Local Bargaining and Supply Chain Instability

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This version: December 2016

Abstract

We analyze the behavior of a competitive n-tier supply chain system, where agents bargain with each other locally. We study the influence of transaction costs on the convergence of the system to a stationary outcome. In particular, we consider a dynamic bargaining game among a finite set of agents and its replications, and use a *limit stationary equilibrium* to examine the system's behavior as the population's size goes to infinity. The convergence of the system to a limit stationary equilibrium would capture the common belief that as the market gets large, it converges to a competitive and stable outcome. However, we prove that depending on the underlying transaction costs, the system might not converge to such an equilibrium. Interestingly, our result shows that a small increase of the transaction cost at one end of the chain can be greatly amplified, and shifts the system away from the steady state. When a limit stationary equilibrium exists, we show that it is unique. We use such an equilibrium outcome to study trade surplus among sellers, buyers and middlemen.

Keywords: Non-cooperative Bargaining, Incentives, Supply Chains

1 Introduction

Multiple tiers of intermediaries are common in several economic environments, ranging from the supply chains of agricultural products to the global financial markets. In the global coffee supply chain, for example, several layers of middlemen such as brokers, wholesalers, and third and fourth party logistic providers engage in getting coffee beans from farmers to end-consumers. Another example is the dark pools of financial markets, where financial intermediaries buy portfolios and assets from other intermediaries. These assets are often repackaged and flipped through many middlemen before being bought by investors. Such intermediary chains, on the one hand, help markets to overcome trade barriers and frictions, but on the other hand, can create incentive misalignment that causes market inefficiency. This has lead to an extensive literature studying several important aspects of supply chain intermediaries such as the double marginalization problem (Lerner (1934) and Tirole (2001)) and the efficiency effect of various forms of contracts (Lariviere and Porteus (2001) and Perakis and Roels (2007)). See also Cachon (2003) for an extensive survey. This literature, however, has mainly used Stackelberg games where agents in upstream markets have full market power to offer take-it-or-leave-it contracts to downstream agents. Despite its wide acceptance, Stackelberg market structures are inconsistent with several pieces of empirical evidence suggesting that bilateral bargaining in vertical relations is, in fact, much more common (see for example, Iyer and Villas-Boas (2003) and Draganska et al. (2010)). Inquiry into the effects of negotiations on the behavior of the whole supply chain system has important implications for supply chain network design. Our paper uses non-cooperative bargaining to model negotiation in vertical relations of an n-tier supply chain. Our results provide new insights into the instability of long supply chains and how sellers, buyers, and middlemen split the trade surplus.

Specifically, we consider a *non-cooperative bargaining game* in an n-tier supply chain system. We assume non-negative transaction costs between the layers of the supply chains. These transaction costs can be thought of as the cost of transportation, or the cost of modifying and updating the good as it goes through the supply chain. For example, in a coffee supply chain, we can think of these costs as the combination of shipping, storing, roasting and packaging the product. When agents interact according to a non-cooperative bargaining game, described formally in the next section, the stability of this supply chain system depends on whether the value of the good is larger or smaller than a weighted sum of the transaction costs. The weight can be as large as the length of the supply chain. This characterization suggests an interesting phenomenon of supply chain instability, in which a small increase of the transaction cost toward the beginning of the chain can be greatly amplified (by a factor of $n+1$) and makes the supply chain system unstable. Moreover, when the supply chain is stable, we show that the equilibrium is unique. We use such an equilibrium outcome to study trade surplus among sellers, buyers and middlemen.

Our paper is organized as follows. Section 2 introduces the model and solution concepts. Section 3 provides the main technical characterization and its interpretations. Section 4 provides several interpretations and applications of our model. In particular Section 4.1 analyzes the (non)existence of the limit stationary equilibrium. Section 4.2 discusses the share of surplus between sellers and buyers. Section 4.3 studies middlemen payoff and their incentive in merger. Section 5 discusses related literature, and the Appendix contains the proofs.

2 The Model and Solution Concept

Consider a supply chain system with n tiers of middlemen, modeled by a line network whose nodes are indexed sequentially by $0, 1, \ldots, n+1$. Let N be $\{0, 1, \ldots, n+1\}$. Each node $i \in N$ represents a population of $m_i \geq 1$ agents. Node 0 and $n+1$ represent sellers and buyers, respectively, while $\{1, \ldots, n\}$ correspond to middlemen. In the remaining of the paper, an agent i is referred to one of the m_i agents at node i.

There is one type of indivisible good in this economy. In every time period each agent can hold at most one unit of the good (an item). This assumption represents the simplest inventory capacity. Because our main focus is to investigate the incentive structure of bargaining even when each agent has a vanishing influence on the economy, this assumption allows us to limit the influence that an agent has on the aggregate economy. We assume agents trade along the chain from node 0 to node $n+1$. Two consecutive agents i and $i+1$ are *feasible trading partners* if i owns an item and $i+1$ does not own one.

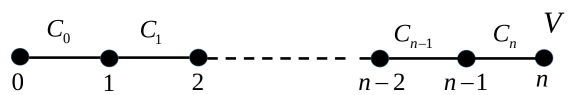

Figure 1: An n-tier supply chain

The transaction cost between two consecutive agents i and $i+1$ is $C_i \geq 0$. The value that a buyer gets for an item is $V \geq 0$. We assume the trade is beneficial, i.e., $V > \sum_i C_i$. C_is and the value V are common knowledge.

The Bargaining Process We consider an infinite horizon, discrete time repeated bargaining game, where an agent i's discount rate is $0 < \delta_i < 1$. Each period is described as follows.

A node i is uniformly picked randomly from the network, then among the m_i agents an agent i is selected uniformly at random. If the selected agent i does not have any feasible trading partner, then the game moves to the next period, starting from Step 1. (Recall that, depending on whether i has an item or not, a feasible trading partner of i is either an agent $i+1$ not owning an item or an agent $i-1$ that owns an item.) If i has several feasible trading partners, then i randomly selects one with which to bargain. To bargain, i suggests a price at which he is willing to trade. If the trading partner refuses, the game moves to the next period. Otherwise, the two agents trade: one gives the item to and receives money from the other and pays the associated transaction cost with this money. We assume middlemen are long-lived and they neither produce nor consume. Buyers and sellers, on the other hand, exit the game if the trade in which they are involved is successful, and they are replaced by clones. The game then moves to the next period.

We denote such a bargaining game by $\Gamma(N, m, C, V, \delta)$.

Solution Concept

To define limit stationary equilibrium, we need several concepts. We first define a (k, T_k)-*replication* of the bargaining game. In the replicated game, the bargaining process is exactly the same as described above, except that the population at each node $i \in \{0, \ldots, n+1\}$ is increased by a factor of k, and the time gap between two consecutive periods is decreased by a factor of T_k. The decrease of the time between two periods is captured by changing the discount rate of i from δ_i to δ_i^{1/T_k}. We assume $T_k \geq 1$, and is nondecreasing in k.

Given the game $\Gamma(N, m, C, V, \delta)$, its replications are defined as follows.

DEFINITION 1 *Given* $\Gamma(N, m, C, V, \delta)$ *and* $k, T_k \in \mathbb{N}^+$, *let* $m'_i = k \cdot m_i$ *and* $\delta'_i = \delta_i^{1/T_k}$. *The* (k, T_k)*-replication of* Γ *is defined as* $\Gamma(N, m', C, V, \delta')$, *and denoted by* $\Gamma^k_{T_k}(N, m, C, V, \delta)$.

