The Role of Managers Revisited
Parinaz Naghizadeh∗ and Thanh Nguyen†
December 10, 2018

Abstract
What do managers do? Why are they well paid? Prior literature has mainly considered these questions from the inside-the-firm perspectives. We instead consider the role of managers in a competitive market. We show that, because of the risk of being fired, managers have an advantage over firm’s owners in maintaining self-enforcing relational contracts with workers. At equilibrium, there are few managing jobs but these jobs are well paid. This is a key element allowing managers to act as middlemen engaging in self-enforcing relational contracts with both workers and firms’ owners. At equilibrium, managers create more jobs and improve the market’s efficiency even though they impose an additional cost on production.

1 Introduction
The CEO-to-workers compensation ratio has skyrocketed over the last 40 years. In the 1950s, a typical CEO made 20 times the salary of his or her average worker. In 2017, CEO pay at an S&P 500 Index firm soared to an average of 361 times more than the average worker. What do managers do and why are they well paid? The literature offers two basic arguments for the essential roles of managing jobs. First managers might possess knowledge and skills

∗College of Engineering, Purdue University, parinaz@purdue.edu
†Krannert School of Management, Purdue University, nguye161@purdue.edu
https://aflcio.org/paywatch Public companies are required for the first time in 2018 to compare the compensation of their chief executive officer to that of the median employee in an annual Securities and Exchange Commission filing.
that add value and (or) reduce the cost of production. For example, through the lens of principal agent models, managers have better information and/or monitoring technology than firms’ owners, and thus, they help firms in contracting with workers. Second, when the competition among firms is imperfect, firms choose to delegate as a commitment device in order to bolster their market power (see for example, Schmalsee et al. (1989), Gibbons et al. (2013) and Legros and Newman (2014)). Both arguments have supporting evidences, but also raise further puzzles. First, why do highly paying CEO jobs persist even when managing skills and knowledge are learnable and even automated? Second, the fact that imperfect competition is the key element of the analysis in the literature on delegation begs the question of whether delegation becomes inefficient in competitive settings. Furthermore, both lines of the literature offer little explanation on the pay gap between CEOs and average workers.

In this paper, we offer a new theory that explains not only the role of managers but also why their pay are necessarily high. We shutdown the channels for both arguments described above. In our model, managers are not better informed than firms, nor they add value to the firms’ production – they might even impose additional costs, and firms behave in a perfect competitive market. In fact, workers can freely switch to managing jobs. At equilibrium, there are few manging jobs but these jobs are well paid. This is a key element allowing managers to engage in self-enforcing relational contracts with both workers and firms’ owners to partially overcome the moral hazard problems. At equilibrium, highly paid managers help to create more jobs and improve the market’s efficiency. The main reason for the effectiveness of managers in our model is that they can be fired, and thus, are in a better position than the firms’ owners in creating self-enforcing relational contracts.

Our paper builds on the classical model of efficiency wages of Shapiro and Stiglitz (1984) and its extension in MacLeod and Malcomson (1998), in which firms hire workers whose

---

2Bridgewater Associates, the world’s largest hedge fund, is building a piece of software to automate the day-to-day management of the firm, including hiring, firing and other strategic decision-making. (https://www.theguardian.com/technology/2016/dec/22/bridgewater-associates-ai-artificial-intelligence-management)
efforts are not verifiable and cannot be contracted on. Firms therefore raise wages above the market clearing price and create involuntary unemployment to eliminate workers’ shirking. We add another type of employment to this model, in the form of managerial positions, for which workers can also apply. Firms provide the budget and delegate the decision of hiring and contracting with workers to their managers. The agreements between firms and managers, as well as those between managers and workers, take on the form of self-enforcing relational contracts. Managers (credibly) promise to pay bonuses to workers. Shirking workers might be fired by managers, and similarly, managers who cheat and privately consume bonuses might be fired by firms. In the absence of managers, firms cannot credibly promise bonus payments to a worker, because they can immediately find a new worker if the relationship with the existing worker is terminated.

We emphasize that our results hold irrespective of the firm’s ability to commit to a particular form of contract between the managers and workers (here, one involving the payment of bonuses). Firms provide their budget to the managers, and fire managers only based of their performance. A key element in our equilibrium analysis is that once fired, it is more difficult for a manager to find a new job, due to the small number and high pay of managing jobs. Therefore, the threat of firing incentivizes the managers to perform the jobs delegated to them. Our findings would apply under alternative interpretations of managers’ cheating on their delegations. For instance, the manager could pay workers’ bonuses, but motivate them to work on projects that privately benefit the manager; this type of cheating would not affect the bonus promised to workers, yet would harm the firm and result in the manager being fired. In a more general terminology, managers can choose to “divert” the budget for personal gains. Such diversion (either direct diversion of the budget or indirect diversion of the company’s resources) is often not observable nor enforceable in court, thus contracts between firms and managers are also relational.

At the core of our analysis are the terms we refer to as moral hazard multipliers, which relate workers and managers’ incentive constraints to basic model parameters including prob-
ability of detection of shirking and size of the firms. These multipliers capture the seriousness of the moral hazard problem for these different groups of agents, with higher multipliers indicating that higher compensation is required to keep an agent from shirking. The inclusion of a bonus scheme in workers’ contracts effectively shifts the moral hazard cost from the workers to the managers. We show that when the firms’ size increases, this will decrease the managers’ moral hazard multiplier, and as a result relational contracts which include performance bonuses help to improve the efficiency of the market through job creation.

Related Literature

The role of managers is usually viewed through principal agent models. In this view, managers are assumed to be better informed or to have better monitoring technology than firms’ owner in contracting with workers. CEO pay is used to reduce the moral hazard problem that arises because CEOs often own very little of the firms they control. The main difference between this standard view and our paper is that CEOs (managers) in our model do not have better information nor monitoring technology. They are better than firms owners because they can lose their job, and high pay motivates them to keep their promise in relational contracts with both workers and firms owners.

The literature on the inequality between CEO and worker pay is extensive (see for example, Gabaix and Landier (2008)). This literature heavily builds on the framework of “superstar economics” (Rosen (1981)), in which agents are endowed with heterogeneous talents that generate value for firms. In contrast, in our model, managers as middlemen between firms and workers do not improve the firms’ production. They even impose additional costs.

Our paper also belongs to the large literature on Industrial Organization (IO), studying the relation between firms’ structure and their behavior. In particular, among many papers, we analyze the role of managers in firms and their effect on firms’ market performance. However, the approach in this literature is dominated by models of firms as a profit maximizing black box in an imperfectly competitive market. As a result, a firm’s organization, including
its decision to hire managers and their compensation, is a direct consequence of the firms’ intent to enhance market power. [Legros and Newman (2014)] write “IO economists have overwhelmingly identified the imperfection in the organizing mechanism with market power”. In contrast to this line of work, our focus is on how imperfections within the firm, in particular moral hazard, influence firms’ internal organization and the market’s efficiency. Our paper focuses on the role of managers within firms using relational contracts, and specifically, their role in the transfer of incentives.

