \qquad\qquad [D, \bar{S}^{\dot\alpha}] = \frac{i}{2} \bar{S}^{\dot\alpha}$$

$$\{Q_\alpha, \bar{Q}_{\dot\beta}\} = 2\sigma^\mu_{\alpha\dot\beta} P_\mu \qquad\qquad \{S_\alpha, \bar{S}_{\dot\beta}\} = 2\sigma^\mu_{\alpha\dot\beta} K_\mu .$$

$\Delta \to$ $1/2$ $\quad 1/2$ $\quad +1$ $\qquad\qquad -1/2$ $\quad -1/2$ $\quad -1$

$$\{Q_\alpha, S_\beta\} = \underset{\sigma^\mu\bar\sigma^\nu}{(\sigma^{\mu\nu})_{\alpha\beta}} M_{\mu\nu} + \epsilon_{\alpha\beta}\Big(\underbrace{\tfrac{3}{2}R}_{Z.} + D\Big)$$

$$\{\bar{Q}_{\dot\alpha}, \bar{S}_{\dot\beta}\} = (\bar\sigma^{\mu\nu})_{\dot\alpha\dot\beta} M_{\mu\nu} + \epsilon_{\dot\alpha\dot\beta}\Big(\tfrac{3}{2}R - D\Big)$$

Superconformal primary

$$[S_\alpha, \phi] = 0 \qquad\qquad (\text{or } \{S_\alpha, \psi\} = 0)$$

$$[\bar{S}_{\dot\alpha}, \phi] = 0 \qquad\qquad \{\bar{S}_{\dot\alpha}, \psi\} = 0$$

Suppose (in addition) $[\bar{Q}_{\dot{\alpha}}, \Phi] = 0$ for some α.

and Φ scalar $[M_{\mu\nu}, \Phi] = 0$.

$$\Rightarrow \left[\{\bar{Q}_{\dot{\alpha}}, \bar{S}_{\dot{\beta}}\}, \Phi\right] + \{\overset{0}{[\phi, \bar{Q}_{\dot{\alpha}}]}, S_\beta\} + \{\overset{0}{[\bar{S}_{\dot{\beta}}, \phi]}, Q_\alpha\} = 0$$

$$[AB+BA, C] + \{CA-AC, B\} + \{BC-CB, A\} = 0$$

$$\cancel{ABC} + \cancel{BAC} - \cancel{CAB} - \cancel{CBA} + \cancel{CAB} + \cancel{BCA} - \cancel{ACB} - \cancel{BAC} + \cancel{ABC} + \cancel{BCA} + \cancel{CBA} + \cancel{ACB} = 0$$

$$[\underbrace{(\bar{\sigma}^{\mu\nu})_{\dot{\alpha}\dot{\beta}} M_{\mu\nu}}_{0} + \epsilon_{\dot{\alpha}\dot{\beta}} (\tfrac{3}{2}R + D), \Phi] = 0$$

$$\Rightarrow \boxed{\tfrac{3}{2}R + \Delta = 0 \quad \text{for this field.}}$$

$$\boxed{\Delta = \tfrac{3}{2}R}$$

$\mathcal{N}=4$

$$\{ S_{\alpha A}, \bar{S}^{B}_{\dot\beta} \} = 2\sigma^{\mu}_{\alpha\dot\beta} K_\mu \delta_A^{\ B}$$

$$\{ Q^A_{\alpha}, S_{\beta B} \} = \epsilon_{\alpha\beta} (\delta^A_{\ B} D + R^A_{\ B}) +$$

$$+ \frac{1}{2} \delta^A_{\ B} M_{\mu\nu} (\sigma^{\mu\nu})_{\alpha\beta}.$$

$$\boxed{\Delta = R}$$

$$\{\gamma^\mu, \gamma^\nu\} = 2\eta^{\mu\nu}$$

$$[\gamma^\mu, \gamma^\nu] = \gamma^\mu\gamma^\nu - \gamma^\nu\gamma^\mu = 2\gamma^\mu\gamma^\nu - 2\eta^{\mu\nu}$$

$$\Sigma^{\mu\nu} = \frac{i}{4}[\gamma^\mu, \gamma^\nu] = \frac{i}{2}\gamma^\mu\gamma^\nu - \frac{i}{2}\eta^{\mu\nu}$$

$$[\Sigma^{\mu\nu}, \Sigma^{\alpha\beta}] = -\frac{1}{4}[\gamma^\mu\gamma^\nu, \gamma^\alpha\gamma^\beta]$$

$$= -\frac{1}{4}\Big(\gamma^\mu 2\eta^{\nu\alpha}\gamma^\beta - \gamma^\mu\gamma^\alpha 2\eta^{\nu\beta} + 2\eta^{\mu\alpha}\gamma^\beta\gamma^\nu$$

$$- \gamma^\alpha\gamma^\nu 2\eta^{\mu\beta}\Big)$$

$$= \frac{ii}{2}\left(\eta^{\nu\alpha}\gamma^\mu\gamma^\beta - \eta^{\nu\beta}\gamma^\mu\gamma^\alpha + \eta^{\mu\alpha}\gamma^\nu\gamma^\beta - \eta^{\mu\beta}\gamma^\alpha\gamma^\nu\right)$$

$$= i\eta^{\nu\alpha}\Sigma^{\mu\beta} - i\eta^{\nu\beta}\Sigma^{\mu\alpha} + i\eta^{\mu\alpha}\Sigma^{\beta\nu} - i\eta^{\mu\beta}\Sigma^{\alpha\nu}$$

$$+ \frac{ii}{2}\frac{i}{2}\left(\cancel{\eta^{\nu\alpha}\eta^{\mu\beta}} - \cancel{\eta^{\nu\beta}\eta^{\mu\alpha}} + \cancel{\eta^{\mu\alpha}\eta^{\nu\beta}} - \cancel{\eta^{\mu\beta}\eta^{\alpha\nu}}\right)$$

$$= i\eta^{\nu\alpha}\Sigma^{\mu\beta} + i\underbrace{\eta^{\mu\alpha}\Sigma^{\beta\nu}}_{} - i\eta^{\nu\beta}\Sigma^{\mu\alpha} - i\eta^{\mu\beta}\Sigma^{\alpha\nu}$$

$$= -i\eta^{\mu\alpha}\Sigma^{\nu\beta} - i\eta^{\nu\beta}\Sigma^{\mu\alpha} + i\eta^{\mu\beta}\Sigma^{\nu\alpha} + i\eta^{\nu\alpha}\Sigma^{\mu\beta}$$

$$\gamma^\mu = \begin{pmatrix} 0 & \sigma^\mu \\ \bar\sigma^\mu & 0 \end{pmatrix} \qquad \sigma^\mu\bar\sigma^\nu + \sigma^\nu\bar\sigma^\mu = 2\eta^{\mu\nu}\,\mathbb{1}_{2\times 2}$$

$00 \to 2\cdot\mathbb{1}$ ✓

$0i \to \sigma^0\bar\sigma^i + \sigma^i\bar\sigma^0 = 0$ ✓

$i0 \to \sigma^i\bar\sigma^0 + \sigma^0\bar\sigma^i = 0$ ✓

$ij \to -\sigma^i\sigma^j - \sigma^j\sigma^i = -2\delta^{ij}\mathbb{1}$ ✓

φ

Review of AdS/CFT

(1)

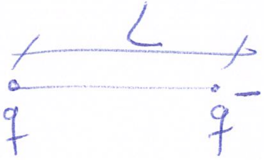
-) Gauge theories

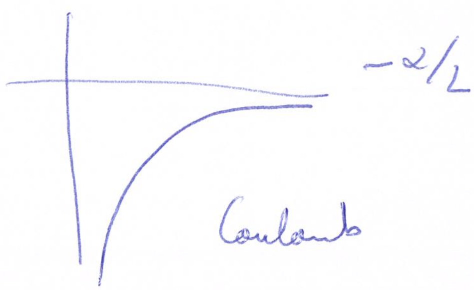
Phenomenologically important, standard model & beyond.

Interesting from theoretical point of view, e.g. AdS/CFT

-) What is interesting to compute?

°) Potential between charges.

