Geophys. J. Int. (1994) **118,** 333–343

Resolving a low-velocity zone with surface-wave data

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Accepted 1994 March 25. Received 1994 March 23; in original form 1993 October 5

SUMMARY

The main purpose of seismic inversion is the retrieval of seismic velocities and densities in the earth. Inversion of body-wave traveltimes cannot uniquely determine the seismic velocities as a function of depth when a low-velocity layer is present. It is generally assumed that surface waves do not suffer from the same non-uniqueness. The issue is addressed whether it is possible to remove the non-uniqueness of traveltime inversions with a realistic set of surface-wave data. This requires the exact determination of the velocity distribution within and around the low-velocity zone. Waveforms, phase velocities as well as group velocities are investigated qualitatively. Synthetic Love-wave phase and group velocities are actually inverted. Waveforms are shown to be sensitive to the exact velocity distribution of a low-velocity layer, but it is concluded that the removal of the non-uniqueness with the use of waveforms is difficult because the differences of the waveforms are generally small and because complications such as lateral heterogeneity and poorly known source parameters reduce the accuracy of waveform inversions. The inversions of Love phase and group velocities indicate that it is difficult to determine the velocity distribution of a low-velocity layer in a statistically significant way with a realistic set of dispersion data. Group velocities are shown to be more sensitive to the low-velocity structure than phase velocities. Unfortunately, group-velocity data suffers from a practical non-uniqueness because in general only fundamental-mode group velocities can be measured. It is concluded that the non-uniqueness in the detailed structure caused by a low-velocity layer cannot readily be resolved by using surface-wave dispersion data.

Key words: inverse problems, low-velocity layer, surface waves.

1 INTRODUCTION

The main purpose of seismic inversion is to determine the distribution of seismic velocities and density in the earth. The best known and most easily applied technique is the inversion of body-wave traveltimes. Gerver & Markushevich (1966) showed that there is no unique solution to this inverse problem when a low-velocity layer (LVL) is present and the source is located above the LVL. They found that there are infinitely many velocity models that give rise to exactly the same traveltime curves when an LVL exists. This non-uniqueness is not just of theoretical importance. In many velocity models, low-velocity structures are present especially at depths between 50 and 300 km (e.g. York & Helmberger 1973; Souriau 1981; Paulssen 1987; Snieder 1988; Spakman 1991). Accurate knowledge of the seismic velocities of an LVL is especially important if we want to link seismological and dynamical earth models.

It is generally believed that surface waves do not suffer from the same non-uniqueness as body waves. Surface-wave phase velocities and group velocities have been used to determine regional *S*-velocity structure of the crust and upper mantle especially, and lately waveform-inversion techniques have been introduced to determine velocity models on a regional and global scale (Nolet 1976; Nolet 1977; Cara, Nercessian & Nolet 1980; Lerner Lam & Jordan 1983; Nakada & Hashizume 1983; Woodhouse & Dziewonski 1984; Mitchell 1984; Cara & Lévêque 1987; Snieder 1988; Montagner & Jobert 1988; Zhang & Tanimoto 1989; Tanimoto 1990; Dost 1990; Nolet 1990; Lévêque, Cara & Rouland 1991; Montagner & Tanimoto 1991; Zielhuis 1992; Stutzmann & Montagner 1993).

The inversion of surface-wave data and its uniqueness has

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been a topic of great interest. Often the inverse problem for Love waves is studied because it is mathematically simpler than the inverse problem for Rayleigh waves. An early study of the inversion of surface-wave data was carried out by Takahashi (1955), who solved the forward and inverse Love-wave problem by using a WKBJ approximation for the modal structure. He studied the problem of recovering the shear-wave velocity as a function of depth from the Love-wave phase velocities for the special case that the modes have only one turning point. Remarkably, his approximate solution is given in terms of an Abel's equation, which is an equation of the same type that Herglotz (1907) and Wiechert (1910) found for the inversion of traveltimes of body waves. This equation cannot be solved uniquely when the velocity is not a monotonous function of depth, which is clearly not the case in the presence of an LVL. However, Takahashi's approximation is not directly applicable to media with LVLs, since an LVL corresponds to three turning points. Nevertheless his result leads us to conjecture that surface waves may suffer from the same non-uniqueness as body waves in the presence of an LVL or that at least surface waves will be insensitive to changes in the shear-wave velocity that do not affect the traveltime curves of body waves.