There is also a large literature on hierarchies and the division of labor, see for example the surveys [Garicano and Zandt (2012) and Mookherjee (2012)]. However, this literature largely relies on the argument that due to limitations of time or expertise, one or several tiers of managers help the firm to overcome the control loss and information processing problems. As discussed in the introduction, our results do not rely on these assumptions. The main rationale behind our results is the fact that firm owners cannot fire themselves and thus cannot make credible promises in relational contracts with workers.

Self-enforcing relational contracts, which are informal agreements that are sustained due to the value of future relationships, are widely observed within firms. We are not the first to study its implications to firms’ structure. See for example, [Bull (1987), Baker et al. (2002) and Gibbons and Henderson (2012)]. The role of managers with respect to existing relational contracts with firm owners has also been discussed in the literature. For example, [Baker et al. (2002)] identify the role of managers as coordinators/facilitators of the relational contracts of firm owners with their workers and other firms. [Gibbons and Henderson (2012)] discuss how the clarity and credibility of relational contracts with managers can lead some firms to have a competitive advantage in the market. However, unlike these papers, our model offers a complementary explanation for the value of managers that cannot be replaced by the firms’ owners.

The theory of relational contracts has also been studied extensively in different problems, including in the presence of moral hazard and adverse selection in firm-worker rela-
tions, Shapiro and Stiglitz (1984); MacLeod and Malcomson (1998); Levin (2003), and in competitive labor markets, Board and Meyer-ter Vehn (2014); see Malcomson (2012) for a recent survey. These papers are concerned with bilateral relational contracts, while we model managers as middlemen between workers and firms. Che and Yoo (2001), Levin (2002) and Rayo (2007), among others, study relational contracts in teams. Their focus, however, are different of ours. In particular, we consider a large competitive market to endogenize the outside options of the agents. This approach allows us to study the role of managers in the market.

2 Model and preliminaries

We start by considering an economy where firms hire workers directly using relational incentive contracts. Each relational contract models the long-term relationship between a risk-neutral worker and a risk-neutral firm over a time horizon $t \in \{1, 2, \ldots\}$ with discount rate $\delta$. We consider scenarios in which the worker has to exert effort to complete the job in each time period, yet the worker’s performance is neither observable, nor contractible, by the firm. Relational contracts are used to mitigate this moral hazard problem. The main idea is to make the contract’s continuation at time $t$ contingent on the relationship’s outcomes from the previous periods. As a result, the prospect of future interactions can stop workers and firms from behaving opportunistically. In this section, we formally describe worker and firm’s incentives, and the terms of the relational contracts. Our model largely follows Shapiro and Stiglitz (1984); MacLeod and Malcomson (1998).\footnote{The only difference is in modeling employment terminations for reasons other than those prescribed by the relational contracts. We consider non-contractual employment terminations that are determined endogenously based on the state of the market, while such separations happen due to exogenous reasons alone in Shapiro and Stiglitz (1984); MacLeod and Malcomson (1998).}
2.1 Economy without managers

**Firms:** There is a continuum of symmetric firms that can create identical jobs in a competitive market. Let $J$ denote the total number of jobs at equilibrium. Firms contract workers to fill these jobs, offering them a compensation package (detailed shortly). We assume that firms can create additional jobs at zero cost, yet the marginal revenue from jobs, $d(J)$, is a decreasing function in the number of jobs in the market. Therefore, at the steady state equilibrium, the total pay to each worker by a firm will be equal to the marginal revenue, $d(J)$.

Let the size of each firm be $k$, that is, each firm can create $k$ jobs. Firms’ size $k$ is an exogenous parameter of the model. We will later study its influence on equilibrium behavior. Given the number of jobs $J$ and firm size $k$, there will be $N_f := \frac{J}{k}$ firms in the market.

**Workers:** There is a continuum of identical workers in the market, measured $N$. In each period, a worker is either employed or unemployed. A worker who is employed can decide whether to exert effort, $e = 1$, or to shirk, $e = 0$. If the worker exerts effort, he generates profit for the firm while incurring a private cost of effort $c$. If the worker shirks, the value of the job to the firm is 0. The compensation of an employed worker depends on his decision and the form of contract that we will explain below. Unemployed workers receive a utility of $\bar{u} = 0$ for the current period.

Using a standard moral hazard model, we assume that the level of effort is neither verifiable nor contractible by the firm, and thus, can not be used in a formal contract. However, workers can be (imperfectly) monitored by firms. If a worker shirks, he may get caught with a probability $\alpha$. Workers who are caught shirking are fired. In addition, there is a probability $p$ that a worker’s employment will be terminated due to non-contractual reasons, that is, due to factors other than those prescribed by the terms of the relational contract. This voluntary turnover rate $p$ captures factors such as workers’ decision to relocate or to quit the job for other personal reasons. A worker’s voluntary decision to leave the current job naturally depends on how hard or easy it is to obtain a similar one. The probability
of getting a new job depends on the number of jobs available relative to the number of unemployed workers, which in turn depends on the ratio between the number of job $J$ and the number of workers $N$ in the market. Thus, we assume

$$p \text{ is increasing in } \frac{J}{N} \text{ and } \lim_{\frac{J}{N} \to 0} p = p_{\min} < \lim_{\frac{J}{N} \to 1} p = p_{\max}.$$  

Both fired workers, as well as those whose jobs have been terminated, enter the unemployment pool and can seek employment at the beginning of each period.

### 2.2 Relational contracts

Firms contract workers directly to fill the available jobs. Following MacLeod and Malcomson (1989, 1998), we consider the following class of self-enforcing bilateral contracts. A firm offers a compensation package $W$ to each worker that they have contracted. This compensation package will in general consist of two terms: a wage $w$, and a contingent bonus or performance pay $b$. The wage $w$ captures the salary promised to the worker; it is enforceable, and is payed to the worker at the beginning of each period. The bonus on the other hand is based on non-contractible performance measures that can be observed after the worker has made an effort decision. These may capture promotions, end-of-year bonuses, commissions, or piece-rates. The worker will receive the bonus if either he does not shirk, or he does, but is not caught shirking.

The timeline of each period is shown in Figure 1 and is as follows:

- **Matching**: Unemployed workers seek jobs at the beginning of each period. Firms
with job vacancies make offers to randomly selected workers from the unemployment pool. Let $q$ denote the probability that a worker is selected. Selected workers decide whether to accept or reject these offers.

- **Formal contract**: If a firm and a worker are matched, they sign a relational contract. The worker is payed a wage $w$ upfront, and is promised a bonus $b$ for exerting effort.

- **Execute**: The worker decides whether to exert effort and incur the cost $c$, or to shirk.

- **Observe**: If the worker shirks, he gets caught with probability $\alpha$, is fired, and thus becomes unemployed.

- **Adjust**: If the worker either does not shirk or is not caught shirking, he is eligible for the bonus $b$. The firm decides whether or not to pay the promised bonus.