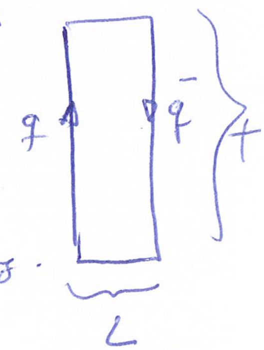


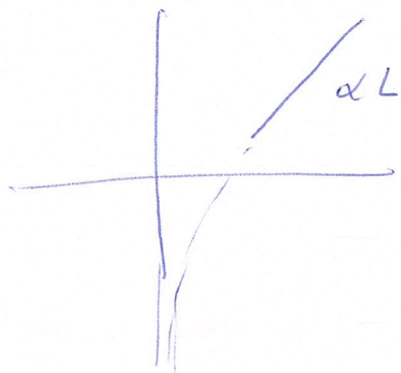


In Lattice gauge theory

It can be computed using Wilson loops

$\langle W_C \rangle = \langle P e^{i\oint_C A_\mu dx^\mu} \rangle$





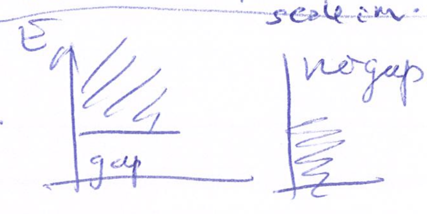
confining.

$\langle W \rangle = e^{-ET}$

$E \sim V_{q\bar{q}}$

area law vs ~~perimeter law~~ scale inv.

°) mass gap (mass of lightest particle).



°) one particle spectrum (M^2 discrete)

°) Scattering amplitudes, bound states, low energy effective theory.

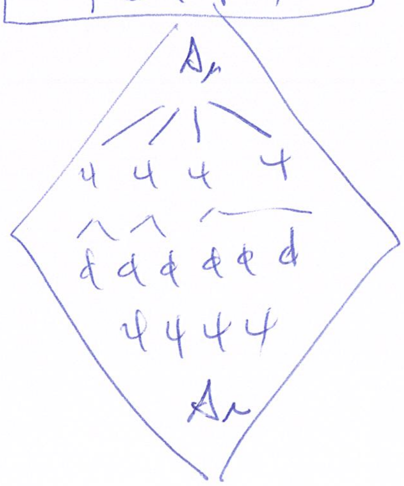
AdS/CFT

Relation between gauge theories and string theory in curved spaces (includes gravity).

Most studied example.

$\mathcal{N}=4$ SYM

$3+1$



Maximal susy

IIB ~~strings~~ strings in

$AdS_5 \times S_5$ $\quad 9+1$

$$AdS_5:\ Y_0^2+Y_{-1}^2-Y_1^2-Y_2^2-Y_3^2-Y_4^2=R^2$$

$$S_5 :\ X_1^2+X_2^2+\dots+X_6^2=R^2$$

$SO(4,2)$ conformal group.

$$ds^2=\frac{1}{z^2}\left(-dt^2+dx_{[3]}^2+dz^2\right)$$

flat space, $3+1$ dim, no gravity

curved space $9+1$ dim gravity

Parameters

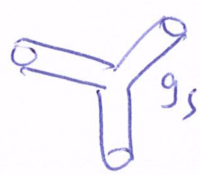
gauge group $SU(N)$

g_{YM} : coupling

$\lambda = g_{YM}^2 N$: 't Hooft coupling

l_s: string length or $\alpha' \sim l_s^{-2}$ string tension.

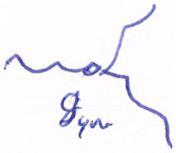
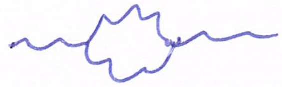
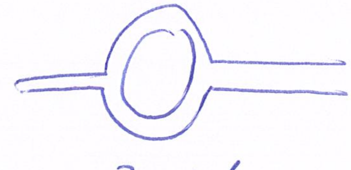
$E = \alpha' L$



probability of a string splitting.

Relation $\quad g_s \sim g_{YM}^2 \qquad R^2/\alpha' = \sqrt{g_s N} = \sqrt{\lambda}$

Meaning of the parameters and limits.

 coupling but  = 

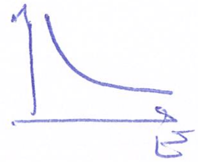
g_{YM} $\qquad\qquad$ $g^2_{YM} N$

't Hooft limit $N \to \infty$ $\quad g^2_{YM} \to 0$ $\qquad \lambda = g^2_{YM} N$ fixed.

$(g^4_{YM} N^2)/N^2$

Planar diagrams dominate.
they can be drawn on a plane without lines crossing.

$\lambda \ll 1$: perturbation in λ $\quad \Big\| \quad$ $\lambda \gg 1$ resum planar diagrams (?)

In pure Yang-Mills coupling runs 

Strings

$g_s \to 0$ $\quad$ Free strings : in 't Hooft limit $g^2_{YM} \to 0$
$\Rightarrow$ free strings.

One has to compute the spectrum of a single string.
(on curved space, otherwise harmonic osc.).

$S = \frac{1}{\alpha'} \text{Area} \sim \frac{R^2}{\alpha'} S_{adim}$ $\qquad \alpha'/R^2 \sim \hbar \sim 1/\sqrt{\lambda}$
$\uparrow$ of quantizing oscillations

$\lambda \ll 1$ large quantum effects $\qquad \lambda \gg 1$: classic strings point-like-gravity + classic strings

Directions of Research

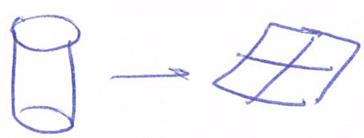
·) Study further $\mathcal{N}=4 \leftrightarrow AdS_5 \times S_5$ case and eventually solve $\mathcal{N}=4$.

$\boxed{\lambda \gg 1}$ easier case (also $\lambda \ll 1$)

One can compute the potential, mass gap, spectrum, some 3-point functions

→ $O(1)$ sphere radius.
$\lambda^{1/4}$ strings
N^2: black holes thermal state.

$\mathcal{N}=4$ is conformal $\Rightarrow$ conf. dim in $\mathbb{R}^{3,1}$ or energies in $\mathbb{R}\times S^3$.



Scattering amplitudes. using Wilson loop w/ light like cusps.

$\boxed{\text{For any } \lambda}$ Much more difficult.

$\mathcal{N}=4$ is integrable in the planar limit $N\to\infty$, $g_{YM}^2 N$ fixed

Integrability: bridge between gauge theories & strings.

Conf. dim., 3-point functions.

Scattering amplitudes → Integrand at each loop
→ Using string theory.

Very little beyond the planar limit (integrable)
→ Riemann surfaces.

-) Look for models w/ phenomenological applications.

·) Construct them using string theory.
Well defined & justified, usually not realistic.

·) Invent models as effective actions.
Not justified, useful as a guide.

There are models for QCD, plasmas, hydrodynamics, superconductors, superfluids, etc.

<u>Interesting aspects</u>:

·) Compute masses for glueballs, mesons, --

·) Symmetries such as $U(1)_A$ become geometrical (rotations)

·) Non-equilibrium dynamics are described by Einstein equations (near-equilibrium they reduce to hydro)
Normally difficult to do (Boltzmann)

