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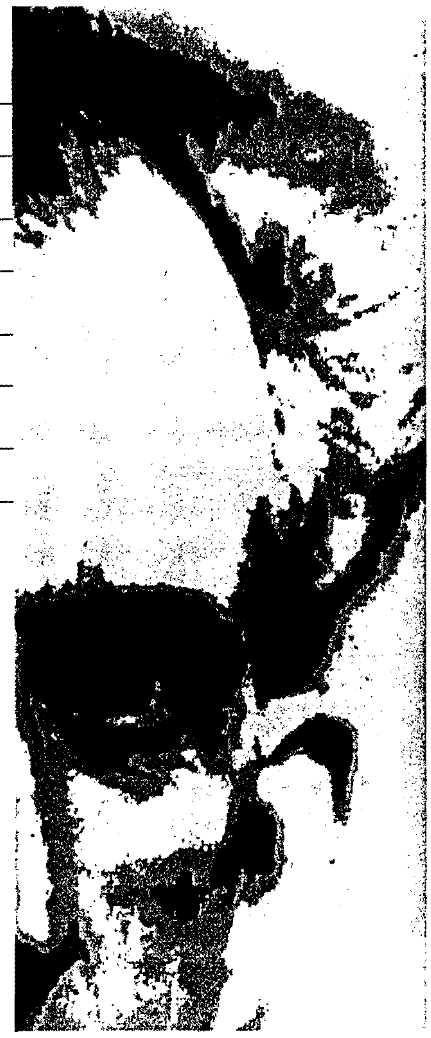

CORNELIUS LANCZOS

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Development of the FFT

Organizer: James W. Cooley

This minisymposium had two speakers concerning the development of the FFT. Professor Robert Nowack of the Department of Earth and Atmospheric Studies at Purdue University discussed applications in Geophysics. He also had some interesting comments to make on the period of time that Lanczos spent at Purdue and his association with Danielson. Professor David W. Kammler of the Mathematics Department of Southern Illinois University is interested in elementary derivations of the FFT and the FHT that are useful in teaching the algorithms and in giving insight into how they work and what they do.

The development of the FFT algorithms has had a long and rather unusual history, filled with strange ironies. The basic constructs are found in works by Gauss and others. Some "discoverers" hit on the idea by using simple insight in the manipulation of algebraic formulas with no more sophistication than is found in high school mathematics. Others, using advanced algebraic methods, approached the problem methodically and derived whole families of algorithms.

Development of the FFT and Applications in Geophysics*

Robert L. Nowack†

Abstract

The development of the fast Fourier transform, or FFT, has had a major impact on most scientific fields. One of the earliest "recent" discoveries of the FFT was by Danielson and Lanczos in 1942, although the method was made widespread through the work of Cooley and Tukey in 1965. In the geophysical sciences, the FFT has been utilized in a wide variety of applications including forward modeling, data processing and inversion methods.

1 Introduction

One of the more important developments in scientific computing has been the ability to rapidly compute discrete Fourier transforms using the FFT. For a time series with length N a power of 2, the number of complex multiplications for the FFT is $(N/2)log_2N$, as opposed to N^2 for direct evaluation [28]. As an example, the FFT of a time series with 1024 points involves (5)(1024) as compared to (1024)(1024) multiplications, a reduction of more than two orders of magnitude. The difference becomes more extreme for longer time series or in higher dimension s. Various specialized FFT algorithms are given in [12]. Within the geophysical sciences, the fast Fourier transform has revolutionized the processing, synthesis, and inversion of geophysical data.

2 The Collaboration of Danielson and Lanczos

The use of the FFT became widespread primarily through the work of Cooley and Tukey [5]. However, shortly after this work, a similar algorithm was described by Rudnick [29] based on earlier work by Danielson and Lanczos [10] (see also, [6]). A geophysical implementation of the FFT prior to 1965 was also noted in [3].

The roots of the FFT have been traced back to Gauss in [18]. However, the reviews in [7], [8] subsequently gave more emphasis to the work of Danielson and Lanczos, and suggested that they independently discovered the doubling algorithm implicit in the FFT. It was noted by Cooley in [8] that, "it appears that Lanczos had the FFT algorithm; and if he had had an electronic computer, he would have been able to write a program permitting him to go to arbitrarily high N."

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There were several early applications of the Danielson and Lanczos algorithm. The algorithm was used in [23] to perform a 64-point Fourier synthesis for stress pulses in a viscoelastic solid. An early digital, electronic computer was used in [2] to compute a 128-point Fourier inversion for neutron scattering using the Danielson and Lanczos algorithm. Nonetheless, to a large extent, the FFT algorithm was brought to the attention of the scientific community by Cooley and Tukey [5] (see also, [19], [20]).

Danielson and Lanczos performed their work in the late 1930's at Purdue University, where Cornelius Lanczos (1893-1974) was a professor of mathematical physics from 1931-1 946 [30]. Gordon Danielson (1912-83) was a graduate student in physics at Purdue working on applications of Fourier analysis to X-ray scattering. Danielson became a professor of physics at Iowa State University in 1948 and a distinguished professor in 1964 [21].

Mrs. Dorothy Danielson recently commented that her husband came to know Lanczos as a graduate student in one of his mathematics classes, and that Lanczos suggested that they work together on an idea for rapid evaluation in Fourier analysis (personal communication, 1993). This became part of Danielson's Ph.D. thesis [9], as well as publications with Lanczos [10], [26].

3 Geophysical Applications of the FFT

The fast Fourier transform has had a major influence in the geophysical sciences. Geophysical applications of the FFT can be divided into the areas of forward modeling, data processing and geophysical inversion. In forward modeling, synthetic results are obtained on the computer and then used for comparison with observations. In the modeling of seismic waves, transient results are often synthesized using the FFT from frequency domain calculations [1]. In other cases, general forcing functions can be incorporated using FFT-based convolution. A number of computer programs for seismic modeling are included in [11].

The pseudospectral method was proposed in [25] for the solution of hyperbolic equations with applications in meteorology and oceanography (see also, [17]). The pseudospectral method has been applied to the forward modeling of seismic data [24] and can often be competitive with finite-difference methods [14].

The processing of geophysical data has also benefited from the efficiency of the FFT algorithm. Analysis of geophysical time series and spatial data using the FFT are described in [3], [4], [22]. The FFT has been used for the analysis of dispersive waves for phase and group velocities in [13]. Deconvolution methods based on the FFT and damped least squares are discussed in [4], and seismic deconvolution using homomorphic signal processing in [32].

The migration of seismic reflection data using the FFT can be very efficient and is described in [15], [31]. Slant stack processing can also be performed using the FFT [16]. Finally, the FFT can be utilized in algorithms for tomography and the inversion of geophysical data. In geophysical inversion, physical parameters of the Earth are inferred from remotely-recorded geophysical data [27].

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