The first rigorous attempt to establish uniqueness in the case of the inverse Love-wave dispersion problem was performed by Gerver & Kazhdan (1972). They showed that the *S* velocity as a function of depth cannot be derived uniquely from only fundamental-mode phase velocities. Barcilon (1976) studied several inverse eigenvalue problems and addressed a problem that is mathematically related to the inverse Love-wave problem. He studied a discrete elastic membrane and found his results 'to point in the direction of uniqueness of the inverse Love problem'. Hald (1977) studied the related inverse eigenvalue problem for an elastic cylinder. He pointed out that it is obvious that we cannot determine the density and the seismic velocities uniquely from a finite amount of data, but that if either ρ or $\lambda + 2\mu$ is known, the other parameter of the two can be determined uniquely from one complete spectrum of eigenfrequencies. Brodskiĭ & Levshin (1979) studied the problem of the inversion of eigenfrequencies of the earth in an asymptotic approximation. They found that a practical non-uniqueness, comparable with that of traveltimes, exists because the 'additional series' cannot be measured. This additional series consists of those eigenfrequencies that are intimately connected with an LVL. Almost all the energy of these eigenvibrations is concentrated in and around the LVL, so that they have very little energy and amplitude at the surface. Consequently, in practice, they cannot be measured. Bruk (1980) derived the theoretical uniqueness of the Love- and Rayleigh-operator eigenvalue inverse problems for a spherical earth. Nadirashvili (1982) also proved the uniqueness of these problems in a half-space. In both papers Borg's theorem (1949) is used. This theorem states that in order to solve an inverse Sturm–Liouville problem uniquely, in general one needs two complete spectra belonging to two different boundary conditions. Sometimes, when the inverse problem is symmetric, the requirement of two boundary conditions can be dropped. However, a unique solution of such an inverse eigenvalue problem requires at least the knowledge of all overtones at all frequencies belonging to one boundary condition. For seismology this is not a realistic requirement and it is evident that the results of Bruk (1980) and Nadirashvili (1982) do not give much information on the degree of uniqueness that can be achieved in inversions with a realistic data set.

The purpose of this paper is to investigate if it is possible to remove the non-uniqueness of traveltime inversions in the presence of an LVL with the use of a realistic set of surface-wave data. For this purpose, we developed velocity models that have an LVL at asthenosphere depths and that give rise to exactly the same traveltimes. These models are indistinguishable with the use of body-wave traveltimes alone. We want to determine if they can be distinguished with the use of surface waves.

2 THE MODELS

We constructed two pairs of laterally homogeneous 1-D shear-velocity models, the A and the B models. The models all have an LVL and are constructed in such way that both A models, as well as both B models, give rise to exactly the same traveltime curves. For reasons of simplicity the density in our models is constant. The density ρ is $4\,\mathrm{g\,cm^{-3}}$ at all depths and the ratio v_p/v_s is taken constant at $\sqrt{3}$. Anelastic damping is not taken into account. Therefore our models only differ in the mutually related *P* and *S* velocities. The *S* velocity of the *A* models, A1 and A2, is shown in Fig. 1. The differences between the models are most pronounced around the LVLs, but it can be seen that the models are also different below the LVLs. We use this pair of A models to check whether the shape of an LVL can be resolved, where shape means the extent of the LVL and the velocity distribution within the LVL. The other two models, B1 and B2, are shown in Fig. 2. Again the *S* velocity is shown as a function of depth. These B models are used to determine if it is possible to find the exact depth of an LVL. It is important to note that the shear velocities in our models differ at some depths by as much as $400\,\mathrm{m\,s^{-1}}$, which is a relative difference of about 9 per cent.