The replication is simply defined as a new finite game in which the population is multiplied by $k > 1$, and the time gap between two consecutive periods reduces to $\frac{\Delta}{T_k}$. The rate $\frac{T_k}{k}$ represents the density of the market transactions in a similar way as changing the time gap Δ in the traditional approach. Namely, when $\lim_{k\to\infty} T_k/k = \infty$, the population grows more slowly than the transaction rates, and agents get selected more and more frequently. This scenario is equivalent to the previous approach when Δ approaches 0. On the other hand, when $\lim_{k\to\infty} T_k/k = 0$, agents will have vanishing opportunities to trade, and the equilibrium converges to a trivial solution, in which each agent's expected payoff is the same as their outside option. The remaining case is when $\lim_{k\to\infty} \frac{T_k}{k}$ is a constant. In this scenario, the rate at which each agent is selected is invariant in the replicated games.

State Space and Stationary Strategy In every period of the game, each agent i either owns an item and tries to sell or does not own an item and is looking to buy. We denote these two states of i by i_1 and i_0, respectively. We represent an aggregate state of the system by an n-dimensional vector ω where its i-th coordinate, ω_i, is the fraction of population of middlemen at node i who currently own an item The set of the aggregate states of Γ is denoted by $\Omega \subset \mathbb{R}^n_+$, and because in Γ the populations of middlemen are $m_1, ..., m_n$,

$$\Omega = \{\frac{0}{m_1}, \frac{1}{m_1}, \frac{2}{m_1}, ..., \frac{m_1}{m_1}\} \times ... \times \{\frac{0}{m_n}, \frac{1}{m_n}, \frac{2}{m_n}, ..., \frac{m_n}{m_n}\}.$$

We are interested in stationary equilibria, and thus focus on stationary strategies defined as follows.

DEFINITION 2 *A* stationary strategy profile σ *is a mapping from the play of the game in a single period to the agents' strategies, and depends on neither the history of the game nor the state of the system. Namely, suppose the current play of the game is such that agent i and j are selected by the bargaining process and i is the proposer, then σ consists of a distribution of prices that i proposes to j and a function indicating the probability of j's accepting the offer for each of the proposed prices.*

To define a limit stationary equilibrium, we will need to introduce the following concept of semi-stationary equilibrium, which captures the incentive structure in finite games.

DEFINITION 3 *Given a finite game Γ, let Ω be the set of possible aggregate states. A pair of state and stationary strategy profile (ω, σ), where $\omega \in \Omega$ and $0 < \omega_i < 1$ for all $0 \leq i \leq n$, is a* **semi-stationary** *equilibrium if σ is a sub-game perfect equilibrium of Γ assuming that agents believe that the state of the economy always remains at ω.*

In our dynamic games, the agents' strategies change the system's state, however, in the definition of semi-stationary equilibrium we do not require any condition on how the state ω is influenced by σ. Next, we incorporate such a condition in the limit when the system gets large.

Finally, to define our solution concept, we will need a *balance condition*, defined in Definition 4. This condition corresponds to the standard "law of motion" in continuum market applied at a steady state. Thus, intuitively, our limit stationary equilibrium is a limit of equilibria in the finite economies that satisfy the balance condition.

In particular, we consider how the trading dynamic influences the aggregate state of the system in the limit. For a given stationary strategy profile σ if in the game $\Gamma^k_{T_k}$, the agents play according to σ, then the game dynamic is a Markov chain on the set of aggregate states

$$\Omega(k) = \{\frac{0}{km_1}, \frac{1}{km_1}, ..., \frac{km_1}{km_1}\} \times ... \times \{\frac{0}{km_n}, \frac{1}{km_n}, ..., \frac{km_n}{km_n}\}.$$

According to the rule of the game, after each period the state $\omega \in \Omega(k)$ of the system either stays unchanged or only two of its coordinates, say ω_i and ω_{i+1}, move up or down by $\frac{1}{km_i}$ and $\frac{1}{km_{i+1}}$, respectively. Suppose the current state of the game is ω, then the probability that ω_i goes down by $\frac{1}{km_i}$ is equivalent to the probability that trade between i and $i+1$ occurs. Specifically, at a given period the probability that an agent i owning an item is selected is $\frac{1}{n+2}\omega_i$, and the probability that some agent $i+1$ not owning an item is selected is $\frac{1}{n+2}(1-\omega_{i+1})$. Therefore, if

ω satisfies $0 < \omega_i < 1$ for $1 \leq i \leq n$, then the probability that in a period an agent i sells an item to an agent $i+1$ is equal to

$$\begin{aligned} \mathbf{P}_{i \to (i+1)}(\omega, \sigma) = & \frac{1}{n+2}\omega_i \times Pr(i+1 \text{ accepts } i\text{'s offer}) \\ & + \frac{1}{n+2}(1-\omega_{i+1}) \times Pr(\ i \text{ accepts } i+1\text{'s offer}). \end{aligned} \tag{1}$$

Hence, the probability that ω_i goes up by one unit is $\mathbf{P}_{(i-1)\to i}$.

To define our solution concept, we first need the following notion.

DEFINITION 4 *A sequence of states and stationary strategies $(\omega(k), \sigma(k))$ in $\Gamma^k_{T_k}$ is said to be* **balanced**, *if*

$$\lim_{k\to\infty} (\mathbf{P}_{(i-1)\to i}(\omega(k), \sigma(k)) - \mathbf{P}_{i\to(i+1)}(\omega(k), \sigma(k))) = 0$$

for every $i \in \{1, \ldots, n\}$, where $\mathbf{P}$ is defined as in (1).

We are now ready to define the concept of limit stationary equilibrium.

DEFINITION 5 *Given a bargaining game Γ and a class of (k, T_k) replications, a pair of an aggregate state $\omega \in [0,1]^n$ and a stationary strategy profile σ is a* **limit stationary equilibrium** *of Γ, if there exist a constant K and a sequence of semi-stationary equilibria $(\omega(k), \sigma(k))$ in $\Gamma^k_{T_k}$ for all $k \geq K$, such that the sequence $(\omega(k), \sigma(k))$ is balanced and converges to (ω, σ).*

Remark. To define the convergence of the sequence $(\omega(k), \sigma(k))$, here we assume that there is a metric on the set of states and strategies. To be concrete, consider, for example, the Euclidean metric for the state space and the Discrepancy metric for the space of stationary strategies.

3 Characterization of Limit Stationary Equilibrium

In this section we prove our main technical characterization. Section 3.1 characterizes the set of semi-stationary equilibria for a finite game, Section 3.2 examines the limit behavior of these equilibria as the population's size goes to infinity.

3.1 Characterization of semi-stationary equilibria

The following characterization for a semi-stationary equilibria is core to our analysis.

LEMMA 1 *Given the game $\Gamma(N, m, C, V, \delta)$, consider the following strictly convex program*

$$\textit{minimize: } \sum_{i=0}^{n}(n+2)m_i\frac{(1-\delta_i)}{\delta_i}x_i^2 + (n+2)m_{n+1}\frac{(1-\delta_{n+1})}{\delta_{n+1}}(\delta_{n+1}V - x_{n+1})^2 + \sum_{i=0}^{n} z_i^2 \tag{A}$$
$$\textit{subject to: } x, z \geq 0 \textit{ and } x_i - x_{i+1} + z_i + C_i \geq 0 \ \forall\ 0 \leq i \leq n.$$

Let x, z be the unique solution of (A). If (ω, σ), where $0 < \omega_i < 1$ for all $i \in \{1, \ldots, n\}$, is a semi-stationary equilibrium, then
(i) *the expected payoffs of sellers and buyers are $\frac{x_0}{\delta_0}$ and $V - \frac{x_{n+1}}{\delta_{n+1}}$, respectively;*

(ii) *if $z_i > 0$, then when trade $(i \to i+1)$ arises, it is accepted with probability 1;*

(iii) *if $x_i - x_{i+1} + C_i > 0$, then i and $i+1$ never trade.*

The proof of Lemma 1 is given in Appendix A.

