ECE 661 Computer Vision: Exam 2, Fall 2006

1. (8 points)

(a) Show that a world plane is imaged by a camera matrix **P** according to the following relationship

$$\mathbf{x} = \mathbf{H}\,\mathbf{x}_{\Pi}$$

where **H** is a 3x3 homography of rank 3.

x is a 3-vector in the homogenous representation of an image point.

$\mathbf{x}_{\Pi}$ is a 3-vector in the homogenous representation of a point in a world plane.

Hint: Assume that the world plane is given by Z=0.

(b) Show that if **P** = **K** [**R t**], and the world plane being imaged is the Z=0 plane, then

$$\mathbf{H} = \mathbf{K}\,[\mathbf{r}_1\ \mathbf{r}_2\ \mathbf{t}]$$

where **K** is the intrinsic camera parameter matrix, $\mathbf{R}=[\mathbf{r}_1\ \mathbf{r}_2\ \mathbf{r}_3]$ and **t** are the extrinsic camera parameters.

2. (7 points)

Show that the set of world points that are mapped to a line **l** in the image plane via the camera matrix **P** constitute the plane $\mathbf{P}^T\mathbf{l}$.

Hint: If a world point **X** is in the world plane $\boldsymbol{\pi}$, then $\mathbf{X}^T\boldsymbol{\pi}=0$.

3. (15 points)

(a) Show that when the camera matrix $\mathbf{P} = \mathbf{K}\,\mathbf{R}\,[\mathbf{I} \mid -\check{\mathbf{C}}]$, a world point $\mathbf{X}_\infty = [\mathbf{d}^T\ 0]^T$ in the plane at infinity ($\boldsymbol{\pi}_\infty$) is mapped to the image point $\mathbf{x} = \mathbf{Hd}$ with the planar homography $\mathbf{H} = \mathbf{KR}$.

(b) Show that with a general camera $\mathbf{P} = \mathbf{K}\,\mathbf{R}\,[\mathbf{I} \mid -\check{\mathbf{C}}]$ the image of the absolute conic Ω_∞ is the conic $\boldsymbol{\omega} = (\mathbf{KK}^T)^{-1}$.

Hints:

- The result from part (a) may be helpful for solving part (b)

- Under 2D point homography $\mathbf{x}' = \mathbf{Hx}$, a conic $\mathbf{C}$ is transformed as $\mathbf{C}' = \mathbf{H}^{-T}\mathbf{C}\,\mathbf{H}^{-1}$.

- Recall that the absolute conic Ω_∞ is a (point) conic on $\boldsymbol{\pi}_\infty$. The points satisfy the equations:

$$X_1^2 + X_2^2 + X_3^2 = 0 \quad \text{and} \quad X_4 = 0$$

- The above equations can be written as

$$(X_1\ X_2\ X_3)\ \mathbf{I}\ (X_1\ X_2\ X_3)^T = 0$$

with the understanding that we are only considering the points on $\boldsymbol{\pi}_\infty$ (that is, $X_4 = 0$).

4. (15 points)

For the purpose of camera calibration, consider using three world planes that are not parallel. Assume that the homography $\mathbf{H}$ that maps from a point $\mathbf{x}_\Pi$ on one of these three planes to the image $\mathbf{x}$ is given by

$$\mathbf{x} = \mathbf{H}\,\mathbf{x}_\Pi = [\mathbf{h}_1\ \mathbf{h}_2\ \mathbf{h}_3]\ \mathbf{x}_\Pi$$

(a) Show that the IAC ($\boldsymbol{\omega}$) obeys the following constraints:

$$\mathbf{h}_1^T \boldsymbol{\omega}\, \mathbf{h}_2 = 0 \quad \text{and} \quad \mathbf{h}_1^T \boldsymbol{\omega}\, \mathbf{h}_1 = \mathbf{h}_2^T \boldsymbol{\omega}\, \mathbf{h}_2$$

(b) Briefly explain how to use the above constraints to determine the intrinsic camera parameter matrix **K**.

Hints:

- A plane $\boldsymbol{\pi}$ intersects $\boldsymbol{\pi}_\infty$ in a line and this line intersects Ω_∞ in two points that are the circular points of $\boldsymbol{\pi}$ i.e. $[1, \pm i, 0]^T$. The images of circular points lie on $\boldsymbol{\omega} = (\mathbf{K}\mathbf{K}^T)^{-1}$.

5. (10 points)

(a) Briefly explain the three criteria that are chosen to characterize the performance of Canny edge detector.

(b) How does one carry out the Hough transformation of an image if the goal is to extract straight line segments from the image?

6. (10 points)

Given two 3x4 camera projection matrices **P** and **P′**, show that the fundamental matrix **F** that relates the corresponding points in two images is given by

$$F = [\mathbf{e}']_x \mathbf{P}' \mathbf{P}^+$$

where $\mathbf{e}'$ is the epipole in the second (right) image, and the notation $[\mathbf{a}]_x$ is to convert a 3-vector $\mathbf{a}=[a_x\ a_y\ a_z]^T$ into the 3x3 skew-symmetry matrix shown below:

$$[\mathbf{a}]_x = \begin{bmatrix} 0 & -a_z & a_y \\ a_z & 0 & -a_x \\ -a_y & a_x & 0 \end{bmatrix}$$

Hint:

- For $\mathbf{x}=\mathbf{P}\mathbf{X}$, the ray back-projected from $\mathbf{x}$ by $\mathbf{P}$ can be expressed as

$$\mathbf{X}(\lambda) = \mathbf{P}^+\mathbf{x} + \lambda\mathbf{C}$$

where **C** is the center of camera **P**.

7. (8 points)

Suppose we have two images acquired by cameras with non-coincident centers, provide answers to the following questions:

(a) The fundamental matrix **F** is a rank ***x*** homogenous 3x3 matrix with ***y*** degree of freedom. What are the values of ***x*** and ***y***?

(b) All corresponding points $\mathbf{x} \leftrightarrow \mathbf{x}'$ in the two images of a stereo pair must satisfy the epipolar constraint that relates **x, x′** and **F**. What is the mathematical expression for the epipolar constraint?

(c) What is the mathematical expression in term of **F** for the epipolar line $\mathbf{l}'$ in the second (right) image for the pixel at **x** in the first (left) image?

(d) What is the mathematical expression in term of **F** for the epipolar line **l** in the first (left) image for the pixel at $\mathbf{x}'$ in the second (right) image?

(e) What is the mathematical relationship between the epipole **e** on the first (left) image and the fundamental matrix **F**?

(f) What is the mathematical relationship between the epipole $\mathbf{e}'$ on the second (right) image and the fundamental matrix **F**?

8. (10 points)

Given the fundamental matrix **F**, show that the pair of camera matrices ($\mathbf{P}$, $\mathbf{P}'$) can only be determined up to an arbitrary projective transformation of 3-space.

9. (5 points)

Briefly explain that why and in which context we would prefer to use 7-point correspondences for computing the fundamental matrix **F** rather than using the 8-point algorithm.

10. (7 points)

(a) Why is the epipolar constraint useful for stereo matching (i.e. finding corresponding points between the first and the second images)?

(b) What happens to the epipoles and the epipolar lines in the rectified images after applying image rectification?

(c) What are the advantages of applying image rectification before we do stereo matching?

11. (5 points)

What are the essential arguments that go into Otsu's algorithm for finding the best gray level threshold for image segmentation?