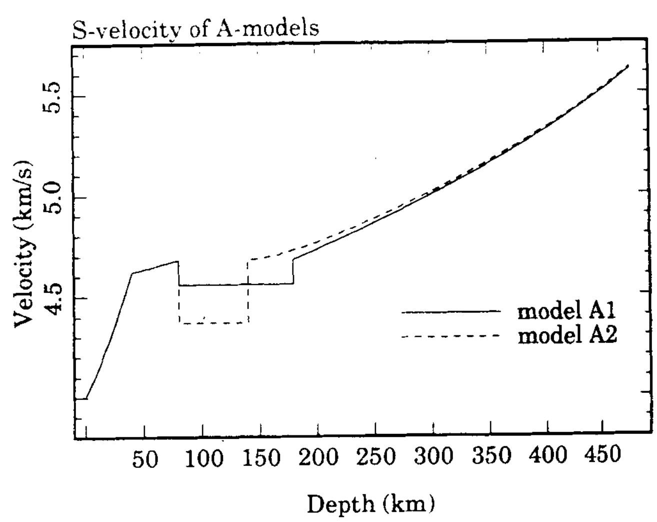

Figure 1. Shear-wave velocity as a function of depth for the A models.

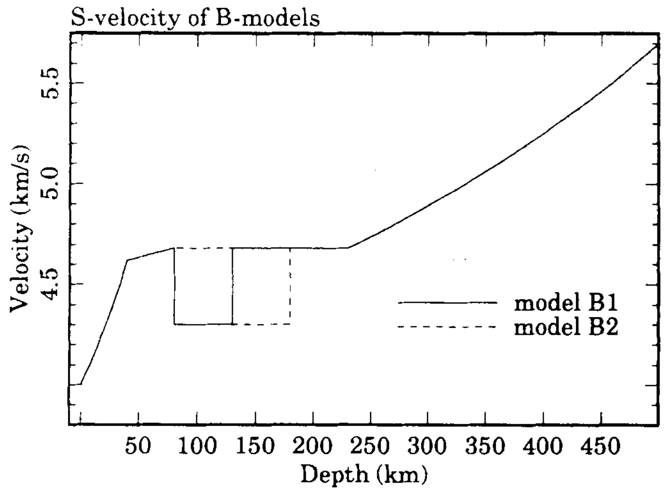

Figure 2. Shear-wave velocity as a function of depth for the B models.

3 WAVEFORMS

The seismogram is the most fundamental datum. All other data such as phase and group velocities have to be derived from the seismograms. In order to check if we can distinguish between our models with the use of surface waves, we first compare synthetic seismograms of surface waves propagated through the models. We calculated synthetic seismograms for Love and Rayleigh waves at several epicentral distances and source depths using the fundamental mode and five higher modes. For Rayleigh waves an explosive source was modelled, whereas in the case of Love waves a horizontal point force perpendicular to the source–receiver line was used. In the Figs 3 and 4, seismograms are shown of Love and Rayleigh waves that have propagated through the A and the B models. For Rayleigh waves the vertical component is shown, and for Love waves the transverse component. In all seismograms a source with the same total seismic moment was used, and the scale, shown at the left of each seismogram, is added for comparison of the amplitudes at the surface. The time-scale is indicated by the begin and end values of the seismograms. Because the differences between the seismograms of the B models are comparable with those found for the A models, we only present three seismograms for the B models.

From the seismograms it is clear that surface waves are sensitive to variations in the LVL which leave the traveltimes of body waves unaffected. In the seismograms, Love and Rayleigh waves display a similar sensitivity to changes in the LVL, although Love waves may be slightly more sensitive to the LVL than Rayleigh waves. If we compare the seismograms generated with the models A1 and A2, we see that for shallow sources the waveforms do not differ very much. Only the longer periods show a noticeable phase difference which is only apparent for long-propagation distance. The same applies to the B models. The reason that these seismograms are very much alike is that shallow sources mainly excite the fundamental mode at short periods. These higher frequencies of the fundamental mode have low amplitude at the depth of the LVL in our models and therefore they are not very sensitive to the exact velocity distribution of the LVL. The differences between the seismograms are most pronounced when the source is located in or around the LVL (Figs 3c, f and 4b). This is due to the fact that higher modes and especially those with a large amplitude in and around the LVL are now strongly excited. Just these modes have the most different modal structures and phase velocities for the different models. Therefore, the differences in the waveforms are pronounced when the source is located in the LVL. If the source is located below the LVL modes are excited that are less sensitive to the structure of the LVL, so that the differences between the seismograms are less pronounced (Fig. 3d). One has to bear in mind that higher modes interfere considerably, so that small differences in phase velocities lead to seismograms that look very different. This makes, especially in single-station methods, these obvious differences less robust features to resolve the LVL than they seem to be at first sight. It is clear that when the distance that a surface wave has travelled grows, the differences between the seismograms increase. This is caused by the fact that dispersion manifests itself stronger when the propagation distance increases.