- **End of period**: The relationship breaks if the worker is caught shirking or if the firm reneges on paying the bonus. In addition, each employed worker might become unemployed with probability $p$. If the relationship is terminated for either of these reasons, the worker joins the unemployment pool and the firm has a job vacancy in the next period.

We assume the market is sufficiently large, so that the probability of firm and worker reencounters are negligible, and the parties do not keep track of each others’ identities. In addition, either due to the large size of the market or lack of reliable information sharing methods, there are no means of maintaining external reputation scores for either firms or workers. That is, both firms and workers are assumed to be anonymous, so that the history of workers’ past employment or firms’ past behavior will not affect their future prospects. Under these assumptions, the decisions to form or terminate contracts in any period, as outlined above, are based solely on that period’s outcomes.

Firms and workers maximize their long-run expected payoffs with discount factor $\delta$. For a relational contract to sustain a self-enforcing long-term relationship, it must (i) satisfy the
individual rationality constraints of both workers and firms, so that they expected long-run payoffs from the contract are at least that of remaining unemployed or having a vacant job, respectively, and (ii) satisfy both parties’ incentive constraints, so that workers choose to exert effort over shirking, and that firms do not cheat on the promised bonuses. We formally characterize these constraints and the equilibrium in the next section.

2.3 Market equilibrium without managers

Firms’ incentives. Let $\Pi$ denote the long-run expected utility of the firm from having one of its $k$ jobs filled, and $\bar{\Pi}$ denote the utility of that job being vacant, at the beginning of a period. Consider a firm who has signed a relational contract with a worker. If the firm pays the performance pay to an eligible worker (i.e. no shirking detected), its expected future payoff will be $\pi - w - b + \delta \Pi$. In contrast, by reneging on the bonus pay, the firm will get, $\pi - w + \delta \bar{\Pi}$. Payment of bonus pays will therefore be incentive compatible for the firm if and only if

$$\delta (\Pi - \bar{\Pi}) \geq b \ .$$  \hspace{1cm} (ICF)

In the current model, firms can immediately find workers to fill vacant jobs, so that $\Pi = \bar{\Pi}$. Together with (ICF), we conclude that firms will not have an incentive to pay bonus, so that $b = 0$. Therefore, without bonuses, firms’ incentive compatibility constraints are trivially satisfied.

Recall also that at equilibrium, the marginal productivity of jobs will equal the pay to the workers. As a result, firms will offer $w = d(J)$. All firms make zero profit, and their individual rationality is satisfied.

Workers’ incentives. Each worker can be either employed or unemployed at any period. Let $E$ and $U$ denote the long-run expected utilities of employed and unemployed workers, respectively, at the beginning of each period.
Consider an employed worker after he has signed the relational contract with a firm. The expected payoff from exerting effort will be given by \( w - c + \delta(pU + (1 - p)E) \), while the expected payoff from shirking will be \( w + \alpha\delta U + (1 - \alpha)\delta(pU + (1 - p)E) \). Therefore, the worker will not shirk if and only if

\[
\alpha\delta(1 - p)(E - U) \geq c .
\]  

For employed workers, their expected payoff, given that (ICW) is satisfied, will be

\[
E = w - c + \delta(pU + (1 - p)E) .
\]  

(1)

For unemployed workers, given the rematch probability \( q \), their expected utility will be,

\[
U = qE + (1 - q)\delta U .
\]  

(2)

Let \( \gamma_x := (1 - \delta)(\frac{1}{x} - 1) \). Then, using (1) and (2), we get the following expected payoffs for employed and unemployed workers:

\[
U = \frac{1}{p}\frac{w - c}{\gamma_p + 1}\frac{1}{\gamma_q + 1} - \delta ,
\]

\[
E = \frac{1}{p}\frac{w - c}{\gamma_q + 1}\frac{1}{\gamma_p + 1}\frac{1}{\gamma_q + 1} - \delta .
\]

From the above, it is clear that \( E \geq U \), so that workers’ individual rationality constraints are satisfied. Substituting these expressions in (ICW), we obtain,

\[
w - c \geq \frac{1}{\alpha\delta}\frac{1}{(1 - p)(1 - q)} - \delta)c .
\]  

(3)

The rate of reemployment \( q \) is endogenously determined as follows. Assume that (ICW) holds and thus, workers only quit jobs voluntarily. The rate at which workers enter the unem-
ployment pool, \( pJ \), should equal the rate at which unemployed workers obtain employment, \( q(N - J + pJ) \) at the steady-state equilibrium. Therefore, \( q = \frac{pJ}{N - J + pJ} \).

Replacing for \( q \) in the constraint (3) we have the following no-shirking condition (NSC) for workers:

\[
q^2 = \frac{pJ}{N - J + pJ} \Rightarrow \frac{q}{1 - p} = \frac{pJ}{N - J}
\]

Replacing for \( q \) in the constraint (3) we have the following no-shirking condition (NSC) for workers:

\[
w - c \geq \frac{1}{\alpha \delta} \left( \frac{N - (1 - p)J}{(1 - p)(N - J)} - \delta \right) c
\]

The above (NSC) provides a lower bound on the wage \( w \) to keep workers from shirking. The right hand side of (NSC) shows that firms offer efficiency wages to workers (a rent in addition to their cost of effort) to prevent them from shirking.

Notice that because the market is competitive, new firms can enter the market if profit is positive; thus, at equilibrium (NSC) should hold with equality so that firms offer only the minimum no-shirking wage. Combining (NSC) with \( w = d(J) \), we obtain the following equilibrium condition, determining the number of jobs created (and consequently the level of employment) in an economy without managers:

\[
d(J) = c \frac{1 - \delta + \frac{p}{1 - p} \cdot \frac{N}{N - J} - \delta}{\alpha \delta} + c
\]

Figure 2 illustrates the equilibrium employment level \( J^* \) obtained from (4). Substituting this solution in (NSC) will determine equilibrium wages \( w^* \). Together \( (w^*, J^*) \) characterize the economy’s equilibrium.

The condition in (4) has a number of intuitive implications on the economy’s employment level. First, note the effect of increasing the number of jobs \( J \). This will increase the term \( \frac{N}{N - J} \). We have further assumed that \( p \), the probability of employment termination, is non-decreasing in \( \frac{J}{N} \). As a result, as more jobs become available, firms have to provide higher wages to workers to prevent them from shirking. Nevertheless, the marginal productivity of jobs is decreasing in \( J \). Therefore, the market will reach equilibrium when per job productivity equals the required no-shirking wages.
We next consider the effect of the model’s exogenous parameters. If monitoring accuracy $\alpha$ decreases, the RHS of (4) increases (or equivalently, the corresponding curve shifts upwards in Figure 2). This will decrease equilibrium employment $J^*$. Intuitively, a decrease in $\alpha$ means that shirking is harder to detect, so that firms pay higher wages to keep workers from shirking, and consequently can create fewer jobs, increasing involuntary unemployment levels. A similar intuition holds as effort becomes more costly (increasing $c$), or if workers and firms are less patient so that relational contracts are less effective (decreasing $\delta$).

