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Book Review

Geophysical Data Analysis: Discrete Inverse Theory by William Menke, International Geophysics Series, vol. 45, Academic Press, 1989; US $44.95.

As stated in the introduction of this book, inverse theory is a set of mathematical techniques for the estimation of model parameters given data and a specific theory or model. Early developments for reconciling data with theory date back to Gauss and Legendre with the origins of the method of least squares. Geophysics has been in an excellent position to make contributions to inverse theory since many inferences about the earth's interior must be made indirectly, using remotely recorded geophysical data. Geophysical inverse theory must, in addition to estimating earth parameters, also provide self-consistent estimates of the uncertainties of these parameters.

This is a review of the revised edition of Menke's book on geophysical inverse theory. Since I have used the original edition for the last several years in a course on inverse theory, this review allows me the opportunity to make some general comments and observations, while noting any changes in the revised edition. This book provides an excellent introductory account of inverse theory with geophysical applications. It is at a more elementary level than the recent book by Tarantola (1987, *Inverse Problem Theory*) and more specialized treatments such as that presented by Aki and Richards (1980, *Quantitative Seismology*).

Menke's book emphasizes discrete inverse theory, however, this does not imply that the models must be discrete, but rather finite dimensional. Bandlimited, continuous models can be related to finite dimensional models through appropriate basis function expansions. Of course, this is not to say that earth parameters are strictly bandlimited in scale. Nonetheless, many of the features of inverse theory can be illustrated using finite dimensional vector spaces without initially addressing the more complex issues of closure and completeness, important for infinite dimensional cases. In the revised edition, Menke attempts to illustrate the relationship between continuous and discrete cases in a newly added chapter.

Chapter 1 of the book is a good introduction to the formulation of an inverse problem. Several illustrative inverse problems are given which are further developed later in the book. These examples show how discrete matrix problems are obtained from the physics of a given problem.

Some elementary comments on probability theory are provided in Chapter 2. While I have found this chapter to be very useful to students, it is not a substitute for some previous familiarity with probability. In addition, no reference are given to assist the student in reviewing this material. Since *a priori* probability distributions are actively applied throughout the book, a somewhat surprising omission is that there is no explicit statement of Bayes' theorem. Although this theorem is straightforward, its ramifications have caused considerable, historical controversy in statistics.

Chapters 3, 4 and 5 in Menke's book provide three viewpoints of the linear inverse problem. The viewpoint in Chapter 3 is called the length method. After giving different possible length measures or norms, this chapter specializes to the least squares case. Since, in the least squares case, an inner product space is implied, arguments about minimization of lengths could have been converted to simpler geometric arguments via the projection theorem. However, the approach followed in this chapter is a more direct minimization of lengths. This is in part required since little development of vector spaces has thus far been given in the book. Even though a working knowledge of linear algebra is assumed in the preface of the book, my experience is that students still need a review. I therefore use supplementary material from Luenberger (1969, *Optimization by Vector Space Methods*) and Strang (1988, *Linear Algebra and its Applications*) for this.

The viewpoint of the linear inverse problem taken in Chapter 4 is that of generalized inverses and a good introduction is given to and resolution covariance for the least squares and minimum-norm solutions. The Backus and Gilbert generalized inverse for the underdetermined problem is described, as well as the simultaneous minimization of resolution spread and model variance size. The discussion of the SVD inverse, called here the natural generalized inverse, is deferred until Chapter 7.

The third viewpoint for the linear inverse problem is the statistical approach given in Chapter 5. Some parts of this Chapter are adapted from the work of Tarantola and Valette (1982). *A priori* distributions and inexact theories are incorporated with very clear figures and illustrations. This chapter also includes a brief section which uses F tests to compare how different models fit the data. This can be used in model building to develop more elaborate models from simpler ones.

A concise but physically understandable introduction of null vectors and nonuniqueness in solutions is given in Chapter 6. Surprisingly, this is discussed after the Backus and Gilbert results described in Chapter 4. In Chapter 7, vector mappings are used to describe Householder transformations and the singular value decomposition. The generalized inverse, based on the SVD, is then developed. The chapter concludes with an overview of linear equality and inequality constraints.

An intuitive approach to linear inverse problems with non-Gaussian distributions is presented in Chapter 8. The L_1 and L_∞ norm problems are described and solved by conversion to a linear programming problem where the simplex method can be used for the solution.

Nonlinear inverse problems are introduced in Chapter 9 and a linearized iterative approach is taken. Since many geophysical problems are highly nonlinear, a number of iterations may be required. Also, determining model uncertainties for nonlinear problems is difficult and approximations must be used. For large-scale problems, multiple iterations may even be prohibitive. Recent examples of this include large-scale tomographic problems, as well as migration of seismic reflection data which is a first iteration of a more general nonlinear inversion. Since multiple minima may exist for nonlinear problems, a global minimum may also not be assured. Recent techniques, such as simulated annealing, could be applied but general solutions to nonlinear inverse problems are still outstanding.

Chapter 12 presents a number of interesting sample inverse problems. This chapter partly, but not entirely, compensates for the lack of problems at the end of each chapter. Several useful numerical codes written in Fortran are provided in Chapter 13, including algorithms with equality and inequality constraints. Although the SVD algorithm is described, no code is given. However, this code and other scientific software are readily available through packages such as LINPACK and EISPACK. The codes provided in this book would be even more useful if sample inputs and outputs were provided.

The revised edition has a newly added Chapter 14 which provides descriptions of further applications of inverse theory to geophysics. It serves primarily to cite a number of additional geophysical references, but could have been more coherently incorporated with the applications given in Chapter 12. The resulting reference list is now larger but is also disjointed since the new references have simply been concatenated onto the end of the original list.

The original edition of the book had a number of typographical errors and, although some remain, most have been corrected in the revised edition. My major comments on the book are mainly preferences on the ordering of topics and some omissions, which may be inevitable in an elementary text. Some of these omissions, in particular the lack of references, have been partly corrected in the revised edition. My experience in using this book, along with supplementary material in a course for the first year graduate students, has been very positive. I unhesitatingly recommend it to any student or researcher in the geophysical sciences.

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