The differences in the waveforms shown in the Figs 3(c), (f) and 4(b) are definitely large. Amplitudes and phase are very different in these seismograms. The higher modes are more strongly excited in the model A2 than in A1 and in B2 than in B1. Probably these differences are large enough to resolve the LVL in a waveform inversion, but high-quality seismograms from such deep sources are not as common since the amplitudes are about 100 times smaller than the amplitudes of the seismograms with a source at 10 km depth. Moreover the differences in the seismograms due to the change in the LVL should be compared with other causes of waveform distortion. Zielhuis (1992) shows the significant influence of an erroneous velocity model at the source location on the phase of the waveform. The phase differences that she finds are comparable to the most pronounced differences we find for our models. This problem, as well as source mislocation, lateral heterogeneity, random noise and errors in the source mechanism, will certainly make it difficult to resolve the differences between the models with a different LVL in a waveform-inversion procedure.

To conclude, it is clear that there are only small differences in the waveforms for shallow sources. Waveforms are only strongly affected by details of the LVL when the source is located at a depth near the LVL. It is definitely more difficult to record waveforms from sources at these depths. Therefore, it is questionable whether the differences in the waveforms are large enough to resolve the velocity structure in the LVL given all other factors that influence the waveform of surface waves.

4 PHASE VELOCITIES

Many of the practical complications of waveform inversions do not apply to the measurement and inversion of phase velocities. Properties like the receiver response and source mechanism do not influence phase velocities, and phase velocities can easily be measured with two or more seismographs without being influenced by other parts of the travel path than that under and between the seismographs.