3.2 Limit Stationary Equilibrium

Next, we consider the limit behavior in the replicated economies. As argued in earlier, in the regime $\lim_{k\to\infty} T_k/k = 0$, agents have vanishing trade opportunities, and their payoffs approach 0. In the remainder of the paper, we consider $\lim_{k\to\infty} T_k/k = \textit{constant} > 0$ and $\lim_{k\to\infty} T_k/k = \infty$. When $\lim_{k\to\infty} T_k/k = \textit{constant} > 0$, for simplicity, we assume $\textit{constant} = 1$. Our results extend naturally to any other constant.

To analyze the limit behavior, we apply Lemma 1 to the game $\Gamma^k_{T_k}$ by replacing m_i with km_i and δ_i with δ_i^{1/T_k} in the convex program (A) . Thus, for each k we obtain the following.

$$\text{minimize: } \sum_{i=0}^{n}(n+2)km_i\frac{(1-\delta_i^{1/T_k})}{\delta_i^{1/T_k}}x_i^2 + (n+2)km_{n+1}\frac{(1-\delta_{n+1}^{1/T_k})}{\delta_{n+1}^{1/T_k}}(\delta_{n+1}^{1/T_k}V - x_{n+1})^2 + \sum_{i=0}^{n} z_i^2$$
$$\text{subject to: } x, z \geq 0 \text{ and } x_i - x_{i+1} + z_i + C_i \geq 0 \ \forall\ 0 \leq i \leq n. \tag{A^k}$$

Observe that $\lim_{T_k\to\infty} T_k\frac{1-\delta_i^{1/T_k}}{\delta_i^{1/T_k}} = \ln(\frac{1}{\delta_i})$. Using this relation, let $(x^{(k)}, z^{(k)})$ be the optimal solution of (A^k), we will show that if $lim_{k\to\infty}T_k/k = 1$, then as k approaches infinity, $(x^{(k)}, z^{(k)})$ approaches the unique solution of

$$\text{minimize: } \sum_{i=0}^{n}(n+2)m_i\ln(\frac{1}{\delta_i})x_i^2 + (n+2)m_{n+1}\ln(\frac{1}{\delta_{n+1}})(V - x_{n+1})^2 + \sum_{i=0}^{n} z_i^2 \tag{B}$$
$$\text{subject to: } x, z \geq 0 \text{ and } x_i - x_{i+1} + z_i + C_i \geq 0 \ \forall\ 0 \leq i \leq n.$$

On the other hand, if $\lim_{k\to\infty}\frac{T_k}{k}=\infty$, the coefficient of x_i, i.e., $(n+2)km_i\frac{1-\delta_i^{1/T_k}}{\delta_i^{1/T_k}}$, approaches 0 as k approaches infinity. Thus, it can be shown that $z^{(k)}$ approaches 0 and $x^{(k)}$ converges to the unique solution of the following convex program.

$$\begin{aligned}&\text{minimize: } \sum_{i=0}^{n}(n+2)m_i\ln(\frac{1}{\delta_i})x_i^2+(n+2)m_{n+1}\ln(\frac{1}{\delta_{n+1}})(V-x_{n+1})^2\\&\text{subject to: } x\geq 0 \text{ and } x_i-x_{i+1}+C_i\geq 0 \ \forall\ 0\leq i\leq n.\end{aligned}\tag{C}$$

We obtain the following necessary conditions for a limit stationary equilibrium in the two regimes. The proof of these results is given in Appendix B.

THEOREM 1 *Consider the two regimes $lim_{k\to\infty}T_k/k=1$ and $lim_{k\to\infty}T_k/k=\infty$, assume a limit equilibrium (ω,σ) of Γ exists. Let x be the unique solution of (B) for the regime $lim_{k\to\infty}T_k/k=1$ or the unique solution of (C) for the regime $lim_{k\to\infty}T_k/k=\infty$, then*

(i) *the limit payoffs of buyer and seller are x_0 and $V-x_{n+1}$, respectively*

(ii) *if $x_i>0$, then when trade $(i\to i+1)$ arises, it is accepted with probability 1*

(iii) *if $x_i-x_{i+1}+C_i>0$, then i and $i+1$ never trade.*

Notice that when $\lim_{k\to\infty}\frac{T_k}{k}=\infty$ agents gets more and more bargaining opportunities, and they are willing to wait for the best deal. (In the bargaining literature agents in this scenario are called "patient".) On the other hand, in the regime $\lim_{k\to\infty}\frac{T_k}{k}=1$, if an agent is proposed and he rejected the offer, then it will cost him to wait for the next available trade. Because of this, compared with the former scenario, equilibrium in the latter will be affected by additional "frictions". Such insights can be seen from the two convex programs (B) and (C). Namely, let x,z be the solution of (B), which characterizes the equilibrium in the regime $\lim_{k\to\infty}\frac{T_k}{k}=1$. Consider now a supply chains with new transaction costs: $C_i'=C_i+z_i$ and assume $\lim_{k\to\infty}\frac{T_k}{k}=\infty$. Because x is the solution of (C) under C', the behavior of the supply chain when $\lim_{k\to\infty}\frac{T_k}{k}=1$ is the same as the behavior of the system when $\lim_{k\to\infty}\frac{T_k}{k}=\infty$ with higher transaction cost, $C_i'=C_i+z_i$. Hence, z_i-s can be interpreted as additional costs caused by bargaining when the agents are not patient.

4 Interpretations and Comparative Analysis

In this section, we apply Theorem 1 to analyze the influence of the supply chain cost structure on several features of the market outcome.

4.1 Supply Chain Instability

We show our first comparative result on the (non)existence of a limit stationary equilibrium. For simplicity, we assume that the population and the discount factor for each i are the same, that is, $m_i = m$, $\delta_i = \delta$. Our main result in this section is the following theorem.

THEOREM 2 *Consider the game* $\Gamma(N, m, C, V, \delta)$*, where* $m_i = m, \delta_i = \delta$ *and a class of* (k, T_k)*-replications such that* $\lim_{k\to\infty} T_k/k = \infty$*, then the following holds.*

(I) *If* $V > (n+1)C_0 + nC_1 + .. + C_n$*, then there exists an unique limit stationary equilibrium.*

(II) *If* $\sum_i C_i < V < (n+1)C_0 + nC_1 + .. + C_n$*, then the game* Γ *does not have a limit stationary equilibrium.*

With the observation at the end of Section 3, we obtain the following corollary of Theorem 2.

COROLLARY 4.1 *Consider the game* $\Gamma(N, m, C, V, \delta)$*, where* $m_i = m, \delta_i = \delta$ *and a class of* (k, T_k)*-replications such that* $\lim_{k\to\infty} T_k/k = 1$*, then there exists* $z \geq 0$ *such that:*

(I) *If* $V > (n+1)(C_0 + z_0) + n(C_1 + z_1) + .. + (C_n + z_n)$*, then there exists an unique limit stationary equilibrium,*

(II) *If* $\sum_i C_i < V < (n+1)(C_0 + z_0) + n(C_1 + z_1) + .. + (C_n + z_n)$*, then the game* Γ *does not have a limit stationary equilibrium.*

Thus, compared with scenario in Theorem 2, when $\lim_{k\to\infty} T_k/k = 1$, V needs to be larger for the system to be stable. As argued earlier, this is because of the additional bargaining cost caused by impatient agents, captured by the variable z of the program (B).