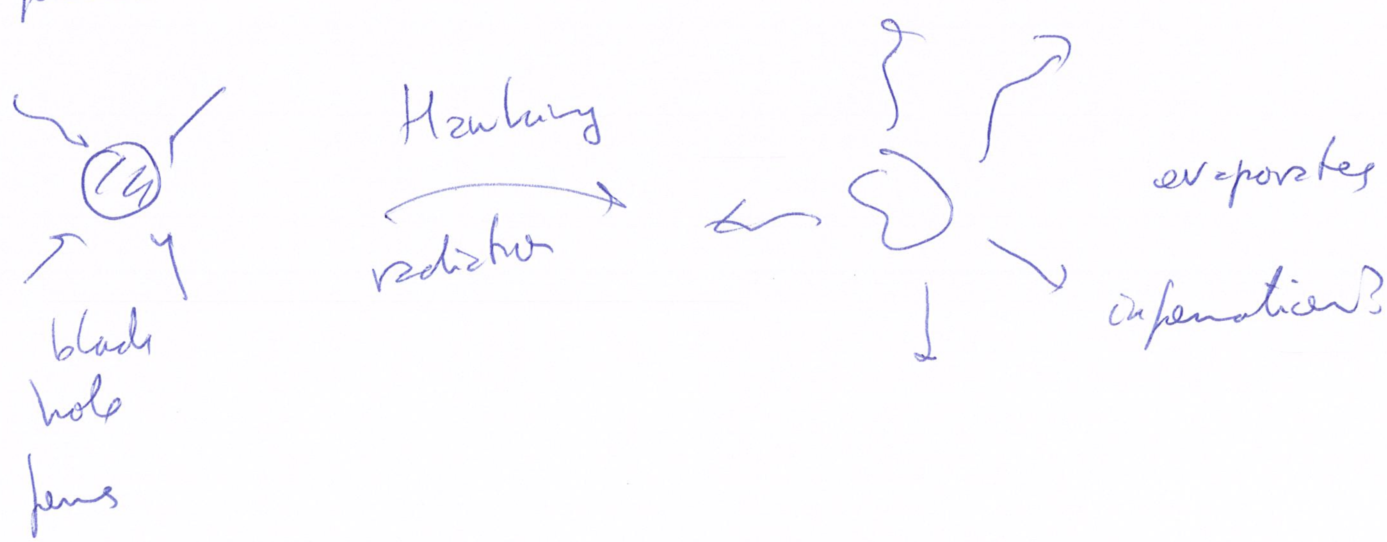
·) η/s small, RHIC plasma (LHC)

·) Superconductors, transport properties (ballistic, conventional)

·) Hydrodynamics, shock waves.

Applications to gravity

Corrections $1/N \sim g_s$ correspond to quantum gravity.
Problems with black holes, origin of space-time, loss of information.
in AdS/CFT.

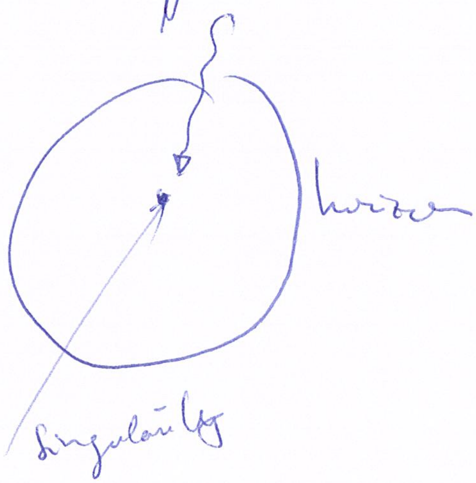


Hawking radiation is emitted at the horizon →
cannot carry any information. → Violates Unitarity.
↓ NO
Gravity in $AdS_5 \times S^5$ ⟷ Unitary field theory. ✓

~~Formation~~ Black Hole Formation ⇒ thermalization of the Field theory. Loss of information only apparent and due to "coarse graining" (ensembles) used in statistical mechanics

AdS/CFT does not explain very well what happens.

Quantum effects.

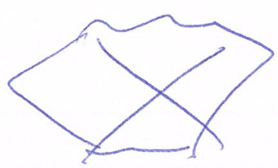


Mixed situation.
Quantum effects large near singularity but not at horizon & beyond (?)

AdS/CFT	$N \to \infty$	quantum effects suppressed	not at the same time.
	$N \gtrsim 1$	" " large	

Some proposal.

Entanglement entropy $\Longleftrightarrow$ space time.



two region are connected by space time if they are entangled, not if they are not.

Reconstruct space time from field theory
"Operator associated to the interior of black hole".

Another theory of gravity?

AdS space

$$X_{-1}^2 + X_0^2 - X_1^2 - \cdots - X_p^2 = R^2$$

AdS_{p+1}

$$\left(EAdS_{p+1} \quad X_0^2 - X_1^2 - \cdots - X_{p+1}^2 = R^2 \right)$$

Coordinates

X_A : embedding coordinates $\quad SO(p,2)$ symmetry manifest.

global coordinates:

$$ch^2\rho - sh^2\rho = 1.$$

$$X_0 = R\, ch\rho\, st \qquad \Big\| \qquad X_1 \cdots X_p \to sh\rho,\ \Omega_{p-1}$$

$$X_{-1} = R\, ch\rho\, ct$$

Ω_{p-1} ↑ sphere.

$$ds^2 = R^2\left((d(ch\rho))^2 + ch^2\rho\, dt^2\right) - R^2\left((d(sh\rho))^2 + sh^2\rho\, d\Omega_{p-1}^2\right)$$

$$dch\rho = sh\rho\, d\rho \qquad dsh\rho = ch\rho\, d\rho$$

$$ds^2 = R^2\left(-d\rho^2 + ch^2\rho\, dt^2 - sh^2\rho\, d\Omega_{p-1}^2\right)$$

↑ 1-time direction

$t : 0 \to 2\pi \to$ universal cover.

Boundary $\rho \to \infty$

$$ds^2 = R^2\left(-d\rho^2 + \frac{1}{4}e^{2\rho}\underbrace{\left(dt^2 - d\Omega_{p-1}^2\right)}_{\text{Boundary}}\right)$$

Boundary $R \times S_{p-1}$

Field theory lives in $R \times S_{p-1}$
↑ Breaks conf. invariance.

$SO(p-1)$ manifest.
$SO(2)$ manifest (translations in t).

Poincare coordinates

$$\frac{t}{z} = X_0 \qquad X_{i=1-p-1} = x^i/z$$

$$X_{-1} + X_p = X_+$$

$$X_{-1} - X_p = X_-$$

$$X_+X_- + X_0^2 - X_1^2 - \dots - X_{p-1}^2 = R^2$$

$$X_+X_- = R^2 - \frac{(t^2 - x_i^2)}{z^2}$$

$$X_- = 1/z \qquad X_+ = z\left(R^2 - \frac{t^2 - x_i^2}{z^2}\right)$$

$$dX_- = -\frac{1}{z^2}\,dz$$

$$dX_+ = dz\left(R^2 - \frac{t^2 - x_0^2}{z^2}\right) + z\left(+\frac{2}{z^3}\,(t^2 - \vec{x}^2)\,dz\right)$$

$$- \frac{1}{z}\left(2t\,dt - 2\vec{x}\,d\vec{x}\right)$$

$$dX_-\,dX_+ = -\frac{dz^2}{z^2}\left(R^2 - \frac{t^2 - \vec{x}^2}{z^2}\right) - \frac{2\,(t^2 - \vec{x}^2)\,dz^2}{z^4} -$$

$$+ \frac{2}{z^3}\,dz\,(t\,dt - \vec{x}\,d\vec{x})$$

$$ds^2 = dX_+\,dX_- + dX_0^2 - dX_c^2$$

$$dX_0^2 - dX_c^2 = \left(\frac{dt}{z} - \frac{t}{z^2}\,dz\right)^2 - \left(\frac{dx_i}{z} - \frac{x_i}{z^2}\,dz\right)^2$$

$$= \frac{dt^2 - dx_i^2}{z^2} - \frac{2\,dz}{z^3}\,(t\,dt - \vec{x}\,d\vec{x}) + \frac{dz^2}{z^4}\,(t^2 - \vec{x}^2)$$

$$ds^2 = -\frac{dz^2}{z^2}R^2 + \cancel{(t^2 - \vec{x}^2)\frac{dz^2}{z^4}} - \cancel{\frac{2(t^2 - \vec{x}^2)}{z^4}\,dz^2} + \frac{dt^2 - d\vec{x}^2}{z^2} + \cancel{\frac{dz^2}{z^4}\,(t^2 - \vec{x}^2)}$$

$$= \frac{dt^2 - d\vec{x}^2 - R^2 dz^2}{z^2} = -R^2\left(\frac{-dt^2 + d\vec{x}^2 + dz^2}{z^2}\right)$$

$z \to z/R$

$$\boxed{\begin{aligned} &\varepsilon_p\, ds = \varepsilon_\infty\, dt \\ &\varepsilon_\infty = \varepsilon_p \frac{R}{z} \end{aligned}}$$

f.t.

manifest $SO(1,p-1)$ symmetry.

scale symmetry $\begin{cases} t \to \lambda t \\ x_i \to \lambda x_i \\ z \to \lambda z \end{cases}$

Boundary at $z \to 0$.

$$ds_b^2 = dt^2 - d\vec{x}^2 \qquad \text{flat space.}$$

conf. dim. rescale in z.