Figure 2: Level of employment in an economy without managers determined by (4).

3 Economy with managers

3.1 Model and relational contracts

We now consider a market in which firms offer two types of positions: managers and workers. Workers perform the jobs (as described in the previous section), while managers act as intermediaries between firms and workers (as detailed shortly). Therefore, there are two job pools, one with $N_w$ workers and the other with $N_m$ managers, with total measure $N = N_m + N_w$. 
Manager: Each manager may be either employed or unemployed. Employed managers have relational contracts with firms, and are in turn tasked with maintaining relational contracts with $k$ workers; see Figure 3.

A manager’s contract with the firm offers a wage $W$ and a contingent bonus $B$. The manager is responsible for the relational contracts of the $k$ workers hired by the firm. This involves monitoring the workers and allocating their wages and bonuses accordingly. Performing these managerial duties has a private cost per job $c_m$ to the manager. Managers may renege on performing these tasks, and deviations are detected with some probability $\alpha_m$ by the firm. Reneging managers who are caught do not receive the bonus pay $B$, are fired, and join the unemployed managers’ pool. In addition, there is a probability $p_m$ that the contract between a manager and a firm is terminated for reasons other than those prescribed by the contract, such as relocation. We again assume that $p_m$ depends on how likely it is for a manager to get a new job following decisions to quit or relocate. This depends on the
ratio between the total number of management jobs, \( \frac{J}{k} \), and \( N_m \). That is, we assume:

\[
p_m \text{ is increasing in } \frac{J}{kN_m} \text{ and } \lim_{\frac{J}{kN_m} \to 0} p_m = p_{\text{min}} < \lim_{\frac{J}{kN_m} \to 1} p_m = p_{\text{max}}.
\]

Managers whose contracts have been terminated due to such voluntary reasons, as well as fired managers, can seek employment at the beginning of each period.

**Relational contracts:** The timeline of events in each period in an economy with managers is shown in Figure 4. The events within each unit of relational contracts is similar to the those in the model without managers described in the previous section, except that here, a unit of relational contracts consists of two types of contracts occurring simultaneously: the contracts between workers and managers, and those between firms and managers.

At the end of the period, matches of either type may be broken with certain probability, and reemployment of both types is possible within the matching phase at the beginning of the next period. During the matching phase, firms with vacant manager positions will first make an offer to randomly selected managers from the unemployment pool. Subsequently, firms with vacant worker positions will make offers to unemployed workers to fill their vacancies. We denote the probability of workers and managers finding employment during the matching period by \( q_w \) and \( q_m \), respectively. Similar to the previous section, we assume the market is large enough, so that firm-manager and manager-worker reencounter probabilities are all negligible, and that all parties are anonymous due to the lack of external reputations. As a result, all decisions to form or terminate relational contracts in any period are based solely on that period’s outcomes.

Finally, in this economy, we allow free movement of managers and workers, so that they can choose to switch their occupation to another type at the end of each period. By opting for switching, a worker/manager will join the unemployment pool of managers/workers at the beginning of the next period.

\footnote{Note that we do not impose an explicit expression for \( p_m \) as a function of the job-manager ratio. Our analysis is robust in the limit regime and is insensitive to the choice of such functions.}
3.2 Market equilibrium with managers

Workers’ incentives. Each worker is either employed or unemployed following the matching phase of each period. Employed workers have a relational contract with wage $w$ and bonus $b$ with a manager. Let $E_w$ and $U_w$ denote the life-time payoff of an employed and unemployed worker, respectively. If an employed worker decides to work, his payoff will be

$$w - c + b + \delta(pU_w + (1 - p)E_w).$$

If he shirks instead, his payoff will be

$$w + \alpha \delta U_w + (1 - \alpha)(b + \delta pU_w + \delta(1 - p)E_w).$$

Thus, the incentive constraint for an employed worker not to shirk is $\alpha(b + \delta E_w - \delta U_w) \geq c$, which can be written as:

$$\alpha \delta(1 - p)(E_w - U_w) \geq c - \alpha b \quad \text{(5)}$$

Let $X_w := w + b - c$; this is the payoff for one period of an employed worker who exerts effort. When the incentive constraint (5) is satisfied, the long-run expected payoff of an employed worker will be given by

$$E_w = X_w + \delta(p_w U + (1 - p_w)E_w). \quad \text{(6)}$$

Where $p_w$ is the probability that a worker voluntarily quits his job. Let $q_w$ denote the probability that an unemployed worker becomes employed by a manager during the matching phase at the beginning of a period. Then, the payoff of an unemployed worker can be written as

$$U_w = q_w E_w + \delta(1 - q_w)U_w \quad \text{(7)}.$$
Using (6) and (7), we can find the expected payoffs $E_w$ and $U_w$:

$$
U_w = \frac{1}{p} \frac{X_w}{(\gamma_p + 1)(\gamma_q + 1) - \delta},
$$

$$
E_w = \frac{1}{p} \frac{X_w}{(\gamma_q + 1)} \frac{X_w}{(\gamma_p + 1)(\gamma_q + 1) - \delta}.
$$

where $\gamma_x := (1 - \delta)(\frac{1}{x} - 1)$.

Substituting these expressions in the incentive constraint (5), we obtain the following condition on the payments to prevent workers from shirking:

If $q_w < 1$, then $X_w \geq \frac{1}{\alpha \delta} \left( \frac{1}{(1 - p_w)(1 - q_w)} - \delta \right) (c - \alpha b)$

If $q_w = 1$, then $c - \alpha b = 0$.

We next find the endogenously determined probability $q_w$ of an unemployed worker finding employment during the matching phase. When (9) is satisfied, workers will leave employment only due to their voluntary decisions. Therefore, in a steady state equilibrium, where the rate of workers leaving and entering the unemployment pool should be equal, we get, $q_w = \frac{p_w J}{N_w - J + p_w J}$.

Substituting this in (9), we obtain the following equilibrium conditions for the workers:

If $N_w > J$, then $X_w = w + b - c \geq \frac{1}{\alpha \delta} \left( 1 - \delta + \frac{p_w}{1 - p_w} \cdot \frac{N_w}{N_w - J} \right) (c - \alpha b)$

If $N_w = J$, then $c - \alpha b = 0$.

The above worker incentive condition, similar to the (NSC) in the economy without managers, imposes a lower bound on the per-period payoff that workers should be offered in order to keep them from shirking. The right-hand side can similarly be interpreted as a rent that is payed to workers over their cost of effort. We observe that the inclusion of a bonus lowers the RHS of this incentive constraint. This means that using bonus pays (as a substitute to wages $w$, which becomes possible due to the presence of managers), allows the
firm to mitigate workers’ moral hazard problem with lower wages. However, as we will see shortly, the economy’s moral hazard problem is not fully removed through these bonuses, but rather is effectively transferred to the manager’s contract design problem.