The proof of Theorem 2 is given in Appendix C, in which we start by assuming that a limit stationary equilibrium exists. Then, by Theorem 1, the payoff and the trade pattern are determined by the solution of the convex program (C). Solving this program, we observe that when $V > (n+1)C_0 + nC_1 + .. + C_n$, the solution x_i are all positive. This means that whenever a possible trade between agent i and $i+1$ arises, agents will trade with probability 1. Furthermore, for this "always trade" dynamic, we can construct a sequence of semi-stationary equilibria that satisfies the balance condition and converges to this limit equilibrium. However, when $V < (n+1)C_0 + nC_1 + .. + C_n$, the solution of the convex program (C) implies that trade between seller and middlemen 1 never occurs, while trade between middlemen n and buyers occurs with probability 1. Naturally, there cannot be any nonzero aggregate state that satisfies the balance condition together with this dynamic because the inflow of goods is zero while the outflow is positive.

The nonexistence of a limit stationary equilibrium implies that no strategy in which agents trade with positive probability can converge to a limit stationary equilibrium. Thus, the system's behavior can be in either a no-trade equilibrium or an equilibrium in which agents' strategies depend on the history of the game and the offered prices and the aggregate state change over time.

The second comparative result can be interpreted from Theorem 2 as follows. Consider an n-tier supply chain system with a large n. A small increase in C_0 can cause a large increase in $(n+1)C_0 + nC_1 + .. + C_n$, and move the cost structure from condition (I) to condition (II). Thus, a small fluctuation at the beginning of the chain can be greatly amplified, moving the system away from the efficient and steady state.

The economic intuition for our result is the following. Consider the case where an agent i in state 1 (owning an item) bargains with an agent $i+1$ in state 0. Because trading with agent $i+1$ is the only option for i, thus even though i has an item, his "bargaining position" is similar to that of $i+1$'s when he does not have an item. Here by bargaining position we mean the expected payoff of an agent in a given state. This is precisely shown in the proof of Lemma 1, which implies $u_{i+1_0} = u_{i_1}$. Now, assume trade occurs between i and $i+1$; agent i's state will be i_0 (not holding an item), while $i+1$'s state will be $(i+1)_1$, and a transaction cost of C_i is paid. The benefit of this trade would be approximately

$$u_{i+1_1} + u_{i_0} - u_{i+1_0} - u_{i_1} - C_i = (u_{i+1_1} - u_{i_1}) - (u_{i_1} - u_{i_0}) - C_i = x_{i+1} - x_i - C_i.$$

In order for trade to be beneficial, this difference must be positive, which implies $x_{i+1} - x_i \geq C_i$. (Here we ignore the discount rates. Precise formulations are given in the proof of Lemma 1 in Appendix A.)

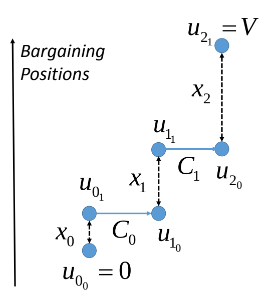

Figure 2: Illustration of a supply chain with 1 tier of middlemen. Bargaining positions go up quickly along the chain.

As illustrated in Figure 2, the transaction cost in the bargaining process between i and $i+1$ also has an influence on the next bargaining step between $i+1$ and $i+2$. In particular, for this trade to be successful, $x_{i+2}-x_{i+1}-C_{i+1}$ needs to be nonnegative. Hence, the difference between the payoff of agent $i+2$ when he owns an item (u_{i+2_1}) and the payoff of agent i when he does not own an item (u_{i_0}) is $x_i+x_{i+1}+x_{i+2} \geq 3x_i+2C_i+C_{i+1}$. As the bargaining continues, the transaction costs at the beginning of the chain repeatedly influence the bargaining process in the later tiers of the supply chain. As a result, the bargaining positions increase very rapidly along the chain. Thus, if we assume that the economy is in a steady state (i.e., u does not change over time), then to support the increase of bargaining positions the value for the good V must sufficiently large enough.

4.2 Buyer-Seller Payoffs

We continue with the scenario in Section 4.1. Assume that $V > (n+1)C_0 + nC_1 + .. + C_n$, in which case a limit stationary equilibrium exists. We will examine the equilibrium payoffs of sellers and buyers in this case.

THEOREM 3 *Consider the game* $\Gamma(N, \vec{m}, \vec{C}, V, \vec{\delta})$*, where* $m_i = m, \delta_i = \delta$ *and a class of* (k, T_k)*-replications such that* $\lim_{k\to\infty} T_k/k = \infty$. *Assume* $V > \sum_{i=0}^{n}(n+1-i)C_i$*, then in the limit equilibrium the expected payoff of a seller* (u_{0_1}) *and the payoff of a buyer* (u_{n+1_0}) *are*

$$u_{0_1} = \frac{(V - \sum_{i=0}^{n} C_i)}{n+2} - \sum_{i=0}^{n} \frac{n-i}{n+2} C_i \text{ and } u_{n+1_0} = \frac{n+1}{n+2}(V - \sum_{i=0}^{n} C_i) + \sum_{i=0}^{n} \frac{n-i}{n+2} C_i.$$

Proof: See Appendix C.2.

To illustrate this outcome, consider a simple example, where $C_0 = C, C_1 = C_2 = .. = C_n = 0$. According to Theorem 3, when $V > (n+1)C$, the expected payoff of sellers and buyers are

$$\frac{1}{n+2}V - \frac{n+1}{n+2}C \text{ and } \frac{n+1}{n+2}V - \frac{1}{n+2}C, \text{ respectively.} \tag{2}$$

From the analysis above, observe that a seller's payoff is smaller than a buyer's. The economic intuition follows a similar argument as in Section 4.1. Namely, when agents bargain along the supply chain, the costs of previous transactions are sunk. Each time a middleman i at state i_1 needs to bargain with a middleman $i+1$ at state $(i+1)_0$, and thus they are in similar bargaining positions. This process increases the agents' payoff as their position gets closer to the buyers. Thus, for example, as illustrated in Figure 2 the payoff of the buyer, u_{2_0}, is larger than the payoff of the seller, u_{0_1}. Similar to the instability effect captured in Theorem 2, the imbalance between the seller's and the buyer's payoffs depends on both the length of the chain and the transaction costs C_i-s.

4.3 Middlemen Incentives

In this section, we analyze the payoff of middlemen and their incentive to change the supply chain structure. Notice that middlemen sometimes hold the good and sometimes not. Let u_{i_1} and u_{i_0} denote the expected payoff of agent i in the two states: holding and not holding an item, respectively. We will use the average $U_i := \frac{u_{i_0}+u_{i_1}}{2}$ as the index of middlemen expected payoff or sometimes we call it bargaining power.

THEOREM 4 *Consider the game* $\Gamma(N, m, C, V, \delta)$, *where* $\delta_i = \delta$ *and a class of* (k, T_k)-*replications such that* $\lim_{k\to\infty} T_k/k = \infty$. *Assume a limit equilibrium exists. Let* x *be the solution of* (C), *then the payoff of middleman* i *is*

$$U_i = \sum_{j=0}^{i-1} \frac{m_j}{m_i} x_j + \frac{x_i}{2}.$$

Proof: See Appendix C.3.