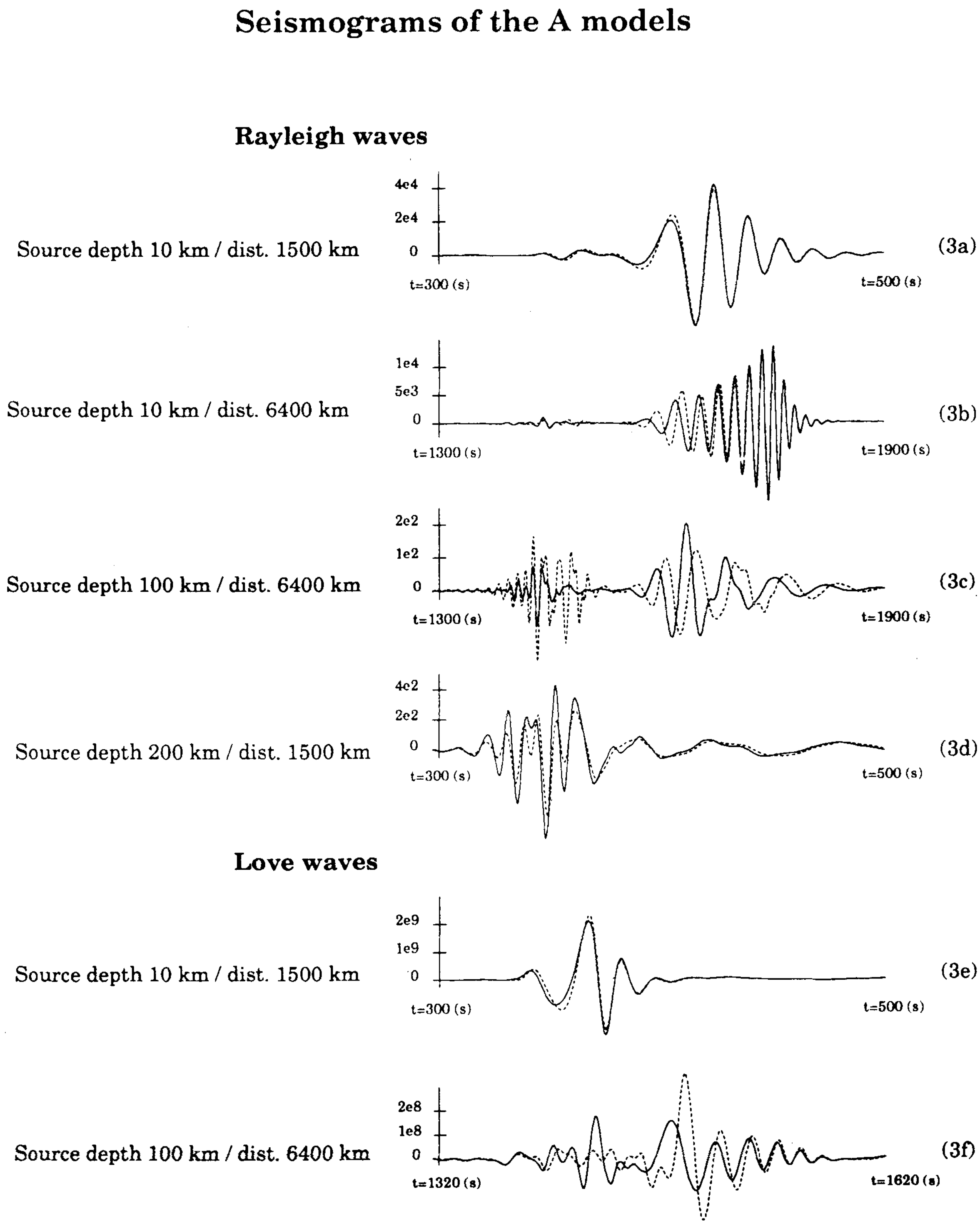

Figure 3. Seismograms of surface waves propagated through the A models for different source depths and epicentral distances. Rayleigh waves are excited with an explosive source and Love waves with a point force. The seismic moment is the same for all sources.

In Fig. 5 the phase velocities of the six lowest Love modes are shown for the models A1 and A2. In the box the high-frequency part is enlarged. This part of the dispersion curves is governed by the mutual alternation of channelled and crustal modes, which is characteristic in the presence of an LVL (e.g. Panza, Schwab & Knopoff 1972; Pěč & Novotný 1976; Kennett 1983). The character of Rayleigh-wave dispersion is very similar and is not presented here. In Fig. 6, the Rayleigh-wave phase velocities of the B models are shown. In this case the Love-wave phase velocities are omitted because they exhibit similar behaviour as the Rayleigh-wave phase velocities. In Figs 5 and 6, one can see that there are only small differences between the phase velocities of models A1 and A2, and between those of