**Firms’ incentives.** Similar to the argument in the environment without managers in Section 2.3, firms can always find a replacement for managers, so that they do not have incentives to pay bonuses to the managers. Thus, $B = 0$. The contract with the manager therefore consists of paying a wage $W$ in return for management of the execution of $k$ jobs. One can think of $W$ as the budget of the firm in one period. As the market is competitive on the firms’ side, the payment per job is equal to the job’s marginal profit. This implies

$$\frac{W}{k} = d(J).$$

**Managers’ incentives.** Each manager may be either employed or unemployed after the matching phase of each period. For employed managers, there is a $c_m$ cost of management, per job, incurred if the manager performs her duties. This cost is private to the managers, and so the manager’s decision to exert the required effort is subject to moral hazard. In addition, this is not the only moral hazard problem that the manager imposes.

A manager in our setting has relational contracts with both workers and the firm. On the contracts with workers, the manager will pay a wage $w$ upfront to each worker, and promises a bonus $b$ if no shirking is detected. Given that workers’ incentive constraints are satisfied, they all exert effort and are eligible to receive their bonuses. Based on the contract with the firm’s owner, the manager’s job is to deliver the production from $k$ non-shirking workers to the firm using the provided budget $W$. The manager is fired if he “cheats” on this task. There are several ways that the manager can cheat. For example, the manager can refuse to pay bonuses to working workers and instead keep the bonus for private consumption. The manager can also pay bonuses to workers, but motivate them to work on projects for her own benefit. These actions, which harm the firm, are often referred to as diversions of
resources. Such an action of the manager is caught by the firm’s owners with probability \( \alpha_m \), and reneging managers are fired.

With these contracts, if the manager decides to “cheat” on paying the bonuses, her payoff will be

\[
W - kw + \alpha_m \delta U_m + (1 - \alpha_m) \delta (p_m U_m + (1 - p_m) E_m).
\]

Otherwise, by performing her duties, the manager’s payoff is

\[
W - k(w + b + c_m) + \delta (p_m U_m + (1 - p_m) E_m).
\]

Therefore, the incentive constraint for the manager is,

\[
\alpha_m \delta (1 - p_m) (E_m - U_m) \geq k(b + c_m). 
\tag{10}
\]

Let

\[
X_m := \frac{W}{k} - (w + b + c_m) = d(J) - (w + b + c_m).
\]

This is the payoff in one period, per job, of a manager who does not cheat on her contracts. Then, the long-run expected payoff of an employed manager, assuming \( \text{(10)} \) is satisfied so that she does not cheat, is

\[
E_m = kX_m + \delta (p_m U_m + (1 - p_m) E_m).
\]

The payoff of an unemployed manager on the other hand is,

\[
U_m = q_m E_m + (1 - q_m) \delta U_m,
\]

where \( q_m \) denotes the probability that an unemployed managers gets matched to a firm during the matching phase at the beginning of a period.

\footnote{See for example Burkart and Ellingsen (2004).}
Solving for $E_m$ and $U_m$ from the last two expressions, we obtain

\begin{align*}
U_m &= \frac{1}{p_m} \frac{kX_m}{(\gamma_p + 1)(\gamma_q + 1) - \delta}, \\
E_m &= \frac{1}{p_m} \frac{\gamma_q + 1}{(\gamma_p + 1)(\gamma_q + 1) - \delta}.
\end{align*}

(11)

where $\gamma_x = (1 - \delta)(\frac{1}{x} - 1)$. Therefore, as long as $X_m \geq 0$, managers’ individual rationality constraints are satisfied.

Substituting in (10), the managers’ incentive constraint becomes

\[ X_m \geq \frac{1}{\alpha_m \delta} \frac{1}{(1 - p_m)(1 - q_m)} - \delta (b + c_m). \]

(12)

We next find $q_m$, the endogenous rate of reemployment of managers. In a steady state equilibrium, when (12) is satisfied, the only unemployments result from managers’ voluntary decisions. Equating the rate of managers leaving and entering the unemployment pool, we have,

\[ q_m = \frac{p_m \cdot J}{k \cdot p_m \cdot J + N_m - J}. \]

Substituting in (12), we find the following no cheating constraint (NCC) for the manager

\[ X_m = d(J) - b - w - c_m \geq \frac{1}{\alpha_m \delta} \frac{1}{1 - p_m} \cdot \frac{kN_m}{kN_m - J} \cdot (b + c_m) \]

(M.I)

Similar to the incentive constraints for the workers, this provides a lower bound on the per-period per-job payoff to a manager to keep her from acting opportunistically and cheating on the contract. Note that the RHS of this constraint is increasing in $b$, that is, when workers’ contracts include bonus pays, firms have to increase the managers’ compensation to keep them from cheating on their contracts. Together with the worker constraint (W.I), this implies that the payment of bonuses $b$ effectively transfers the moral hazard problem from workers to their managers.

We next show that both incentive constraints (W.I) and (M.I) will be binding. First, we argue that (W.I) is binding. Assume in contradiction, that $q_w < 1$ and (9) does not bind. If
bonus $b > 0$, then the managers have incentive to decrease $b$. This change still keeps workers from shirking and will only make (M.I) more slack. On the other hand, if $b = 0$, then (9) implies that $X_w > 0$, this means that $w > 0$. In this case the managers will decrease the wage $w$ to bind (W.I) without violating (M.I).

We next argue that (M.I) also binds. As in Shapiro and Stiglitz (1984), we assume a competitive market with no entry barrier for firms. Thus, if (M.I) does not bind, then more firms can enter the market (we can increase $J$ by $\epsilon$), which even though lowers firms’ marginal revenue, $d(J)$, but still satisfies (M.I). This change will increase the RHS of (W.I) as well, and therefore to restore the workers’ incentives to binding, we can increase $w$. This will in turn decrease the LHS of (M.I). Firms’ will continue entering the market (increase in $J$) and increasing wages $w$ until both constraints bind at equilibrium.

Finally, we allow for free movement of managers and workers between the two types of jobs. Any worker can decide to become a manager, and vice versa. Naturally, when either workers or managers decide to switch jobs, they will first enter the unemployed pool in the new occupation.

Putting all these together, we have the following definition of equilibrium in an economy with managers.

**Definition 3.1 (Stationary equilibrium)** A stationary equilibrium of an economy with managers is determined by $(J, w, b, N_w, N_m)$ satisfying (W.I) and (M.I) with equality, and so that the payoff of unemployed workers equal to the payoff of unemployed managers (free movement), that is $U_w = U_m$, with $U_w$ and $U_m$ given in (8) and (11), respectively.

Remark: The model can be easily modified to the case where there is a switching cost for workers to managerial positions (e.g. due to education investment), and by allowing for intermediate outside options for unemployed managers (e.g. due to availability of lower rank managerial positions). We have adopted the current assumption for simplicity. Our obtained insights will continue to hold with the addition of these parameters as well at the expense of additional notation.
We now characterize the stationary equilibrium of the economy with managers. First, based on (W.I) and (M.I), we define the following *moral hazard multipliers* for workers and managers.