The term $\frac{m_j}{m_i}$ in the payoff of middlemen above reflects the relative size of each middlemen population. If m_i is large, there will be more competition among middlemen at tier i, therefore their payoff will be low.

Furthermore, according to Theorem 4 a middleman i's bargaining position depends on his position in the supply chain. In particular, a middleman that is closer to the buyers will obtain a higher payoff. The intuition again comes from the sunk cost problem, as illustrated in Section 4.1. When middleman i holding an item bargains with $i+1$, who does not have an item, because i does not consume, his only option is to continue trading with $i+1$. Thus, even though $i+1$ does not hold the good, his bargaining position is as high as i who does hold the good. This makes the bargaining position of the middlemen increase along the chain.

The Persistence of Long Chains

Finally, we use a simple example to illustrate whether the middlemen have incentives to change the supply chain structure. In the first scenario, the market has two tiers of middlemen and

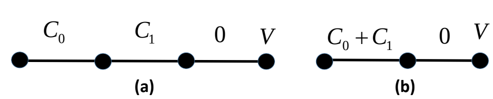

Figure 3: Merging middlemen

the transaction costs are C_0, C_1 and 0. We assume the populations at each node of the supply chain are equal to one another. In the second scenario, pairs of middlemen in the different tiers are partnering and work together as one agent. This is equivalent to a supply chain with one tier of middlemen and transaction costs are of $C_0 + C_1, 0$. See Figure 3 for an illustration. We will consider the limit regime in which $\lim_k T_k/k = \infty$.

By Theorem 2, the second supply chain structure is more robust. Namely, the condition for an efficient limit equilibrium exists in scenario (a), and scenario (b) are $V > 3C_0 + 2C_1$ and $V > 2C_0 + 2C_1$ respectively. Thus, the stability of the longer supply chain is more sensitive to fluctuations in C_0. Furthermore, according to Theorem 3, the share of surplus between sellers and buyers is also more unbalanced in the longer chain. Hence, from a social planner's perspective, the shorter chain in scenario (b) is preferred.

Assume now that the market is in scenario (a), and $V > 3C_0 + 2C_1$ so that there exists a limit stationary equilibrium. We are interested in whether the middlemen have incentive to merge to obtain the more stable supply chain, (b).

It is straightforward to check whether this is the case in the first scenario, by solving the corresponding convex program (C), and let $x^{(a)}$ be its solution. We obtain:

$$x_0^{(a)} = \frac{(V - 3C_0 - 2C_1)}{4} \text{ and } x_1^{(a)} = x_0^{(a)} + C_0; x_2^{(a)} = x_1^{(a)} + C_1; x_3^{(a)} = x_2^{(a)}.$$

According to Theorem 4 the payoff of a middleman 1 is $U_1^{(a)} = x_0^{(a)} + \frac{x_1^{(a)}}{2} = \frac{3}{2}\frac{(V-3C_0-2C_1)}{4} + \frac{C_0}{2}$. The payoff of middleman 2 is $U_2^{(a)} = x_0^{(a)} + x_1^{(a)} + \frac{x_2^{(a)}}{2} = \frac{5}{2}\frac{(V-3C_0-2C_1)}{4} + \frac{3}{2}C_0 + \frac{C_1}{2}$. The total payoff of the two middlemen before merging is $U_1^{(a)} + U_2^{(a)} = (V - 3C_0 - 2C_1) + 2C_0 + \frac{C_1}{2} = V - C_0 - \frac{3}{2}C_1$.

On the other hand, when pairs of middlemen 1 and 2 merge, they act like a single agent. Let $x^{(b)}$ be the optimal solution of the corresponding convex program (C). We have

$$x_0^{(b)} = \frac{1}{3}(V - 2C_0 - 2C_1) \text{ and } x_1^{(b)} = x_0^{(b)} + C_0 + C_1; x_2^{(b)} = x_1^{(b)}.$$

Thus, in this scenario, the payoff of the merged middlemen 1-2 is

$$U_{12}^{(b)} = x_0^{(b)} + \frac{x_1^{(b)}}{2} = \frac{3}{2}\frac{(V - 2C_0 - 2C_1)}{3} + \frac{C_0 + C_1}{2} = \frac{(V - C_0 - C_1)}{2}.$$

However, it is easy to see that if $V > 3C_0 + 2C_1$, then $U_1^{(a)} + U_2^{(a)} > U_{12}^{(b)}$. This suggests that after merging, the two middlemen get lower payoff, thus they have no incentive to merge. We interpret this as an incentive constraint that keeps the chain long and suboptimal.

5 Related Literature

Our paper is closely related to the bargaining framework of Rubinstein and Wolinsky (1987) and other follow-up papers, including Duffie et al. (2005); Rocheteau and Wright (2005), and Nguyen et al. (2015). Our concept of limit equilibrium is similar to stationary equilibrium in those papers. However, our framework is significantly different. Specifically, in prior literature, it is usually assumed that there is a continuum of agents who belong to a set of finite types and in each period, and there is a match that simultaneously assigns each agent with another one. The pairs of matched agents, then involve in a bargaining game. Such a centralized matching process does not exist in many real markets. In our model, we instead select a single agent in very period and let this agent choose her trading partner. However, a technical problem arises because the probability that an agent is selected from a continuum of agents is 0. We overcome this by considering the solution concept as the limit of a sequence of finite games. An important implication from our approach is that while all the papers mentioned above show that a stationary equilibrium exists, here we show that market sometimes fail to converge to a stationary solution.

Our paper also extends the models in Rubinstein and Wolinsky (1987) and the papers mentioned above to consider multiple layers of middlemen. This extension allows us to obtain additional insights on the instability of a long supply chain.

From a technical point of view, our paper further develops the convex program technique developed in Nguyen (2015). An important difference between the current work and that paper is that in Nguyen (2015) all agents are assumed to be short-lived, i.e, they exit the market after trading. Because agents (middlemen, in particular) are, in reality, typically long-lived, the current paper offers more valuable insights into their equilibrium behavior.

Recently, there has been an expanding literature on bargaining games in supply chain. Wu (2004); Nagarajan and Bassok (2008); Feng et al. (2014) and Wan and Beil (2014) are just few examples. However, these papers have either used axiomatic bargaining solution concepts or only considered supply chains without middlemen to avoid the complexity of dynamic bargaining games.

More generally, supply chain stability due to mis-coordination through the tiers is well studied in the literature. See for example, Lariviere and Porteus (2001); Wu (2004) and Perakis and Roels (2007). Our paper provides new insights into the problem that incentives in local bargaining can lead to severe inefficiency. Specifically, the costs at the beginning of a supply chain contribute more to this problem.

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APPENDIX

A Proof of Lemma 1

We start with notation. Consider the game $\Gamma(N, m, C, V, \delta)$, and let (ω, σ) be a semi-stationary equilibrium in this game. Denote by $(i \rightarrow i+1)$ the type of trade between i owning an item and $i+1$ who does no own one. We say trade $(i \rightarrow i+1)$ arises when an agent i owning and item and an agent $i+1$ not owning an item are selected in the process and about to bargain. For every $i \in \{0, 1, .., n+1\}$, let i_0, i_1 denote the state variables of agents of type i when he holds or does not hold an item, respectively.