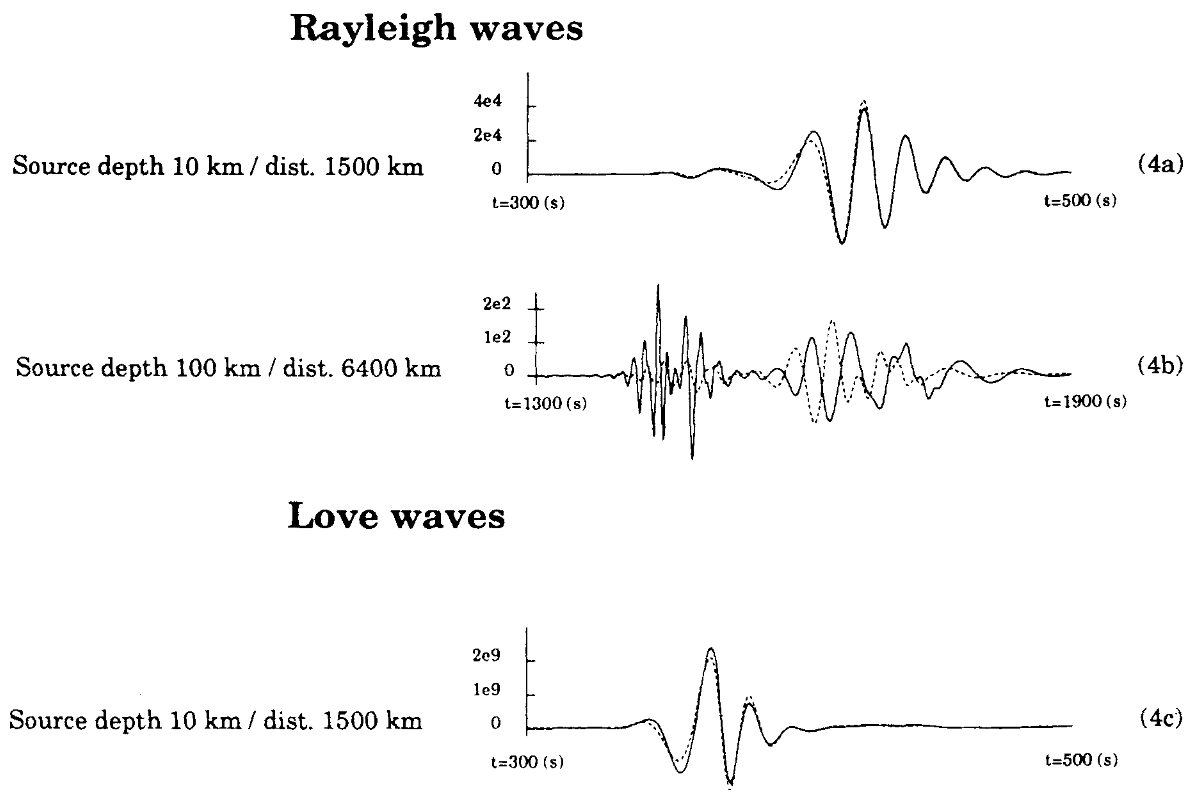

Figure 4. As Fig. 3 but for the B models. Only three seismograms are shown because seismograms for the B models are very similar to those of the A models.

models B1 and B2. If we want to know whether we can discriminate between the models with the use of phase velocities, we clearly need an estimate of error from a realistic data set. In this study, an optimistic error level of 1 per cent for the phase velocities is assumed, which is indicated by the error bars in the Figs 5 and 6. Nolet & Panza (1976) estimated the errors of phase-velocity measurements using seismic arrays at 1 to 2 per cent, which is of the same order magnitude as the estimate of Knopoff (1972) for certain two-station methods. With current data