Conf. map (~~Euclidean~~) Euclidean.

$$\mathbb{R}^d \qquad dx_1^2 + \dots + dx_d^2 = dr^2 + r^2 d\Omega_{d-1}^2$$

$$= r^2 \left(\frac{dr^2}{r^2} + d\Omega_{d-1}^2 \right) = \underset{\substack{\uparrow \\ \Omega(\mu)}}{e^{2\mu}} \underbrace{\left(d\mu^2 + d\Omega_{d-1}^2 \right)}_{\mathbb{R} \times S^{d-1}}$$

$r = e^{\mu}$

$$\mathbb{R}^d \underset{\substack{\uparrow \\ \text{conformal}}}{\longleftrightarrow} \mathbb{R} \times S^{d-1}$$

rescaling. $r \to \lambda r \equiv \mu \to \mu + \ln \lambda$.

translations in $\mu \equiv$ dilatations.

↑

eigenvalues of Hamiltonian $\leftrightarrow$ conf. dim.

map. operators $\leftrightarrow$ fields
$\uparrow$ CFT; in bulk.

$$\Box = -\frac{1}{\sqrt{g}} \partial_\mu (\sqrt{g}\, g^{\mu\nu} \partial_\nu)$$

$$(\Box + m^2)\phi = 0.$$

$$ds^2 = \frac{dz^2 + d\vec{x}^2}{z^2}$$

$$g = \frac{1}{z^{2d}} \qquad \sqrt{g} = z^{-d}$$

$$\Box = -z^d \left(\partial_z (z^{-d} z^2 \partial_z) + \partial_i (z^{-d} z^2 \partial_i)\right)$$

$$= -z^d \partial_z (z^{2-d} \partial_z) - z^2 \partial_i^2$$

$$\phi = \phi(z)\, e^{i\vec{k}.\vec{x}}$$

$$\Box\phi = -z^d \partial_z (z^{2-d} \partial_z)\phi + z^2 k^2 \phi = -m^2 \phi$$

$$-z^2 \phi'' - (2-d)\, z\, \phi' + z^2 k^2 \phi = -m^2 \phi$$

$z \to 0$
$\phi \sim z^\Delta$

$$-\cancel{z^2}\, \Delta(\Delta-1) \cancel{z^{\Delta-2}} - (2-d)\, \Delta\, \cancel{z^\Delta} = -m^2 \cancel{z^\Delta}$$

$$\Delta(\Delta-1)+2\Delta-d\Delta=m^2$$

$$\Delta^2+\Delta-d\Delta=m^2$$

$$\Delta^2+(1-d)\Delta-m^2=0 \qquad m^2=\Delta(\Delta+1-d) \qquad \left(\begin{array}{c}\Delta(\Delta-4)\\ AdS_5\end{array}\right)$$

$$\Delta=\frac{d-1\pm\sqrt{(d-1)^2+4m^2}}{2}$$

$$\Delta=\frac{d-1}{2}\pm\sqrt{\left(\frac{d-1}{2}\right)^2+m^2}$$

Take

$$\Delta=\frac{d-1}{2}+\sqrt{\left(\frac{d-1}{2}\right)^2+m^2}$$

$$z^{\Delta}, \qquad z^{d-\Delta}$$

does not depend on k.

$$\phi\sim z^{\Delta}\underbrace{\phi(\vec{x})}_{\langle\mathcal{O}(\vec{x})\rangle}+z^{d-\Delta}\underbrace{\tilde{\phi}(\vec{x})}_{\text{source.}}$$

$$Z_j=\int e^{i\int \underbrace{j(x)}_{d-\Delta}\underbrace{\mathcal{O}(x)}_{\Delta}d^dx}\mathcal{D}\phi \simeq Z+i\int j(x)\,\mathcal{O}(x)+\ldots$$

$$\langle\mathcal{O}(y)\rangle_x=i\int j(x)\langle\mathcal{O}(x)\mathcal{O}(y)\rangle=i\int j(x)e^{ik(x-y)}G(k)$$

$$\langle\mathcal{O}(x)\mathcal{O}(y)\rangle=\int e^{ik(x-y)}G(k) \qquad \Bigg\| \qquad =i\int_k e^{-iky}G(k)\,j(k)$$

$$\langle \mathcal{O}(k) \rangle = i\, G(k)\, j(k)$$

$$G(k) = -i \frac{\langle \mathcal{O}(k) \rangle}{\langle j(k) \rangle}$$

$$\phi(k,z) \sim z^{\Delta} \phi_1(k) + z^{d-\Delta} \tilde{\phi}_2(k)$$

$$\boxed{G(k) = \frac{\phi_1(k)}{\tilde{\phi}_2(k)}}$$

$$-z^2 \phi'' - (2-d)\, z \phi' + z^2 k^2 \phi + m^2 \phi = 0$$

$$\phi'' + \frac{2-d}{z} \phi' - \left(k^2 + \frac{m^2}{z^2}\right) \phi = 0$$

$$f = C_1 z^{\frac{d-1}{2}} I_\nu(kz) + C_2 z^{\frac{d-1}{2}} K_\nu(kz)$$

$$\nu = \frac{1}{2} \sqrt{m^2 + \left(\frac{d-1}{2}\right)^2} \qquad I_\nu \sim z^{\nu} \qquad K_\nu \sim z^{-\nu}$$

$|z| \to \infty$

$$I_\nu \sim e^{z}$$

$$K_\nu \sim e^{-z}$$ ← in Euclidean.

$K_\nu(kz) \sim_* k^{-\nu} z^{-\nu} + * k^{\nu} z^{\nu}$

$\phi \sim_* z^{\frac{d-1}{2}-\nu} k^{-\nu} + * z^{\frac{d-1}{2}+\nu} k^{\nu}$

$G(k) \sim$

$\Delta = \frac{d-1}{2} + \nu$

$\phi \sim c_2 z^{d-\Delta} k^{-\nu} + c_1 z^{\Delta} k^{\nu}$

$G(k) \sim \frac{c_1 k^{\nu}}{c_2 k^{-\nu}} = \frac{c_1}{c_2} k^{2\nu} = \frac{c_1}{c_2} k^{2(\Delta - \frac{d-1}{2})} = \frac{c_1}{c_2} k^{2\Delta-(d-1)}$ ✓

$G(k) = \int e^{ikx} \frac{d^{d-1}x}{x^{2\Delta}} \sim k^{-(d-1)+2\Delta} = k^{2\Delta-(d-1)}$ ✓

$$-\frac{1}{\sqrt{g}}\partial_\mu\left(\sqrt{g}\,g^{\mu\nu}\partial_\nu\right)\phi + m^2\phi = 0$$

$$\phi = \phi(z)\,e^{ikx}$$

$$ds^2 = \frac{dz^2 + d\vec{x}^2_{(p)}}{z^2} \qquad \boxed{d = p+1}$$

$$g = \left(\frac{1}{z^2}\right)^{p+1} \qquad \sqrt{g} = z^{-d}$$

$$\Box = -z^d\,\partial_\mu\left(\cancel{g}\,z^{-d+2}\,\partial_\nu\right)\phi$$

$$= -z^d\,\partial_z\left(z^{2-d}\partial_z\right)\phi - z^2\,\partial_i^2\phi$$

$$\Box = -z^2\,\partial_z^2\phi - (2-d)\,z\,\partial_z\phi - z^2\,\partial_i^2\phi$$

$$m^2 + \Box = -z^2\phi'' - (2-d)\,z\,\phi' + k^2z^2\phi + m^2\phi = 0.$$

$$\left(-k_0^2 + \vec{k}^2\right)z^2\phi + m^2\phi - (2-d)\,z\phi' - z^2\phi'' = 0$$

$$\boxed{\phi'' + \frac{(2-d)}{z}\phi' + \left(k_0^2 - k^2\right)\phi - \frac{m^2}{z^2}\phi = 0}$$

$$u'' + \frac{1-2\alpha}{z}u' + \left(\beta^2 + \frac{\alpha^2 - \nu^2}{z^2}\right)u = 0$$