Let u_{i_1} and u_{i_0} be the expected payoff of agent i in state i_1 and i_0, respectively. Consider

$$x_i = \delta_i(u_{i_1} - u_{i_0}). \tag{3}$$

We will first derive a condition for x based on the recursive formulation of a stationary equilibrium. We then show that these conditions correspond to the complementary slackness conditions of the convex program (A), which proves our lemma.

Now, consider a situation when an agent i is selected to be the proposer and he has an item to sell. Because $\omega_{i+1} < 1$, i has a feasible trading partner, i.e., an agent $i+1$ who currently does not have an item. If trade between i and $i+1$ is successful, then in the next period $i+1$ moves to state $(i+1)_1$, resulting an expected payoff of $\delta_{i+1}u_{(i+1)_1}$, otherwise he stays in the same state with the payoff of $\delta_{i+1}u_{(i+1)_0}$. Hence, in order for $i+1$ to accept the offer, i will need to ask $i+1$ to pay for the difference: $\delta_{i+1}u_{(i+1)_1} - \delta_{i+1}u_{(i+1)_0}$. Furthermore, at equilibrium, i will never offer a lower price. After trade i moves from i_1 to i_0. Therefore, if trade is successful, i's expected payoff is

$$\delta_i u_{i_0} + \delta_{i+1}(u_{(i+1)_1} - u_{(i+1)_0}) - C_i. \tag{4}$$

However, i also has an option to decline to propose (or to suggest an offer that will be rejected), in which case i's payoff is $\delta_i u_{i_1}$. Thus, when i has an item and he is the proposer, i's expected payoff is

$$\max\{\delta_i u_{i_0} + \delta_{i+1}(u_{(i+1)_1} - u_{(i+1)_0}) - C_i, \delta_i u_{i_1}\}.$$

To simplify, we introduce the following variable

$$z_i = \max\{\delta_{i+1}(u_{(i+1)_1} - u_{(i+1)_0}) - \delta_i(u_{i_1} - u_{i_0}) - C_i, 0\} = \max\{x_{i+1} - x_i - C_i, 0\}. \tag{5}$$

With this notation, the payoff of agent i who owns an item and is selected as a proposer, represented by (4), is

$$\max\{\delta_i u_{i_0} + \delta_{i+1}(u_{(i+1)_1} - u_{(i+1)_0}) - C_i, \delta_i u_{i_1}\} = \delta_i u_{i_1} + z_i.$$

Intuitively, $z_i \geq 0$ can be interpreted as the extra surplus that i obtains when he is the proposer.

Now, given an agent i owning an item, the probability of i being selected as the proposer is $\frac{1}{(n+2)m_i}$, and his expected payoff is given as above. On the other hand, when i is not selected as the proposer, his payoff is $\delta_i u_{i_1}$. Thus, over all agent i's expected payoff is $\frac{1}{(n+2)m_i}(\delta_i u_{i_1} + z_i) + \left(1 - \frac{1}{(n+2)m_i}\right)\delta_i u_{i_1}$. For u to be the payoffs of a semi-stationary equilibrium, it needs to satisfy the following recursive Bellman equations.

$$u_{i_1} = \frac{1}{(n+2)m_i}(\delta_i u_{i_1} + z_i) + \left(1 - \frac{1}{(n+2)m_i}\right)\delta_i u_{i_1},$$

which is equivalent to

$$(n+2)m_i(1-\delta_i)u_{i_1} = z_i. \tag{6}$$

Now, if i does not have an item and is selected as the proposer, then i will propose to an agent $i-1$ who has an item. By a similar argument as above, we denote

$$z_{i-1} = \max\{\delta_i(u_{i_1} - u_{i_0}) - \delta_{i-1}(u_{(i-1)_1} - u_{(i-1)_0}) - C_{i-1}, 0\} = \max\{x_i - x_{i-1} - C_{i-1}, 0\}.$$

i's payoff in this case is $\delta_i u_{i_0} + z_{i-1}$. Hence, u_{i_0} must satisfy the following recursive formula.

$$u_{i_0} = \frac{1}{(n+2)m_i}(\delta_i u_{i_0} + z_{i-1}) + \left(1 - \frac{1}{(n+2)m_i}\right)\delta_i u_{i_0},$$

which is equivalent to

$$(n+2)m_i(1-\delta_i)u_{i_0} = z_{i-1}. \tag{7}$$

Subtracting (7) from (6), we get $(n+2)m_i(1-\delta_i)(u_{i_1}-u_{i_0}) = z_i - z_{i-1}$. Replacing $x_i = \delta_i(u_{i_1}-u_{i_0})$, we have

$$\frac{(n+2)m_i(1-\delta_i)}{\delta_i}x_i = z_i - z_{i-1}. \tag{8}$$

We have two special cases, where i is a seller or a buyer, that is, $i = 0$ and $i = n+1$. Because after trade a seller exits the game with no item, $u_{0_0} = 0$, while a buyer will own an item, and therefore $u_{(n+1)_1} = V$. Hence, we have

$$x_0 = \delta_0(u_{0_1}) \text{ and } x_{n+1} = \delta_{n+1}(V - u_{(n+1)_0}). \tag{9}$$

Applying this to (6) and (7), we obtain

$$\begin{aligned} &(n+2)m_0\frac{1-\delta_0}{\delta_0}x_0 = z_0 \\ &(n+2)m_{n+1}\frac{1-\delta_{n+1}}{\delta_{n+1}}(\delta_{n+1}V - x_{n+1}) = z_{n+1}. \end{aligned} \tag{10}$$

So far, given a stationary equilibrium payoff u, we define x as in (3), z as in (5), and derive (10) and (8). We will show that x, z is the unique solution of (A) by considering $\lambda_i = 2z_i$ as dual variables of the convex program. We need to check that x, z and λ satisfy the first order and the complementary-slackness conditions of (A).

First, by replacing $z_i = \lambda_i/2$ in (10) and (8), we obtain the desired first order conditions. Second, the complementary slackness conditions also follow. In particular, if $x_i - x_{i+1} + z_i + C_i > 0$, then $z_i > x_{i+1} - x_i - C_i$, and because of (5), $z_i = 0$, which means the dual variable $\lambda_i = 0$. Furthermore, if $\lambda_i > 0$, then $z_i > 0$; therefore, according to (5) $z_i = x_{i+1} - x_i - C_i$, implying that the constraint i binds.

From here, the proof of the theorem is straightforward. In particular,

(i) The expected payoff of a seller and a buyer are u_{0_1} and $u_{(n+1)_0}$, respectively. From (9), we obtain $u_{0_1} = \frac{x_0}{\delta_0}$ and $u_{(n+1)_0} = V - \frac{x_{n+1}}{\delta_{n+1}}$.

(ii) When $z_i > 0$, either agent i that owns an item or $(i+1)$ that does not own one, when selected as the proposer, will earn an extra payoff of z_i. The other agent will accept the offer with probability 1 in any equilibrium.

(iii) If $x_i - x_{i+1} + C_i > 0$, then the gain from trade is insufficient, and in this case, the proposer is better off declining to propose.

By this we conclude the proof. ∎

B Proof of Theorem 1

Let $(x^{(k)}, z^{(k)})$ is the optimal solution of (A^k). We first show that in each of the limiting regimes, $(x^{(k)}, z^{(k)})$ converges to the optimal solution of the corresponding program.

