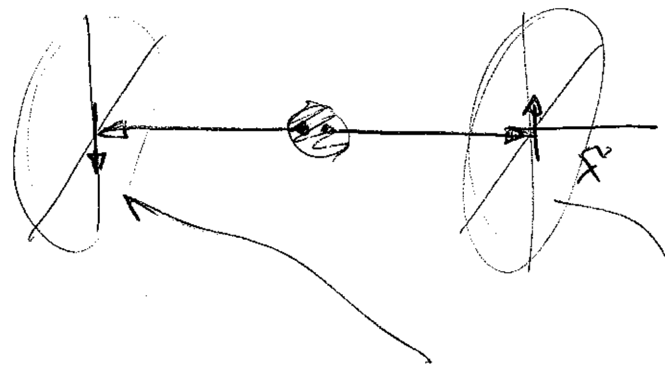
Bell inequalities.



$$\frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)$$

If we measure $\uparrow$ here then $\downarrow$: Same for any direction.

$$|\uparrow_{\hat n}\rangle = c_{\frac{\theta}{2}}\, e^{-i\varphi/2}\,|\uparrow\rangle + s_{\frac{\theta}{2}}\, e^{i\varphi/2}\,|\downarrow\rangle \;;$$

$$\cancel{|\uparrow^{1}_{\hat n}\downarrow^{2}_{\hat n}\rangle = c^2_{\frac{\theta}{2}}\, e^{-i\varphi}\,|\uparrow\uparrow\rangle + s^2_{\frac{\theta}{2}}\, e^{i\varphi}\,|\downarrow\downarrow\rangle} + \quad \theta \to \pi-\theta$$

$$|\downarrow_{\hat n}\rangle = -i s_{\frac{\theta}{2}}\, e^{-i\varphi/2}\,|\uparrow\rangle + c_{\frac{\theta}{2}}\; i\, e^{i\varphi/2}\,|\downarrow\rangle$$

$$|\uparrow_{\hat n}\downarrow_{\hat n}\rangle = -i s_{\frac{\theta}{2}} c_{\frac{\theta}{2}}\, e^{-i\varphi}\,|\uparrow\uparrow\rangle + i c^2_{\frac{\theta}{2}}\,|\uparrow\downarrow\rangle - i s^2_{\frac{\theta}{2}}\,|\downarrow\uparrow\rangle + i s_{\frac{\theta}{2}} c_{\frac{\theta}{2}}\, e^{i\varphi}\,|\downarrow\downarrow\rangle$$

$$|\downarrow_{\hat n}\uparrow_{\hat n}\rangle = -i s_{\frac{\theta}{2}} c_{\frac{\theta}{2}}\, e^{-i\varphi}\,|\uparrow\uparrow\rangle + i c^2_{\frac{\theta}{2}}\,|\downarrow\uparrow\rangle - i s^2_{\frac{\theta}{2}}\,|\uparrow\downarrow\rangle + i s_{\frac{\theta}{2}} c_{\frac{\theta}{2}}\, e^{i\varphi}\,|\downarrow\downarrow|$$

$$\frac{1}{\sqrt{2}}\left(|\uparrow_{\hat n}\downarrow_{\hat n}\rangle - |\downarrow_{\hat n}\uparrow_{\hat n}\rangle\right) = \frac{i}{\sqrt{2}}\,|\uparrow\downarrow\rangle - \frac{i}{\sqrt{2}}\,|\downarrow\uparrow\rangle = \frac{i}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)$$

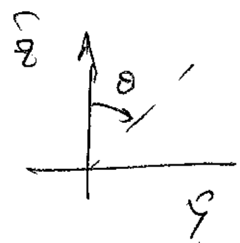
$$P_{\uparrow \hat z} = c^2\frac{\theta}{2} = |\langle \uparrow | \uparrow_{\hat n}\rangle|^2$$

in state $|\uparrow_{\hat n}\rangle$

$$P_{\downarrow \hat z} = s^2\theta/2 = |\langle \uparrow_{\hat z} | \downarrow_{\hat n}\rangle|^2$$

$$P_{\underbrace{\hat z,+}_{1},\ \underbrace{\hat y,+}_{2}} = \left|\langle \uparrow_{\hat z} \uparrow_{\hat y} | \frac{1}{\sqrt2}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)\right|^2 =$$

$$= \left|\frac{1}{\sqrt2}\langle \uparrow_{\hat y}|\downarrow_{\hat z}\rangle\right|^2 = \frac12 \sin^2\left(\frac{\pi}{4}\right) = \frac14 = \frac12\times\frac12$$



$$P_{\hat z,+;\hat n,+} = \frac12 |\langle\uparrow_{\hat n}|\downarrow_{\hat z}\rangle|^2 = \frac12 \sin^2\frac{\theta}{2}$$

$$P_{\hat y,+,\hat n,+)} = \frac12 |\langle \uparrow_{\hat n}|\downarrow_{\hat y}\rangle|^2 = \frac12 \sin^2\left(\frac{\pi}{4}-\frac{\theta}{2}\right)$$

$c2\theta = c^2 - s^2 = 1 - 2s^2$

$s^2 = \frac{1-c2\theta}{2}$

if $\theta = \pi/4 \Rightarrow$
$$P_{\hat z,+,\hat n,+} = \frac12 \sin^2\frac{\pi}{8} = \frac14\left(1 - c\frac{\pi}{4}\right) = \frac14\left(1-\frac{\sqrt2}{2}\right)$$
$$P_{\hat y,+\,\hat n+} = \frac12 \sin^2\frac{\pi}{8} = \frac14\left(1-\frac{\sqrt2}{2}\right)$$

$$\frac12\times\frac{1}{\cancel{2}} \ ?> \frac{1}{\cancel{2}}\left(1-\frac{\sqrt2}{2}\right)$$

$$\frac12 + \frac{\sqrt2}{2} > 1$$

$$1+\sqrt2 > 1$$

Suppose result is predetermined.

$N_{\hat{z},\hat{y},\hat{n}}$ ----
$+ \; + \; +$

particle 1. undetermined

$$P_{\hat{z},+;\hat{y},+} = \frac{N_{(+-+)} + N_{(+--)}}{N_T}$$

$$P_{\hat{z},+;\hat{n},+} = \frac{N_{++-} + N_{+--}}{N_T}$$

$$P_{\hat{n},+;\hat{y}+} = \frac{N_{+++} + N_{-++}}{N_T}$$

$$P_{\hat{z},+;\hat{y},+} = \frac{N_{+++} + N_{+--}}{N_T} \; ? \; P_{z+\hat{n}+} + P_{n+,y+} =$$

$$= \frac{N_{++-} + N_{+--} + N_{+++} + N_{-++}}{N_T}$$

$$P_{z+,y+} \leq P_{z+\hat{n}+} + P_{n+y+}$$

For particle 1

$$P_{z+,y-} \leq P_{z+,n-} + P_{n+,y-}$$

$$P_{z+y-n+} + P_{z+y-n-} \leq$$
$$\leq P_{z+n-} + P_{n+,y-}$$