617, Homework III

Problem 1

For a free gas, compute the following (in both cases, Bose and Fermi):

a) The fluctuations of n_p, the occupation number of a state of given momentum p.

b) The fluctuations of

$$n(p, \Delta p) = \frac{V}{h^3} \int_{p<|\vec{p}|<p+\Delta p} d^3p \; n_p \tag{0.1}$$

for a finite Δp such that $\Delta p << p$

c) Determine if the fluctuations in a) and b) are small (compared to the fluctuating quantity) in the thermodynamical limit ($N, V \to \infty$).

Problem 2

Compute the grand partition function for a system of N non-interacting quantum harmonic oscillators, all with the same frequency ω_0. Determine the free energy, energy, entropy and specific heat of the system as functions of (T, N).

Problem 3

Consider a simple model of a solid as a set of N harmonic oscillator of frequency ω_1 (use results from the previous problem) and total (macroscopic) ground state energy E_1 (including the zero point energy of the oscillators). The solid can also be in another phase with ground state energy $E_2 > E_1$ and $\omega_2 < \omega_1$ representing a less tightly bound crystal.

a) Compute the partition function of both phases.

b) What is the preferred phase at low temperature and what the one at large temperature. Show that there is a phase transition.

c) Assuming that the phase transition happens in the regime of large temperature, determine the temperature of the transition.

Problem 4

Treating all cases as free particles and using standard data, determine the Fermi energy of

a) electrons in a metal

b) nucleons in a heavy nucleus

c) He^3 atoms in liquid He^3 ($v = 46\mathring{A}^3$/atom)

Problem 5

Compute the Helmoltz free energy per particle for a free fermion gas at low temperature (using the Sommerfeld expansion).

Problem 6

Consider a typical metal and compare the specific heats of the electron and phonon gases at room temperature. Treat both as free gases and use your previous estimate for the Fermi energy and estimate the Debye temperature for phonons as done in class.

Problem 7

A cylinder is divided by a piston separating two free fermion gases with particles of the same mass. The only difference is that on one side the particles have spin 1/2 and on the other spin 3/2. Find the relative density of the two gases at equilibrium in the limits of small and large temperatures.

Problem 8

Compute the entropy per photon in blackbody radiation.

Problem 9

Compute the grand canonical partition function for a **two dimensional** free Bose gas $\epsilon_p = \frac{p^2}{2m}$. Give formulas for the number of particles N, pressure and entropy as a function of (μ, T, V) where in this case the volume is the area available to the system. Show that there is no Bose-Einstein condensation.