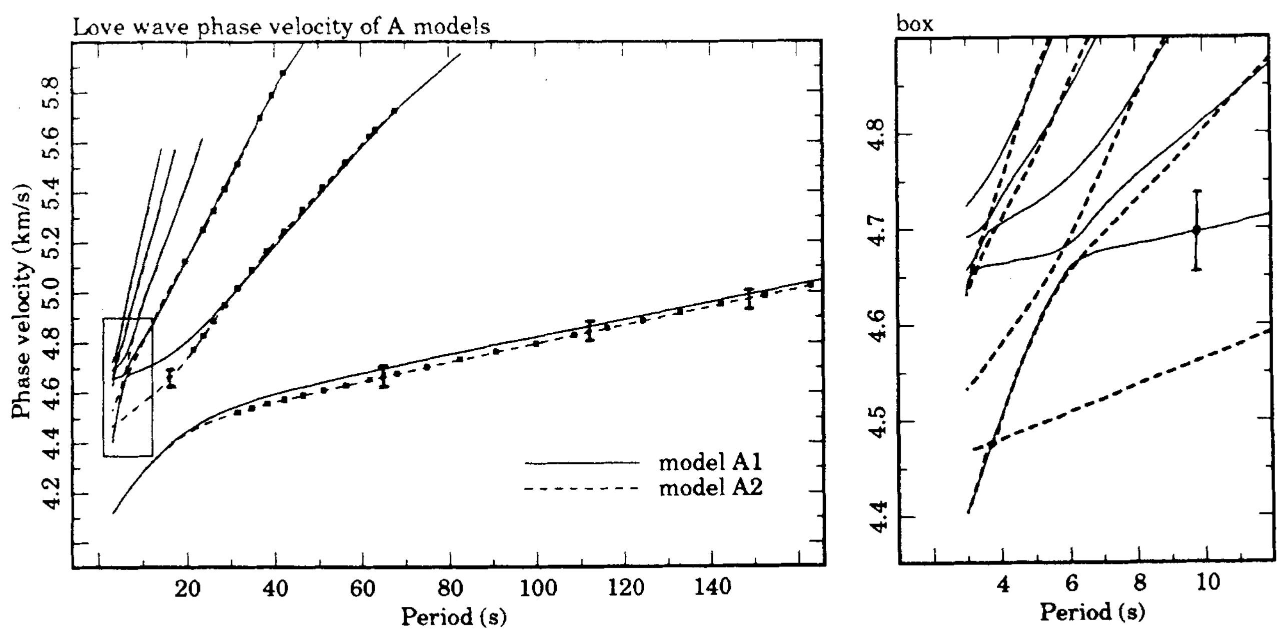

Figure 5. Love-wave phase velocities of the A models. The box shows the high-frequency part with the alternation of crustal and channelled modes. The error bars indicate a realistic experimental error. The points indicate the dataset used in a linear inversion described in Section 4.

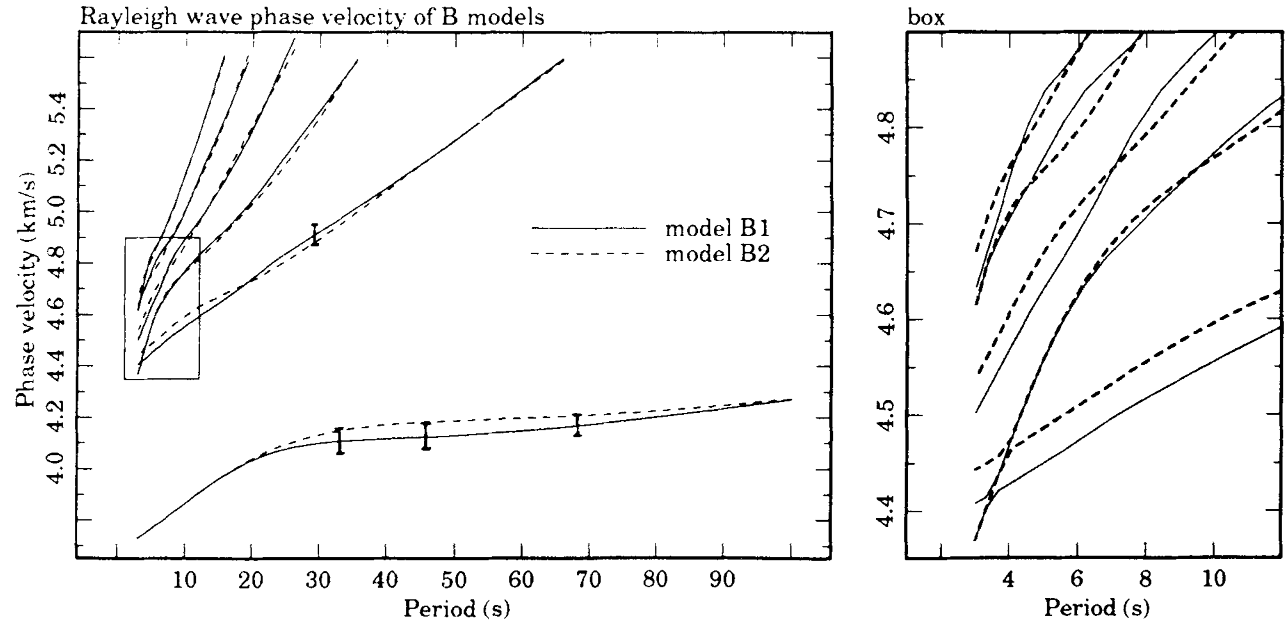

Figure 6. As Fig. 5, but Rayleigh-wave phase velocities for the B models.

these error estimates are still valid (Nolet & Dost, private communication 1993).