$1-2\alpha = 2-d$

$-2\alpha = 1-d \qquad \alpha = \frac{d-1}{2}$

$\beta^2 = k_0^2 - \vec{k}^2 \qquad \alpha^2 - \nu^2 = -m^2$

$\nu^2 = \alpha^2 + m^2 = \left(\frac{d-1}{2}\right)^2 + m^2$

$$\boxed{\nu = \sqrt{m^2 + \left(\frac{d-1}{2}\right)^2}}$$

$\beta = \sqrt{k_0^2 - \vec{k}^2}$

$$\boxed{\phi = z^{\frac{d-1}{2}} J_\nu\left(\sqrt{k_0^2 - \vec{k}^2}\, z\right)}$$

or $J_{-\nu}$ $(\nu \notin \mathbb{Z}$

Euclidean $\sqrt{k_0^2 - k^2} \in i\mathbb{R} \to I_\nu, \boxed{K_\nu}$

finite $z \to \infty$

$$\Phi = z^{\frac{d-1}{2}} \left(A_\nu J_\nu\left(\sqrt{k_0^2 - \vec{k}^2}\, z\right) + B_\nu J_{-\nu}\left(\sqrt{k_0^2 - \vec{k}^2}\, z\right) \right)$$

$z \to 0$

$$\phi'' + \frac{(2-d)}{z}\phi' - \frac{m^2}{z^2}\phi = 0$$

z^Δ $\quad \Delta(\Delta-1)\cancel{z^{\Delta-2}} + (2-d)\Delta \cancel{z^{\Delta-2}} - m^2 \cancel{z^{\Delta-2}} = 0$

$$\Delta^2 \cancel{+\Delta} + \cancel{2}\Delta - d\Delta - m^2 = 0$$

$$\Delta^2 - (d-1)\Delta - m^2 = 0$$

$$\Delta = \frac{(d-1) \pm \sqrt{(d-1)^2 + 4m^2}}{2} \to \frac{d-1}{2} \pm \sqrt{m^2 + \left(\frac{d-1}{2}\right)^2}$$

$$\phi = \phi_1(k)\, z^\Delta + \phi_2(k)\, z^{d-\Delta}$$

$z \to \infty$

$$\phi'' + \frac{(2-d)}{z}\phi' + k^2\phi = 0 \qquad \text{(Minkowski)}$$

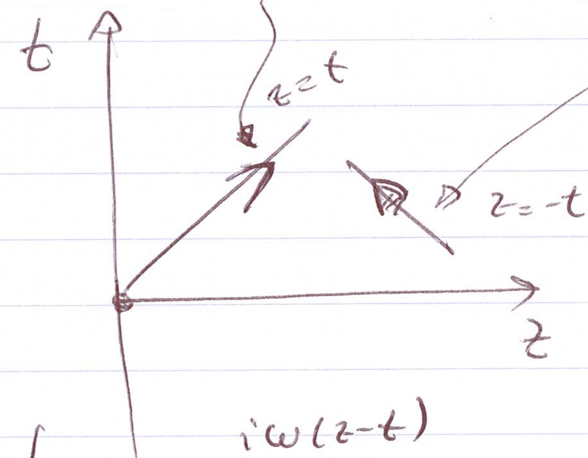
$$\phi'' + \frac{(2-d)}{z}\phi' - k^2 z = 0 \qquad \text{(Euclidean)}$$

$$\phi'' + k^2\phi = 0 \qquad \phi = e^{\pm ikz} \qquad \phi'/z \to 0 \checkmark$$

$$\phi'' - k^2 z = 0 \qquad \phi = e^{\pm kz}$$

$$\phi(x) = e^{\pm ikz - ik_0 t + i\vec{k}\vec{x}}$$

$$\vec{k} = 0 \qquad e^{i\omega(z-t)} \quad ; \quad e^{i\omega(-z-t)}$$



$$\phi \sim e^{i\omega(z-t)}$$

$$J_{\pm\nu} \underset{z\to\infty}{\approx} \sqrt{\frac{2}{\pi k z}} \cos\left(kz \mp \frac{\pi}{2}\nu - \frac{\pi}{4}\right) + \cdots$$

$$H_\nu^{(1)}(z) \approx \sqrt{\frac{2}{\pi k z}}\, e^{i\left(kz - \frac{\pi}{2}\nu - \frac{\pi}{4}\right)} \qquad (J_\nu + i Y_\nu)$$

$$\phi = z^{\frac{d-1}{2}} H_\nu^{(1)}\left(\sqrt{k_0^2 - k^2}\; z\right) \qquad \text{Minkowski}$$

$$\phi = z^{\frac{d-1}{2}} K_\nu\left(\sqrt{k_0^2 + k^2}\; z\right) \qquad \text{Euclidean.}$$

$$H_\nu \underset{z\to 0}{\sim} \frac{k^\nu z^\nu}{2^\nu} \frac{1}{\Gamma(1+\nu)} + i \frac{\cos\nu\pi}{\sin\nu\pi} \frac{k^\nu z^\nu}{2^\nu} \frac{1}{\Gamma(1+\nu)} - \frac{1}{\sin\nu\pi} \frac{2^\nu}{z^\nu k^\nu} \frac{1}{\Gamma(1-\nu)}$$

$$= \frac{i e^{-i\nu\pi}}{\sin\nu\pi} \frac{z^\nu k^\nu}{2^\nu \Gamma(1+\nu)} - \frac{1}{\sin\nu\pi} \frac{2^\nu k^{-\nu} z^{-\nu}}{\Gamma(1-\nu)}$$

$$\phi \sim \frac{i e^{-i\nu\pi}}{\sin\nu\pi} \frac{k^{\nu} z^{\frac{d-1}{2}+\nu}}{2^{\nu}\Gamma(1+\nu)} - \frac{1}{\sin\nu\pi}\frac{2^{\nu}}{\Gamma(1-\nu)} z^{\frac{d-1}{2}-\nu} k^{-\nu}$$

$$\phi \sim \frac{i e^{-i\nu\pi}}{\sin\nu\pi} \frac{k^{\nu} z^{\Delta}}{2^{\nu}\Gamma(1+\nu)} - \frac{1}{\sin\nu\pi}\frac{2^{\nu}}{\Gamma(1-\nu)} z^{-\Delta+\frac{d-1}{2}} k^{-\nu}$$

$$G(k) = \frac{i e^{-i\nu\pi}}{\cancel{\sin\nu\pi}\, 2^{\nu}\Gamma(1+\nu)} \cancel{\sin\nu\pi} \frac{k^{2\nu}}{2^{\nu}} \Gamma(1-\nu)$$

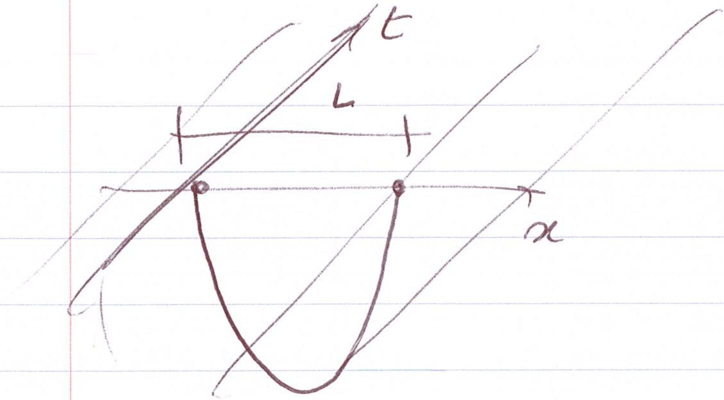
$$= i\, 2^{-2\nu} e^{-i\nu\pi} \frac{\Gamma(1-\nu)}{\Gamma(1+\nu)} k^{2\nu}$$

$$\Delta = \frac{d-1}{2} + \nu$$

$$\langle \mathcal{O}(x)\, \mathcal{O}(y) \rangle = \frac{1}{|x-y|^{2\Delta}} \quad \checkmark$$

$$\int \frac{e^{ikx}}{|x|^{2\Delta}} d^{d-1}x \sim k^{-(d-1)+2\Delta} \sim k^{2\nu}$$