**Definition 3.2 (Moral-hazard multiplier)** The moral hazard multiplier for workers and managers, $\Delta_w$ and $\Delta_m$, are defined as,

$$
\Delta_w := \frac{1}{\alpha \delta} (1 - \delta + \frac{p_w}{1 - p_w} \cdot \frac{N_w}{N_w - J})
$$

$$
\Delta_m := \frac{1}{\alpha_m \delta} (1 - \delta + \frac{p_m}{1 - p_m} \cdot \frac{k N_m}{k N_m - J}).
$$

Intuitively, the moral hazard multipliers, which appear on the right hand sides of workers and managers incentive constraints, determine how the economy’s state and underlying parameters (such as monitoring accuracy, turnover rates, or number of jobs) affect the minimum compensation required for preventing shirking. As shown in the next theorem, the relative behavior of $\Delta_w$ and $\Delta_m$ will have a key role in determining the economy’s equilibrium.

A key difference between the moral hazard multipliers of workers and managers is how they depend on the population of workers and managers. In particular, because of the factor $k$, the moral hazard multiplier of the managers is more sensitive to the population size than that of the workers. This has an important implication. As shown in Section 4, as workers switch to managing jobs, the managers’ moral hazard multipliers decrease quickly, allowing the managers to take over the moral hazard problem of the workers.

The following Theorem shows how the bonus $b$ should be allocated.

**Theorem 3.1** Let $(J, w, b, N_w, N_m)$ be a stationary equilibrium. Then, the following holds:

$0 \leq b \leq \frac{c}{\alpha}$, with $b = \frac{c}{\alpha}$ implying full employment for workers, i.e., $N_w = J$. Furthermore,

- If $\alpha \Delta_w < \Delta_m$, then $d(J) \geq (\Delta_w + 1)c + (\Delta_m + 1)c_m$, and equality is achieved when $b = 0$,
• If $\alpha \Delta_w > \Delta_m$, then $d(J) \geq (\frac{\Delta_m}{\alpha} + 1)c + (\Delta_m + 1)c_m$, and equality is achieved when $b = \frac{c}{\alpha}$.

Proof. Assume first that the equilibrium is such that $N_w > J$, that is, there is some worker unemployment. The following [W.I] and [M.I] conditions should be satisfied at equilibrium:

1. $X_w = w + b - c = \Delta_w(c - \alpha b)$ (13)
2. $X_m = d(J) - b - w - c_m = \Delta_m(b + c_m)$ (14)

First notice that by (13) and $X_w \geq 0$, we have $b \leq \frac{c}{\alpha}$. Moreover, if $b = \frac{c}{\alpha}$, and $N_w > J$, then $X_w = 0$. This means that workers’ utility is 0. However, because of free movement, this means that unemployed managers should also have a utility of 0. When $p_m \in (0, 1)$, by (11), this is impossible. Thus, $\alpha b = c$ implies that $N_w = J$.

Adding (13) and (14), we obtain

$$d(J) - c - c_m = X_m + X_w = \Delta_w(c - \alpha b) + \Delta_m(b + c_m) = \Delta_w c + \Delta_m c_m + (\Delta_m - \alpha \Delta_w) b.$$  

Thus, if $\alpha \Delta_w < \Delta_m$, then $d(J)$ is minimized when $b = 0$. In contrast, if $\alpha \Delta_w > \Delta_m$, then it is optimal to choose $b = \frac{c}{\alpha}$ to minimize $d(J)$.

Remark: Notice that maximizing the number of jobs, $J$, is equivalent to minimize $d(J)$. Thus, Theorem 3.1 illustrates two possible regimes of equilibria depending on the moral hazard multipliers from Definition 3.2. When $\Delta_m > \alpha \Delta_w$, in an equilibrium that maximizes $J$, $d(J) = (\Delta_w + 1)c + (\Delta_m + 1)c_m$, managers do not provide bonuses, and therefore will not help firms overcome the moral hazard problems. We can see this by comparing it with the equilibrium condition of the
economy without managers \( [4] \), which can be rewritten as

\[
d(J) = (\Delta w + 1)c.
\]

In this scenario, managers actually reduce the number of jobs because of the managing cost \( c_m \).

On the other hand, when \( \Delta m < \alpha \Delta w \), in an equilibrium that maximizes \( J \), managers will offer the maximum bonus to workers. This shifts the moral hazard cost of workers from \( c \) to \( (c - \alpha b) = 0 \) as in \( [13] \), but increases the moral hazard cost of managers from \( c_m \) to \( c + c_m \). As we show in the next section, when the size of the firm is large enough, the moral hazard multiplier of managers is small, and despite the increase in cost, the presence of managers will help create more jobs compared to the economy without managers analyzed in Section 2.3.

4 Analysis

4.1 When do managers create more jobs?

We first analyze the conditions under which managers can lead to the creation of more jobs. We show that as the size of firms grows, if the management cost is not too large and monitoring technology is sufficiently accurate, managers will create more jobs and improve market efficiency. The intuition is as follows.

First, as the size of firms grows, managers have increased tasks, and will be offered higher payoffs. This motivates some workers to switch to managing jobs. However, larger firms also means fewer managing positions are available, which implies that finding a new managing job from within the unemployed managers’ pool now becomes harder. Therefore, the voluntary turnover rate for managers, \( p_m \), decreases. This in turn makes the moral hazard multiplier for managers smaller. Now, recall that using relational contracts with bonuses, managers
can motivate workers better than firms. By doing so, the moral hazard problem effectively only gets transferred from workers to managers. When manager’s moral hazard multiplier are small (for instance when firms are large, and monitoring is sufficiently accurate) and their managing cost are small enough, the firms can better manage managers’ moral hazard. This increased efficiency leads to the creation of more jobs.

This intuition is formalized in the following theorem.

**Theorem 4.1** Let $p$ be the voluntary turnover rate of workers in the economy without managers, if

$$\alpha_m > \frac{1 - p_{\min} - \delta}{1 - p - \delta},$$

then there exists $c_0$, $K$ such that if the managing cost, $c_m \leq c_0$ and the firms’ size, $k > K$, then there will be more jobs in the economy with managers.

**Proof.** First recall that by Theorem 3.1 there are two possible regimes of equilibria depending on the moral hazard multipliers from Definition 3.2, with either $b = 0$ or $b = \frac{c}{a}$ being the optimal choice of bonus pays in the equilibrium that maximizes $J$, depending on the relative size of the multipliers.

Assume the managers set $b = \frac{c}{a}$; then, by the first part of Theorem 3.1 $N_w = J$. As a result, $\Delta_w \to \infty$, and therefore we have $\Delta_m \leq 0\Delta_w$. Hence, the choice of $b = \frac{c}{a}$ is optimal, irrespective of the firm size $k$.

