617, Homework II

Problem 1

Let

$$H = \int d^3p\, f(p)\, \ln f(p) \tag{0.1}$$

where $f(p,t)$ is arbitrary except for the conditions

$$N = \int d^3p f(p) \tag{0.2}$$

$$E = \frac{1}{N}\int d^3p \frac{p^2}{2m} f(p) \tag{0.3}$$

for some given values N, E. Find the function $f(p)$ that minimizes H. How do you interpret the result?

Problem 2

A room of volume $V = 10m^3$ is under standard conditions of pressure and temperature (atmospheric pressure and $T = 300^o K$)

1. Estimate the probability that, at an instant of time a $1cm^3$ of volume anywere in the room is totally devoid of air due to a statistical (or thermal) fluctuation.

2. The same for a volume 1 $\mathring{A}^3$.

Problem 3

Compute the density fluctuations in the grand canonical ensemble in a similar way as we did the fluctuations of energy in the canonical one. Write your answer using the isothermal compressibility

$$\kappa_T = -\frac{1}{v}\frac{\partial v}{\partial P} \tag{0.4}$$

where $v = \frac{V}{N}$ and P is the pressure. Argue that the fluctuations are small (compared to what?)

Problem 4

Consider the classical ideal gas in the grand canonical ensemble. Compute the grand partition function Ξ and derive the thermodynamic quantities including the equation of state, specific heat, entropy, chemical potential, and energy. Compare with the results from the canonical ensemble.