$$2\Delta - (d-1) = \cancel{d-1} + 2\nu - \cancel{(d-1)} = 2\nu$$



$$\text{Area} = \int d\sigma d\tau \sqrt{\dot{X}^2 X'^2 - (\dot{X} X')^2}$$

$t = \tau \qquad x = \sigma \qquad z = z(\sigma)$

$\dot{X} = (1, 0, 0) \qquad X' = (0, 1, z')$

$\dot{X}^2 = \frac{1}{z^2} \qquad X'^2 = \frac{1 + z'^2}{z^2} \qquad \dot{X} \cdot X' = 0$

$$\text{Area} = \int d\sigma d\tau \sqrt{\frac{1}{z^4}(1 + z'^2)}$$

$$= 2 \int_0^{L/2} d\sigma \, \frac{\sqrt{1 + z'^2}}{z^2}$$

$z(0) = 0 \qquad z'(L/2) = 0$

$$d\,\frac{\partial L}{\partial z'} - \frac{\partial L}{\partial z} = 0$$

$$d\left(L - \frac{\partial L}{\partial z'} z' \right) = z'\,\frac{\partial L}{\partial z} + z''\,\cancel{\frac{\partial L}{\partial z'}} - d\,\frac{\partial L}{\partial z'}\, z' - \cancel{\frac{\partial L}{\partial z'}\, z''}$$

$$= 0$$

$$\frac{\partial L}{\partial z'} = \frac{1}{z^2}\,\frac{1}{\cancel{2}}\,\frac{\cancel{2} z'}{\sqrt{1+z'^2}}$$

$$\frac{\sqrt{1+z'^2}}{z^2} - \frac{z'^2}{z^2\sqrt{1+z'^2}} = \frac{1+\cancel{z'^2}-\cancel{z'^2}}{z^2\sqrt{1+z'^2}} = C$$

$$\sqrt{1+z'^2} = \frac{1}{Cz^2} \qquad 1+z'^2 = \frac{1}{C^2 z^4}$$

$$z' = \sqrt{\frac{1}{c^2 z^4} - 1} = \frac{\sqrt{1-c^2 z^4}}{c z^2}$$

$$\int_0^z \frac{c z^2\, dz}{\sqrt{1-c^2 z^4}} = \int_0^\sigma d\sigma$$

(3)

$$C\int_0^{z_0} \frac{z^2\,dz}{\sqrt{1-c^2z^4}} = L/2$$

$z'(L/2) = 0 \qquad \dfrac{1}{C z_0^2} = 1 \qquad z_0^2 = 1/C \qquad C^2 = 1/z_0^4$

$$\frac{1}{z_0^2}\int_0^{z_0} \frac{z^2\,dz}{\sqrt{1-z^4/z_0^4}} = L/2$$

$u = z/z_0$

$$z_0\int_0^1 \frac{u^2\,du}{\sqrt{1-u^4}} = L/2 \qquad L\uparrow,\ z_0\uparrow$$

$y = u^4 \qquad u = y^{1/4} \qquad du = \frac{1}{4}y^{-3/4}\,dy$

$$\frac{z_0}{4}\int_0^1 \frac{y^{1/2}\,y^{-3/4}\,dy}{\sqrt{1-y}} = L/2$$

$$z_0\int_0^1 y^{-1/4}(1-y)^{-1/2}\,dy = 2L$$

$$z_0 \int_0^1 y^{\frac{3}{4}-1} (1-y)^{\frac{1}{2}-1}\, dy = 2L$$

$$z_0 \frac{\Gamma(3/4)\,\Gamma(1/2)}{\Gamma(5/4)} = 2L \qquad \Bigg| \qquad z_0 = \frac{2L\,\Gamma(5/4)}{\sqrt{\pi}\,\Gamma(3/4)}$$

$$\underline{\text{Area}} = 2\int_0^{L/2} d\sigma \frac{1}{Cz^4} = 2z_0^2 \int_0^{L/2} \frac{d\sigma}{z^4}$$

$$= 2z_0^2 \int_0^{L/2} \frac{d\sigma}{dz}\frac{dz}{z^4} = 2z_0^2 \int_0^{z_0} \frac{dz}{z^4}\frac{Cz^2}{\sqrt{1-C^2z^4}} =$$

$$= \frac{2\cancel{z_0^2}}{\cancel{z_0^2}} \int_\epsilon^{z_0} \frac{dz}{z^2}\frac{1}{\sqrt{1-z^4/z_0^4}} = \frac{2}{z_0}\int_{\epsilon/z_0}^1 \frac{du}{u^2}\frac{1}{\sqrt{1-u^4}}$$

$$y = u^4 \qquad dy = 4u^3 du = 4y^{3/4} du$$

$$\text{Area} = \frac{2}{z_0}\int_{(\epsilon/z_0)^4}^1 \frac{dy}{4y^{3/4}}\frac{1}{\sqrt{y}}\frac{1}{\sqrt{1-y}} = \frac{1}{2z_0}\int_{(\epsilon/z_0)^4}^1 dy\, y^{-\frac{5}{4}}(1-y)^{-\frac{1}{2}}$$

$$= \frac{1}{2z_0}\int_{(\epsilon/z_0)^4}^1 dy\, y^{-\frac{5}{4}}(1-y)^{-1/2}$$

$$\int dy\; y^{-5/4}(1-y)^{-1/2} = \int dy\, \frac{1}{y^{1/4}}\left(\frac{1}{\sqrt{1-y}}\right)\left(\frac{1}{y}-1+1\right)$$

$$= \int dy\; y^{-1/4}\,\frac{1}{\sqrt{1-y}}\left(\frac{1-y}{y}+1\right)$$

$$= \int dy\; y^{-5/4}\sqrt{1-y} + \int dy\; y^{-1/4}(1-y)^{-1/2}$$

$$= \int dy\,\left(-4\,\partial_y\, y^{-1/4}\right)\sqrt{1-y} + \int dy\; y^{-1/4}(1-y)^{-1/2}$$

$$= -4\, y^{-1/4}\sqrt{1-y}\,\Big|_{(\varepsilon/z_0)^4}^{1} \cancel{+} \overset{2}{\cancel{4}} \int dy\, y^{-1/4}\,\frac{1}{\cancel{2}}(1-y)^{-1/2} + \int dy\, y^{-1/4}(1-y)^{-1/2}$$

$$= 4\,\frac{z_0}{\varepsilon} - \int_0^1 dy\; y^{\frac{3}{4}-1}(1-y)^{\frac{1}{2}-1}$$

$$\text{Area} = \frac{2}{\varepsilon} - \frac{1}{2z_0}\,\frac{\Gamma(3/4)\,\Gamma(1/2)}{\Gamma(5/4)}$$

$$V = -\frac{1}{2z_0}\,\frac{\Gamma(3/4)\sqrt{\pi}}{\Gamma(5/4)} = -\frac{\Gamma^2(3/4)\,\pi}{4L\,\Gamma^2(5/4)}$$

$$V = -\frac{\alpha}{L} \qquad \alpha = \frac{\pi\,\Gamma^2(3/4)}{4\,\Gamma^2(5/4)}$$

Other backgrounds

Finite temperature

$$ds^2 = \frac{R^2}{U^2 f(U)}\, dU^2 - \frac{U^2 f(U)}{R^2}\, dt^2 + \frac{U^2}{R^2}\, dx_i^2 + R^2\, d\Omega_5^2$$

$$f(U) = 1 - \left(U_T/U\right)^4$$

$$U = R^2/z \qquad dU = -\frac{R^2}{z^2}\, dz \qquad \frac{dU}{U} = -\frac{dz}{z}$$

$$ds^2 = R^2\left(\frac{dz^2}{z^2 f(z)} - \frac{f(z)\, dt^2}{z^2} + \frac{dx_i^2}{z^2} + d\Omega_5^2\right)$$

$$f(z) = 1 - \left(z/z_0\right)^4 \qquad z = z_0 \text{ horizon} \qquad \boxed{0 < z < z_0}$$

Confinement

2+1

$$ds^2 = R^2\left(\frac{dz^2}{z^2 f(z)} + \frac{f(z)\, d\tau^2}{z^2} + \frac{(dx_1^2 + dx_2^2 - dt^2)}{z^2}\right)$$

periodic.