We start with the regime in which $\lim_{k\to\infty} \frac{T_k}{k} = 1$. First, observe that $(x^{(k)}, z^{(k)})$ is inside a bounded set. If $(x^{(k)}, z^{(k)})$ does not converge to (x, z) (the optimal solution of (B)), then there exists a subsequence of $(x^{(k)}, z^{(k)})$ that converges to $(x^+, z^+) \neq (x, z)$. Without loss of generality (by re-indexing) we assume $\lim_{k\to\infty}(x^{(k)}, z^{(k)}) = (x^+, z^+)$.

Let $f_k(x, z)$ be the objective function of (A^k), and $f(x, z)$ be the objective function of (B). When $\lim_{k\to\infty} \frac{T_k}{k} = 1$, we have

$$\lim_{k\to\infty} k \frac{1 - \delta_i^{1/T_k}}{\delta_i^{1/T_k}} = \ln(\frac{1}{\delta_i}).$$

Thus, $f_k(x, z)$ will converge to $f(x, z)$ as k goes to infinity. Furthermore, because both f_k and f are continuous, we have

$$\lim_{k\to\infty} f_k(x^k, z^k) = f(x^+, z^+). \tag{11}$$

Since $(x^{(k)}, z^{(k)})$ is the optimal solution of (A^k), $f_k(x^k, z^k) \leq f_k(x, z)$. Taking the limit we have $\lim_{k\to\infty} f_k(x^k, z^k) \leq \lim_{k\to\infty} f_k(x, z) = f(x, z)$. From this and (11), we have $f(x^+, z^+) \leq f(x, z)$. However, this is a contradiction, since $(x^+, z^+) \neq (x, z)$ and (x, z) is the *unique* optimal solution of (B). Therefore, we conclude that

$$\lim_{k\to\infty}(x^k, z^k) = (x, z).$$

Now, consider $\lim_{k\to\infty} \frac{T_k}{k} = \infty$. Observe that the solution of (A^k) does not change if we multiply the objective function by $\frac{T_k}{k\cdot(n+2)}$. Therefore, we obtain the following program, which has the same optimal solution.

$$\begin{aligned}
&\text{minimize: } \sum_{i=0}^{n} T_k m_i \frac{(1 - \delta_i^{1/T_k})}{\delta_i^{1/T_k}} x_i^2 + T_k m_{n+1} \frac{(1 - \delta_{n+1}^{1/T_k})}{\delta_{n+1}^{1/T_k}} (\delta_{n+1}^{1/T_k} V - x_{n+1})^2 + \sum_{j=0}^{n} \frac{T_k}{k(n+2)} z_j^2 \\
&\text{subject to: } x, z \geq 0 \text{ and } x_i - x_{i+1} + z_i + C_i \geq 0 \;\; \forall \;\; 0 \leq i \leq n.
\end{aligned} \tag{C^k}$$

We will now show that $z^{(k)}$ converges to 0. Assume the contrary, that $z_j^{(k)}$ does not converge to 0, then there exists an $\epsilon > 0$ such that $z_j^{(k)} > \epsilon$ for infinitely many k-s. In this case, the objective of (C^k) can be arbitrarily large because $\lim_{k\to\infty} \frac{T_k}{k(n+2)} = \infty$. This implies that $z_j^{(k)}$ were not the optimal solution of the program. (For example choosing $x_i = 0, z_i = 0$ for all i gives a better solution.) Therefore, $\lim_{k\to\infty} z^{(k)} = 0$.

Let $\mathcal{P} = \{x : x_i - x_{i+1} + C_i \geq 0, x_i \geq 0\}$, and $\mathcal{P}^{(k)} = \{x : x_i - x_{i+1} + z_i^{(k)} + C_i \geq 0, x_i \geq 0\}$. We have $\mathcal{P} \subset \mathcal{P}^{(k)}$. Furthermore, because $z^{(k)}$ approaches 0, $\mathcal{P}^{(k)}$ converges to $\mathcal{P}$ (in Hausdorff metric).

The remainder of the proof is similar to the argument given for the case $\lim_{k\to\infty} \frac{T_k}{k} = \infty$. In particular, assume $x^{(k)}$ does not converge to x (the optimal solution of (C)), then because the sequence $x^{(k)}$ is in a bounded set, there exist $x^+ \neq x$ and a subsequence of $x^{(k)}$ that converges to x^+. Without loss of generality (by re-indexing), we assume $\lim_{k\to\infty} x^{(k)} = x^+$.

Furthermore, because $\mathcal{P}^{(k)}$ converges to $\mathcal{P}$ and $\mathcal{P}$ is a closed set, we have $x^+ \in P$. Denote

$$f(x) = \sum_{i=0}^{n} m_i \ln(1/\delta_i) x_i^2 + m_{n+1} \ln(1/\delta_{n+1})(V - x_{n+1})^2, \text{ and}$$

$$f_k(x) = \sum_{i=0}^{n} T_k m_i \frac{(1-\delta_i^{1/T_k})}{\delta_i^{1/T_k}} x_i^2 + T_k m_{n+1} \frac{(1-\delta_{n+1}^{1/T_k})}{\delta_{n+1}^{1/T_k}} (\delta_{n+1}^{1/T_k} V - x_{n+1})^2.$$

Notice that $\lim_{T_k\to\infty} T_k m_i \frac{(1-\delta_i^{1/T_k})}{\delta_i^{1/T_k}} = m_i \ln(\frac{1}{\delta_i})$ and $\lim_{k\to\infty} \delta_{n+1}^{1/T_k} = 1$. Furthermore, both f_k and f are continuous, therefore, $\lim_{k\to\infty} f_k(x^{(k)}) = f(x^+)$.

Because $x^{(k)}$ is the minimizer of $f_k(x)$ for $x \in \mathcal{P}^{(k)}$ and $x \in \mathcal{P} \subset \mathcal{P}^{(k)}$, therefore, $f_k(x^{(k)}) \leq f_k(x)$. Taking the limit we have $f(x^+) \leq f(x)$. However, this is a contradiction because x is the *unique* minimizer of $f(x)$ for $x \in \mathcal{P}$. From this we obtain

$$\lim_{k\to\infty} x^{(k)} = x.$$

It remains to prove **(i), (ii)** and **(iii)** in both Theorems. The proofs of **(i)** and **(iii)** follow from the convergence of $x^{(k)}$ to x. To prove **(ii)** , we need to show that if $x_i > 0$, then there exists K such that $z_i^{(k)} > 0$ for all $k > K$. In this case according to **(ii)** in Lemma 1, trade between $(i \to i+1)$ will occur with probability 1.