In this equilibrium, given the available pool of employees $N$, the total number of (employed and unemployed) managers, $N_m$, will be $N - N_w = N - J$. Replacing $b = \frac{c}{a}$ and $N_m = N - J$ in (14), we have

$$X_m = \frac{1}{\alpha_m \delta}(1 - \delta + \frac{p_m}{1 - p_m} \cdot \frac{k(N - J)}{kN - (k + 1)J})(\frac{c}{a} + c_m). \quad (15)$$

Because there are no unemployed workers at equilibrium, workers whose contract gets terminated for any reason can find jobs immediately. As a result, the lifetime utility of
workers is

\[ U_w = E_w = \frac{1}{1 - \delta} X_w. \]

Recall also that by (11), an unemployed manager’s expected payoff is,

\[ U_m = \frac{1}{p_m \gamma_{pm} + 1} \frac{kX_m}{(\gamma_{qm} + 1) - \delta} = \frac{kX_m}{\alpha_m \delta(1 - p_m) \gamma_{qm} \Delta_m}. \]

Because of free movement, the utility of unemployed managers should be equal to that of unemployed workers, that is \( U_w = U_m \). This implies

\[ X_w = (1 - \delta) \frac{kX_m}{\alpha_m \delta(1 - p_m) \gamma_{qm} \Delta_m} = \frac{k(\frac{c}{\alpha} + c_m)}{\alpha_m \delta(1 - p_m) \frac{1}{\gamma_{qm}} - 1} = \frac{k(\frac{c}{\alpha} + c_m)}{\alpha_m \delta(1 - p_m) kN_m - J}. \]

Replacing \( N_m = N - N_w = N - J \), we have

\[ X_w = \frac{1}{\alpha_m \delta} (p_m \frac{kJ}{1 - p_m} \frac{kJ}{kN - (k + 1)J}) (\frac{c}{\alpha} + c_m). \]  

(16)

Adding the per-period per-job pay for workers and managers, given in (16) and (15), we get,

\[ X_w + X_m = \frac{1}{\alpha_m \delta} (1 - \delta + p_m \frac{kN}{1 - p_m} \frac{kN}{N - k + 1 J}) (\frac{c}{\alpha} + c_m). \]  

(17)

Substituting the above in \( d(J) = X_w + X_m + c_m + c \) determines the number of jobs created at equilibrium for any given \( k \).

Specifically, let \( k \) go to infinity. By (14), as \( k \to \infty, kX_m \to \infty \). That is, the per-period pay of employed managers (from all jobs they manage) goes to infinity. By free movement, the expected payoff of unemployed managers is equal to that of unemployed workers. Thus \( q_m \), the probability that an unemployed manager can get a job needs to go to 0; otherwise, the payoff of unemployed managers would also grow to infinity. Recall that \( q_m = \frac{p_m J}{kN - J + p_m J} \).

Therefore, \( q_m \to 0 \), implies that the ratio between the number of management jobs, \( \frac{J}{k} \) and \( N_m \) approaches 0. Hence, the probability that an employed manager voluntarily gives up his
job, $p_m$, also gets smaller, and approaches $p_{\text{min}}$.

Therefore, as $k$ goes to infinity, and $p_m \to p_{\text{min}}$, from (17) we have,

$$\lim_{k \to \infty} X_w + X_m = \frac{1}{\alpha \delta} (1 - \delta + \frac{p_{\text{min}}}{1 - p_{\text{min}}} \cdot \frac{N}{N - J})(\frac{c + \alpha c_m}{\alpha_m}).$$

The equation above determines the number of jobs at equilibrium when $k \to \infty$. Specifically, if we let $c_m \to 0$, and using $X_w + X_m = d(J) - c_m - c$, we get,

$$d(J) = \frac{1}{\alpha \delta} (1 - \delta + \frac{p_{\text{min}}}{1 - p_{\text{min}}} \cdot \frac{N}{N - J})(\frac{c}{\alpha_m}) + c \quad \text{(M.E.)}$$

Compare this with the equilibrium equation of the economy without managers given in (4):

$$d(J) = \frac{1}{\alpha \delta} (1 - \delta + \frac{p}{1 - p} \cdot \frac{N}{N - J})c + c \quad \text{(No.M.E)}$$

The right hand side of (M.E.) is smaller than the right hand side of (No.M.E) if $\alpha_m$ satisfies the condition in Theorem 4.1. This relation continues to hold as long as $c_m$ is sufficiently small. This shows that the number of job, $J$, solving (M.E.) is larger than that of (No.M.E), that is, managers help create more jobs as long as the monitoring technology of the firm to detect their cheating is sufficiently accurate; See Figure 5 for an illustration.

Notice that the condition in Theorem 4.1 only requires a lower bound on the monitoring technology of managers. In particular, if $\alpha_m = 1$, that is if managers always get caught if they cheat, then, the condition in Theorem 4.1 will hold. In particular, assume $\alpha = \alpha_m = 1$, that is the monitoring technology for detecting shirking of workers and managers are both perfect. We can then obtain the following condition on the cost imposed by managers under which managers help create jobs.

**Corollary 4.1** Assume $\alpha = \alpha_m = 1$. Then, there exists $K > 0$ such that if $k > K$ and

$$c_m \leq \frac{p - p_{\text{min}}}{1 - p} c$$
managers help create more job compared to the economy without managers.

Proof. Following steps similar to the proof of Theorem \[4.1\] and substituting for \(\alpha = \alpha_m = 1\), we get,

\[
\lim_{k \to \infty} X_{w} + X_{m} = d(J) - c_m - c = \frac{1}{\delta} (1 - \delta + \frac{p_{\text{min}}}{1 - p_{\text{min}}} \cdot \frac{N}{N - J})(c + c_m).
\]

The equation above determines the number of jobs at the equilibrium with perfect monitoring. Specifically,

\[
d(J) = \frac{1}{\delta} (1 + \frac{p_{\text{min}}}{1 - p_{\text{min}}} \cdot \frac{N}{N - J})(c + c_m).
\]

(M.PM)

Compare this with the equilibrium equation of the economy without managers given in \[4\], assuming perfect monitoring:

\[
d(J) = \frac{1}{\delta} (1 + \frac{p}{1 - p} \cdot \frac{N}{N - J})c.
\]

(No.M.PM)

Thus, having

\[
c_m \leq \frac{p - p_{\text{min}}}{1 - p} c
\]
is a sufficient condition to get

\[
\frac{1}{\delta}(1 + \frac{p_{\text{min}}}{1 - p_{\text{min}}} \cdot \frac{N}{N - J})(c + c_m) < \frac{1}{\delta}(1 + \frac{p}{1 - p} \cdot \frac{N}{N - J})c.
\]

Consequently, the number of jobs in (M.PM) will be greater than that from (No.M.PM).

**Remark:** We can compare the equilibrium expressions for the economies with and without managers, (M.PM) and (No.M.PM), respectively, for comparative analysis. We observe the following trade-off between the two economies. The effective cost term in (M.PM), \((c + c_m)\), is higher than that of (No.M.PM), \(c\); this means that managers increase job creation costs. In contrast, the coefficient (which is related to the moral hazard multiplier) is smaller in (M.PM) as \(p_{\text{min}} < p\); this implies that managing moral hazard on the managers’ end is less costly to firms than managing workers’ moral hazard. If \(c_m\) grows large, the number of jobs in (M.PM) decreases, and the added management cost will eventually dominate the aforementioned tradeoff. In this scenario, managers effectively take away workers’ jobs.