$$z \to z_0 \qquad f(z) \simeq -4\, \cancel{z_0^3/z_0^4}\, (z - z_0) = -\frac{4}{z_0}(z - z_0) = \frac{4}{z_0}(z_0 - z)$$

$$ds^2 \simeq R^2\left(+\frac{dz^2}{\cancel{z_0^2}\,\frac{4}{z_0}(z - z_0)} + \frac{4}{z_0}\,\frac{(z_0 - z)\, d\tau^2}{z_0^2} + \frac{(dx_1^2 + dx_2^2 - dt^2)}{z_0^2}\right)$$

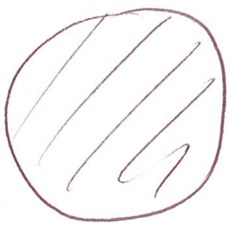
$z_0 - z = u = r^2 \qquad dz = -du = -2r\,dr$

$$\frac{du^2}{4z_0 u} + \frac{4}{z_0^3}\, u\, d\tau^2 = \frac{\cancel{4}\, r^{\cancel{2}} dr^2}{\cancel{4}\, z_0\, r^{\cancel{2}}} + \frac{4r^2}{z_0^3}\, d\tau^2$$

$$= \frac{1}{z_0}\left(dr^2 + r^2\left(d\left(\frac{2\tau}{z_0}\right)\right)^2\right)$$

$$\theta = \frac{2\tau}{z_0}$$

$$ds^2 \simeq R^2\left[\frac{1}{z_0}\left(dr^2 + r^2 d\theta^2\right) + \frac{dx_1^2 + dx_2^2 - dt^2}{z_0^2}\right]$$



τ : disk is filled.

In finite temperature case. we do $t \to i\tau$ and get $\theta = \frac{2\tau}{z_0}$ $\qquad \theta \equiv \theta + 2\pi$

there is also periodic AdS.

$$\tau \equiv \tau + \frac{z_0}{\cancel{2}}\cancel{2}\pi = \tau + \pi z_0 \qquad \beta = \frac{1}{T} = \pi z_0$$

β : periodicity in Euclidean time

$\mathrm{tr}\left(e^{-\beta H}\right)$

$$\boxed{T = \frac{1}{\pi z_0}}$$

Confinement <u>3+1</u> ③

$$ds^2 = \left(\frac{R}{U}\right)^{3/2} \frac{dU^2}{f(U)} - \left(\frac{U}{R}\right)^{3/2} f(U) dt^2 + \qquad \text{black hole}$$

$$+ \left(\frac{U}{R}\right)^{3/2} \sum_{i=1}^{4} dx_i^2 + R^{3/2} U^{1/2} d\Omega_{4}^2$$

Double Wick rot. $\qquad f(U) = 1 - U_T^3/U^3$

$$e^{\phi} = g_s \alpha'^{1/2} \left(R^3/U^3\right)^{-1/4} = g_s\sqrt{\alpha'} \left(U/R\right)^{3/4}$$

$$R^{3/2} = g_{YM}\sqrt{N} \times \#$$

$\uparrow$ number.

$$ds^2 = \left(\frac{R}{U}\right)^{3/2} \frac{dU^2}{f(U)} + \left(\frac{U}{R}\right)^{3/2} f(U) dz^2 +$$

$$+ \left(\frac{U}{R}\right)^{3/2} \left(-dt^2 + dx_1^2 + dx_2^2 + dx_3^2\right) + R^{3/2} U^{1/2} d\Omega_4^2$$

action of a string at $U = U_T$

$\sim\!\!\!\!S = \int\sqrt{g}$ $\qquad S = \alpha'$ Area

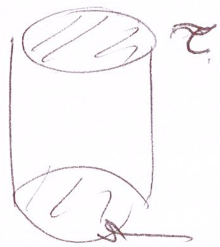
$t = \tau \qquad x = \sigma \qquad ds^2 = \left(\frac{U_T}{R}\right)^{3/2} (-d\tau^2 + d\sigma^2)$

$U = U_T \qquad \sqrt{h} = \left(U_T/R\right)^{3/2} \qquad S = \left(\frac{U_T}{R}\right)^{3/2} \Delta x \Delta t = 2\pi\tilde{\alpha}'^{-1} \Delta x \Delta t$

String tension.

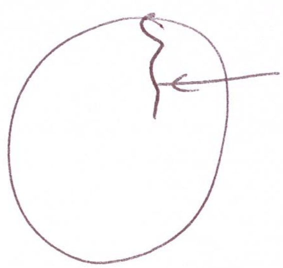
$$\tilde{\alpha}' = \sigma = \frac{1}{2\pi}\left(\frac{U_T}{R}\right)^{3/2}$$

glueballs.



normal modes give massive particles

Confinement



string has nowhere to go

$$ds^2 = -ch^2\rho\, dt^2 + d\rho^2 + sh^2\rho\, d\theta^2$$

$$X = (t, \rho, \theta)$$

gauge choice $t = \tau \quad \rho = \sigma \quad \theta = \theta(\sigma, \tau)$

$$X = (\tau, \sigma, \theta)$$

$$\dot{X} = (1, 0, \dot{\theta})$$

$$X' = (0, 1, \theta')$$

$$\dot{X}^2 = -ch^2\sigma + sh^2\rho\, \dot{\theta}^2$$

$$X'^2 = 1 + sh^2\rho\, \theta'^2$$

$$\dot{X}X' = sh^2\rho\, \dot{\theta}\theta'$$

$$\dot{X}^2 X'^2 - (\dot{X}X')^2 = -ch^2\rho + sh^2\rho\, ch^2\rho\, \theta'^2 + sh^2\rho\, \dot{\theta}^2 \cancel{+ sh^4\rho\, \dot{\theta}^2\theta'^2}$$
$$\cancel{- sh^4\rho\, \dot{\theta}^2\theta'^2}$$

$$S = \frac{1}{l_s^2} \int \sqrt{ch^2\rho - sh^2\rho\, ch^2\rho\, \theta'^2 - sh^2\rho\, \dot{\theta}^2}$$

$$\frac{\partial h}{\partial \dot\theta} = \frac{1}{\cancel{k}\sqrt{\ }}\left(-\,sh^2\rho\,\cancel{k}\,\dot\theta\right)$$

$$\frac{\partial h}{\partial \theta'} = \frac{1}{k\sqrt{\ }}\left(-\cancel{2}\,sh^2\rho\,ch^2\rho\,\theta'\right)$$

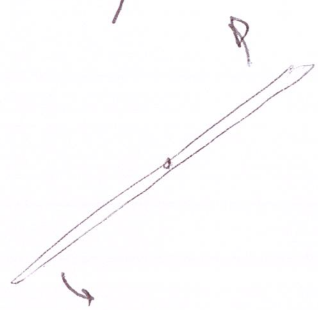
$$\frac{\partial h}{\partial \theta} = 0$$

$$\partial_\tau\left(\frac{sh^2\rho\,\dot\theta}{\sqrt{\ }}\right) + \partial_\sigma\left(\frac{sh^2\rho\,ch^2\rho}{\sqrt{\ }}\,\theta'\right) = 0$$

Solution $\underline{\theta' = 0}$

$$\partial_\tau\left(\frac{sh^2\rho\,\dot\theta}{\sqrt{\ }}\right) = 0 \quad\Rightarrow\quad \dot\theta = \text{indep. of } \tau$$

$$\boxed{\theta = \omega\tau}$$



$$S = \frac{1}{l^2}\int \sqrt{ch^2\rho - \omega^2 sh^2\rho}\; d\rho\, d\tau$$

$$ch^2\rho_0 - \omega^2 sh^2\rho_0 = 0 \qquad \omega^2 = \frac{ch^2\rho_0}{sh^2\rho_0} > 1$$