By considering the first order condition on the variable x_i in the convex program (A^k), we obtain a relation between $x_i^{(k)}$ and $z_i^{(k)}$. In particular, according to (8) and (10) we have

$$(n+2) k m_i \frac{(1-\delta_i^{1/T_k})}{\delta_i^{1/T_k}} x_i^{(k)} = z_i^{(k)} - z_{i-1}^{(k)} \text{ and } (n+2) k m_0 \frac{1-\delta_0^{1/T_k}}{\delta_0^{1/T_k}} x_0 = z_0^{(k)}.$$

Therefore, if $\lim_k x_i^{(k)} = x_i > 0$, then there exists K such that for all $k > K$, $x_i^{(k)} > 0$. This implies $z_i^{(k)} > 0$, which is what we need to prove. ■

C Proof of Theorem 2, Theorem 3 and Theorem 4

To prove these theorems, we first need to consider the convex program (C). When $m_i = m, \delta_i = \delta$, (C) becomes

$$\text{minimize: } \sum_{i=0}^{n} x_i^2 + (V - x_{n+1})^2 : \text{s.t } x \geq 0 \text{ and } x_i - x_{i+1} + C_i \geq 0 \ \forall \ 0 \leq i \leq n. \tag{D}$$

Let $0 \leq l \leq n$ be the smallest index such that $\sum_{j=l}^{n}(n+1-j)C_j \leq V$,. We first show that the solution of (D) is the following

$$\begin{aligned} x_j &= 0 \text{ for } j < l, \\ x_l &= \frac{V - \sum_{j=l}^{n}(n+1-j)C_j}{n+2-l}, \\ x_j &= x_{j-1} + C_{j-1} \text{ for } l < j \leq n+1. \end{aligned} \tag{12}$$

The optimality of this solution can be easily checked together with dual variables that satisfy both the first order and the complementary-slackness conditions.[1] In particular, let the following λ_j be the dual variable for the constraint $x_j - x_{j+1} + C_j \geq 0$:

$$\lambda_j = 0 \text{ for } j < l; \lambda_l = 2x_l \text{ and } \lambda_j = \lambda_{j-1} + 2x_j \text{ for } l < j \leq n.$$

C.1 Proof of Theorem 2

We first prove **(II)** (the nonexistence of a stationary equilibrium.) Assume that there exists a limit stationary equilibrium (ω, σ). Let x be the unique solution of the convex program (D). We will show that $x_n > 0$ and there exists $0 \leq j < n$ such that $x_j - x_{j+1} + C_j > 0$. According to Theorem 1, n and $n+1$ trade with probability 1 whenever this type of trade arises. However, trade between j and $j+1$ never happen. Therefore, there must be a tier of middlemen between j and $n+1$, where the balance condition is violated. This shows that (ω, σ) cannot be a limit stationary equilibrium.

Because of (12), $x_n = x_l + C_l + \cdots + C_{n-1}$. Thus, $x_n = 0$ only when $x_l = 0$ and $C_l = C_{l+1} =$

[1]We leave this as an exercise.

$\cdots = C_{n-1} = 0$. In this case $x_l = \frac{V - C_n}{n+2-l} = 0$, which implies $V = C_n$. This is a contradiction because we assume that $V > \sum_{i=0}^{n} C_i$. Therefore, $x_n > 0$.

It remains to show that there exists $0 \leq j < n$ such that $x_j - x_{j+1} + C_j > 0$. From (12), we have

$$x_l = \frac{V - \sum_{i=l}^{n}(n+1-i)C_i}{n+2-l} \text{ and } x_0 = 0.$$

Consider

$$\sum_{j=0}^{l-1}(x_j - x_{j+1} + C_j) = \sum_{i=0}^{l-1} C_i + x_0 - x_l = \sum_{i=0}^{l-1} C_i + \frac{\sum_{i=l}^{n}(n+1-i)C_i - V}{n+2-l}.$$

For each $0 \leq i \leq l-1$, we have $C_i \geq \frac{(n+1-i)}{n+2-l} C_i$. Together with the above equality we obtain

$$\sum_{j=0}^{l-1}(x_j - x_{j+1} + C_j) \geq \frac{\sum_{i=0}^{n}(n+1-i)C_i - V}{n+2-l} > 0.$$

Therefore, there exists $0 \leq j \leq l-1$ such that $x_j - x_{j+1} + C_j > 0$.

Now, to prove part **(I)** of the theorem, observe that by (12), $x_i > 0$ for all $0 \leq i \leq n+1$. Therefore, if a limit stationary equilibrium exists, all trade $(i \to i+1)$ occurs with probability 1. Thus, this shows the uniqueness of the limit stationary equilibrium if it exists. Notice that the convex programs that characterize semi-stationary equilibria in the finite replications of the game approach (D) as given in (12). Furthermore, we show that in the limit all trade $(i \to i+1)$ occurs with probability 1, whenever either i with an item or $i+1$ without an item is selected to be the proposer in the bargaining process. Thus, for all k large enough this "all-trade" strategy is a semi-stationary equilibrium.

We will need to show that there exists a sequence of aggregate states that satisfies the balance condition. Let ω be the state in the limit. Since all trades are accepted with probability 1, the probability that in a period an agent i sells an item to an agent $i+1$ is equal to

$$\mathbf{P}_{i \to i+1} = \frac{1}{(n+2)}(\omega_i + 1 - \omega_{i+1}).$$

It is not hard to show that, if $\omega_i = \frac{n+1-i}{n+1}$ is the unique solution that satisfies $\mathbf{P}_{i-1 \to i} = \mathbf{P}_{i \to i+1}$. This is because $\mathbf{P}_{i \to i+1} = \frac{1}{(n+2)}(1 + \frac{1}{n+1})$. Thus, any sequence of $\omega(k)$ that converges to $\omega = (\frac{n}{n+1}, \frac{n-1}{n+1}, ..., \frac{1}{n+1})$ will satisfy the balance condition. By this we conclude the proof. ■

C.2 Proof of Theorem 3

As shown above, when $V > (n+1)C_0 + nC_1 + .. + C_n$, there exists a limit stationary equilibrium, and the payoff can be computed by the convex program D. (12) provides such a solution. According to Theorem 1 the payoffs of sellers and buyers are given by

$$u_{0_1} = \frac{(V - \sum_{i=0}^{n} C_i)}{n+2} - \sum_{i=0}^{n} \frac{n-i}{n+2} C_i \text{ and } u_{n+1_0} = \frac{n+1}{n+2}(V - \sum_{i=0}^{n} C_i) + \sum_{i=0}^{n} \frac{n-i}{n+2} C_i, \text{ respectively.}$$

■

C.3 Proof of Theorem 4

We use Lemma 1 to prove this theorem. Recall that to prove the convex program characterization in Lemma 1, we define $x_i = \delta_i(u_{i_1} - u_{i_0})$. (This is equation (3) in Appendix A.)

From (6) and (7), we obtain $\frac{(n+2)(1-\delta_i)}{\delta_i} m_i x_i = z_i - z_{i-1}$. According to (10) in Appendix A, we also have $(n+2)m_0 \frac{1-\delta_0}{\delta_0} x_0 = z_0$

Because of the assumption $\delta_i = \delta$, $z_i = (z_i - z_{i-1}) + (z_{i-1} - z_{i-2}) + ... + (z_1 - z_0) + z_0 = (n+2) \sum_{j=0}^{i} \frac{(1-\delta)}{\delta} m_j x_j$. With this calculation, replacing z_{i-1} and z_i to the payoff of middlemen i, we obtain

$$U_i = \frac{u_{i_0} + u_{i_1}}{2} = \frac{1}{2(n+2)m_i} \frac{(z_i + z_{i-1})}{(1-\delta)} =$$

$$= \frac{1}{2(n+2)m_i(1-\delta)} \Big((n+2) \sum_{j=0}^{i} \frac{(1-\delta)}{\delta} m_j x_j + (n+2) \sum_{j=0}^{i-1} \frac{(1-\delta)}{\delta} m_j x_j \Big) = \frac{1}{\delta} (\sum_{j=0}^{i-1} \frac{m_j}{m_i} x_j + \frac{x_i}{2}).$$

Notice that this equation holds for the original finite game. To apply to the replicated games, we need to replace m_j with km_j and δ with δ^{1/T_k}.

Because $\frac{km_j}{km_i} = \frac{m_j}{m_i}$; and $\lim_{k\to\infty} \delta^{1/T_k} = 1$, we obtain $U_i = \sum_{j=0}^{i-1} \frac{m_j}{m_i} x_j + \frac{x_i}{2}$. ■