Figure 6 compares the jobs created at (No.M.PM) with that at (M.PM) for three values of \(c_m\). We observe that the equilibrium with managers leads to fewer (more) jobs than the equilibrium without managers when the managers’ cost \(c_m\) is high (low). Note that the small value of \(c_m = 0.5\) satisfies the sufficient condition of Corollary 4.1. For intermediate values of \(c_m\), whether the managers can help the economy by creating additional jobs will depend on the production function \(d(J)\) of the economy.

### 4.2 The impact of firms’ size

Next, we analyze the effect of finite, small firm size \(k\), on the ability of the economy to generate more jobs with the help of managers. Let \(\alpha = \alpha_m = 1\). Notice that (M.PM) captures the limit regime when firms’ size \(k\) grows unbounded. When \(k\) is finite, the comparison between
Figure 6: An example with $\delta = 0.5, \alpha = \alpha_m = 1, p_{\text{min}} = 0.1, p = 0.5, N = 8, c = 1, d(J) = 7 - J$, comparing the number of jobs created at (No.M.PM) vs (M.PM) for small, intermediate, and large $c_m$ values of 0.5, 1, and 2, respectively.

The two scenarios (with and without managers) is based on (17), which implies

$$d(J) = \frac{1}{\delta}(1 + \frac{p_m}{1 - p_m} \cdot \frac{N}{N - \frac{k+1}{k}J})(c + c_m).$$  \hspace{1cm} (18)$$

Here, as $k \to \infty$, two effects drive the decrease of the right hand side of (18): the decrease of $p_m \to p_{\text{min}}$ on one hand, and the decrease of $\frac{N}{N - \frac{k+1}{k}J} \to \frac{N}{N-J}$ on the other hand. The decrease of $d(J)$ in turn translates into additional jobs; see Figure 7 for an illustration.

Figure 7: As $k \to \infty$ (i.e., firm size increases), $p_m \to p_{\text{min}}$ and $\frac{N}{N - \frac{k+1}{k}J} \to \frac{N}{N-J}$, leading to increase in the number of jobs.

However, when $k$ is small, $p_m$ might be significantly larger than $p_{\text{min}}$, and further, $\frac{k+1}{k}J$ will be larger than its limit, $J$. As a result, the right hand side of the equation above may
be larger than that of $\text{(No.M.PM)}$. Thus, managers will not be able to generate more jobs if the size of firm is small.

To further illustrate the impact of firms’ size on the ability of creating more jobs in the economy with managers, we assume a specific function for the voluntarily turnover rate. Let

$$p = p_0 \frac{\# \text{jobs}}{\# \text{population}}.$$ 

Normalize the employee measure to $N = 1$. In the economy without managers we will have,

$$d(J) = \frac{1}{\delta} (1 + \frac{p}{1 - p} \cdot \frac{1}{1 - J})c = \frac{c}{\delta} (1 + \frac{p_0 J}{1 - p_0 J} \cdot \frac{1}{1 - J}) \quad (19)$$

In the economy with managers, if there is no unemployment for workers, then $p_m = p_0 \frac{J}{N_m} = p_0 \frac{J}{k(1 - J)}$. Replacing this in (18), we have

$$d(J) = \frac{c + cm}{\delta} (1 + \frac{p_0 \frac{J}{k(1 - J)}}{1 - p_0 \frac{J}{k(1 - J)}} \cdot \frac{1}{1 - \frac{k+1}{k} J}). \quad (20)$$

We are interested in the smallest $k$ such that the solution $J$ satisfying (20) is at least that of (19).

![Figure 8: Minimum size of firms for creating jobs as function of managing cost](image)

In Figure 8 we illustrate a numerical example for $d(J) = 2 - J, c = 1, \delta = 1, p_0 = 0.5$. The figure shows the minimum size of firms needed for managers to create more jobs as a
function of different $c_m$ values. In this example, once $c_m > 0.545$, managers take away jobs.

### 4.3 Comparative analysis on wages and manager’s payoff

We next analyze the worker and manager’s payoff at equilibrium. Let $\alpha = \alpha_m = 1$. For finite firm size $k$, from (15), for every completed job, the manager earns,

$$X_m = \frac{c + c_m}{\delta} (1 - \delta + \frac{p_m}{1 - p_m} \cdot \frac{N - J}{N - \frac{k+1}{k} J}).$$

Also, from (16) the wage of workers is given by:

$$X_w = \frac{c + c_m}{\delta} \left( \frac{p_m}{1 - p_m} \cdot \frac{J}{N - \frac{k+1}{k} J} \right).$$

In the limit regime, when $k \to \infty$, The wage of workers is $\frac{c + c_m}{\delta} \left( \frac{p_m}{1 - p_m} \cdot \frac{1 - J}{N} \right)$, while the manager earns a fee of $\frac{c + c_m}{\delta} (1 - \delta + \frac{p_m}{1 - p_m} \cdot \frac{J}{N})$ per completed job.

The total payoff of managers and workers for both the finite and limit regimes are shown in Figure 9. The total workers’ payoff are the shaded upper rectangular area in the figures (shown for both finite and limit regimes), while that the total managers’ payoff are the lower rectangular areas in Figure 9.

In the limit regime, when $p_{min}$ increases, the number of jobs, $J$, decreases. This will however increase the managers’ fee. The workers’ wage on the other hand, depending on the changes in $p_{min}J$, can either increase or decrease. We illustrate the total surplus of workers and managers, as a function of $p_{min}$, in Figure 10.

### 5 Discussion

We have considered a market model where the owners of firms, managers and workers interact through self-enforcing relational contracts. We showed that if the firm’s size is large and the managing cost is small enough, then managers help the firms to overcome workers’ moral
Managers' total payoff ($3 \rightarrow \infty$)
Managers' total payoff (finite $k$)
Workers' total payoff ($3 \rightarrow \infty$)
Workers' total payoff (finite $k$)

Figure 9: Total Payoff of workers and managers

Figure 10: How surplus is shared between workers and managers as a function of managers' turnover rate (with $d(J) = 2 - J$, $c = 1, c_m = 0.1, \delta = 1$).
hazard problem, create more jobs, and improve efficiency. Our theory explains not only why managers exist in a competitive market, but also why they are well paid.

The importance of managers and relational contracts in the theory of the firm has been addressed by several researchers. However, in these arguments, the manager’s tasks can be executed by the firm’s owners, and the roles of non-owner managers are justified by reasons outside the models. We showed an irreplaceable managing role in maintaining relational contracts. Non-owner managers are better than firms’ owners in designing and executing relational contracts for a simple reason: they can be fired.

Our proposed approach of embedding relational contracts into a competitive market is tractable. It can potentially allow us to study further questions about firms’ internal structure as well as external networks among firms.

References


[Baker et al. 2002] argue that in practice, all non-contractible rights of control become too numerous and complex, and therefore firm must have some mechanism to coordinate their exercise.