$\rho_0 \to \infty \qquad \omega^2 \to 1.$

$$\partial_z \left(\frac{\operatorname{sh}^2\rho\, \dot\theta}{\sqrt{}} \right) = 0$$

$$J = 4 \int_0^{\rho_0} \frac{\operatorname{sh}^2\rho\, \omega}{\sqrt{\operatorname{ch}^2\rho - \omega^2 \operatorname{sh}^2\rho}}\, d\rho$$

$$\mathcal{H} = p_\theta \dot\theta - L = - \frac{\operatorname{sh}^2\rho\, \dot\theta^2}{\sqrt{}} - \sqrt{} = - \frac{\cancel{\operatorname{sh}^2\rho\, \dot\theta^2} + \operatorname{ch}^2\rho - \cancel{\omega^2 \operatorname{sh}^2\rho}}{\sqrt{}}$$

$$E = 4 \int_0^{\rho_0} \frac{\operatorname{ch}^2\rho\, d\rho}{\sqrt{\operatorname{ch}^2\rho - \omega^2 \operatorname{sh}^2\rho}} \qquad \operatorname{ch}^2 - \operatorname{sh}^2 = 1$$

$$\rho_0 \to \infty \qquad \operatorname{ch}^2\rho - \frac{\operatorname{ch}^2\rho_0}{\operatorname{sh}^2\rho_0} \operatorname{sh}^2\rho = 1 + \operatorname{sh}^2\rho \left(1 - \frac{\operatorname{ch}^2\rho_0}{\operatorname{sh}^2\rho_0} \right)$$

$$= 1 - \frac{\operatorname{sh}^2\rho}{\operatorname{sh}^2\rho_0}$$

$$J = 4\omega \int_0^{\rho_0} \frac{\operatorname{sh}\rho_0\, \operatorname{sh}^2\rho\, d\rho}{\sqrt{\operatorname{sh}^2\rho_0 - \operatorname{sh}^2\rho}}$$

$$\rho = \rho_0 - \xi \qquad sh^2\rho = sh^2\rho_0 - 2sh\rho_0 ch\rho_0\, \xi$$

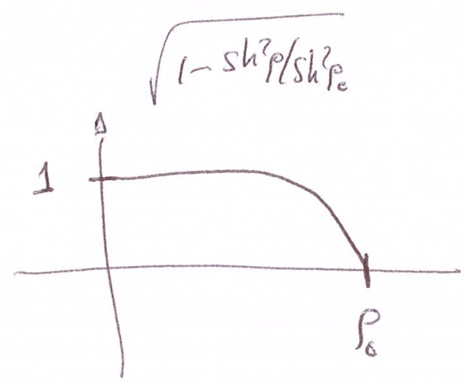
(4)

$$J \simeq 4\, ch\rho_0 \int_0^{\xi} \frac{sh^2\rho_0\, d\xi}{\sqrt{2 sh\rho_0\, ch\rho_0\, \xi}}$$

$$J \simeq \frac{4}{\sqrt{2}} \sqrt{ch\rho_0}\; sh\rho_0^{3/2} \int_0^{\xi} \frac{d\xi}{\sqrt{\xi}}$$

$$\simeq 2^{3/2} \sqrt{ch\rho_0}\; sh\rho_0^{3/2}\, 2\sqrt{\xi}$$

$$E \simeq 4\, ch^2\rho_0 \int_0^{\xi} \frac{sh\rho_0\, d\xi}{\sqrt{2sh\rho_0\, ch\rho_0\, \xi}} \simeq \frac{4}{\sqrt{2}} ch^{3/2}\rho_0\; sh^{1/2}\rho_0\, \sqrt{\xi}$$



$$E - \omega J = 4 \int_0^{\rho_0} \sqrt{ch^2\rho - \omega^2 sh^2\rho}\; d\rho$$

$$= 4\, \frac{1}{sh\rho_0} \int_0^{\rho_0} \sqrt{sh^2\rho_0 - sh^2\rho}\; d\rho \underset{\rho_0 \to \infty}{\simeq} 4\rho_0$$

$\rho_0 \to \infty$

approx.

$$J \simeq \frac{\cancel{4}\omega}{\cancel{2}} \int_0^{\rho_0} e^{\rho_0}\, \frac{e^{2\rho}}{\cancel{4}}\, \frac{d\rho}{\frac{1}{\cancel{2}}\sqrt{e^{2\rho_0} - e^{2\rho}}}$$

$$J \simeq \frac{1}{2}\int_{1}^{e^{2\rho_0}} e^{\rho_0}\,\frac{dx}{\sqrt{x_0 - x}} = - e^{\rho_0}\sqrt{x_0 - x}\,\Big|_{1}^{x_0}$$

$x = e^{2\rho}$

$$= e^{\rho_0}\sqrt{x_0 - 1} \cong e^{2\rho_0}$$

$$E = e^{2\rho_0} \qquad \ln J = 2\rho_0$$

$$E = J + 4\rho_0 = J + 2\ln J$$

$$\boxed{E = J + 2\sqrt{\lambda}\,\ln J}$$

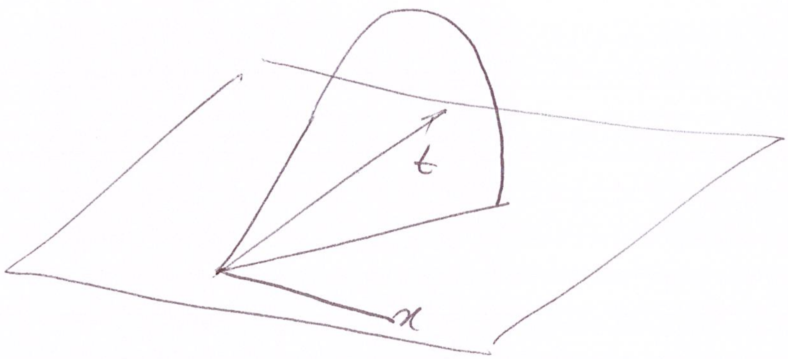
$E(S)$

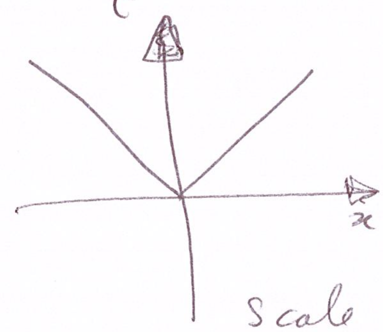
energy

spin

light-like cusp.

(6)





Scale & Lorentz symmetry.

$$\boxed{z = a\sqrt{t^2 - x^2}}$$

?

$$X_{-1}^2 + X_0^2 - X_1^2 - X_2^2 = 1$$

$$z = \frac{1}{X_{-1} - X_2} \qquad X_0 = t/z \qquad X_1 = x/z$$

$$X_0^2 - X_1^2 = \frac{t^2 - x^2}{z^2} = \frac{1}{a^2}$$

$$\left.\begin{aligned} X_0^2 - X_1^2 &= 1/a^2 \\ X_{-1}^2 - X_2^2 &= 1 - 1/a^2 \end{aligned}\right\} \quad \text{symmetric} \quad a = \sqrt{2}$$

$$z = \sqrt{2}\sqrt{t^2 - x^2}$$

$$ds^2 = \frac{-dt^2+dx^2+dz^2}{z^2}$$

$t = \rho\, ch\xi \qquad x = \rho\, sh\xi \qquad t^2 - x^2 = \rho^2$

$dt = d\rho\, ch\xi + \rho\, sh\xi\, d\xi$

$dx = d\rho\, sh\xi + \rho\, ch\xi\, d\xi$

$-dt^2 + dx^2 = -d\rho^2 + \rho^2 d\xi^2$

$$ds^2 = \frac{-d\rho^2 + \rho^2 d\xi^2 + dz^2}{z^2}$$

$z = \sqrt{2}\rho$

$$ds^2 = \frac{-d\rho^2 + \rho^2 d\xi^2 + 2d\rho^2}{2\rho^2} = \cancel{\frac{1}{2}d\xi^2 + \frac{3}{2}\frac{d\rho^2}{\rho^2}}$$

$$= \frac{d\rho^2}{\rho^2} + \frac{1}{2}d\xi^2$$

$\sqrt{h} = \frac{\sqrt{3}}{4}$

$\sqrt{h} = \frac{1}{\sqrt{2\rho^2}} = \frac{1}{\sqrt{2}\rho}$

$$\int \frac{d\xi\, d\rho}{\sqrt{2}\rho} = \frac{\Delta\xi}{\sqrt{2}} \ln\rho \Big|_{\varepsilon}^{L}$$

$$\boxed{A = \frac{1}{\sqrt{2}}\Delta\xi \ln\frac{L}{\varepsilon}}$$