The difference in phase velocities between the models B1 and B2 (Fig. 6) nowhere really exceeds the error level of 1 per cent. It is clear that it is very difficult to distinguish between the B models with the use of phase velocities alone. The differences between the phase velocities of the A models are also small (Fig. 5) except for the higher modes at short periods, where the differences clearly exceed the estimated observational error. The phase velocities of higher modes at higher frequencies could allow us to distinguish the models. Unfortunately, it is very difficult to obtain accurate measurements of higher mode phase velocities for short periods. This is mainly due to the fact that higher modes are channelled waves in the LVL at higher frequencies. As a check we calculated and plotted the depth-dependent normalized amplitude of the surface-wave eigenfunctions. This shows that the higher modes for our model are alternately of the channelled and crustal type at short periods. For instance the first higher mode is channelled for periods between 6 and 12 s. This means that its energy is concentrated strongly in and around the LVL and that amplitudes at the surface are negligible. The channelled modes we found are the same as those shown in Panza *et al.* (1972), who states that these modes should be neglected in an inversion procedure because they can hardly be measured. Given the fact that the channelled modes cannot be detected at the surface, it is unlikely that it is possible to distinguish our A and B models with phase-velocity data, unless the experimental error could be significantly reduced. However, one should note that the differences between the phase-velocity curves show a systematic pattern and do not behave like random errors. This could potentially allow us to resolve the different LVLs despite the small differences in the dispersion curves.

To verify this we performed several inversions with synthetic data. By doing this we also address the question of whether a linear inversion of phase velocities is capable, apart from observational errors, of resolving LVL structures at all. To invert phase velocities we use a linear-inversion technique developed by Nolet (1981) that employs the formulation of Wiggins (1972) and Jackson (1972). For reasons of brevity we only inverted Love-wave dispersion data. Resolution kernels of Love and Rayleigh waves are approximately the same (Der, Massé & Landisman 1970), which indicates that the resolving power of Love and Rayleigh waves is similar. In our linearized inversion we use an eigenvalue decomposition with a sharp eigenvector cut-off. The cut-off, which determines the number of eigenvectors used in the inversion, also defines the resolution and the variance of the solution. The variance associated with each eigenvector is proportional to $1/\lambda_j$ where λ_j is the eigenvalue that corresponds to an eigenvector $\mathbf{u}_j$. Therefore, the smaller the eigenvalues of eigenvectors used in the inversion, the larger the variance in the solution.

In our inversions we use exact phase velocities, that are directly calculated from the 'real' model. Real data always contains errors. To quantify the dependence of the resulting models on measurement errors, we put an imaginary measurement error of 40 m s^{-1} into the inversion algorithm. From the linearized mapping the standard deviation in the model due to this error is determined. This produces an estimate of the band wherein the inversion result will lie when real data with errors of this magnitude are used. The data points that we used in the inversions are indicated in Fig. 5 with black dots. This is a realistic data set that corresponds to the data set obtained by Dost (1987) for the lowest three modes with the use of the NARS array in Western Europe. The error of 40 m s^{-1}, which we assumed in the phase velocities at all periods amounts to somewhat less than 1 per cent, which is very optimistic. In Fig. 7(a) the inversion results are shown. A1 is used as starting model and A2 as true model. This corresponds to a situation where A1 is our best guess as derived from traveltimes and A2 represents the real velocity distribution in the earth. Results

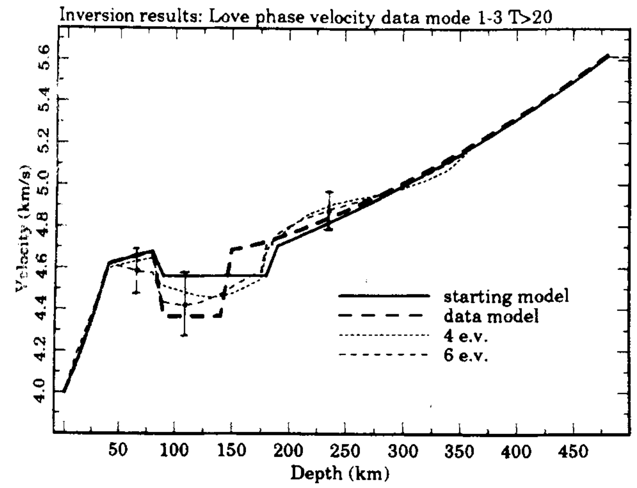

Figure 7(a). Results of linear inversions for the *S* velocity of a synthetic realistic data set of Love-wave phase velocities. A1 is used as a starting model and A2 as a true model. Results are shown for two numbers of eigenvectors. The error bars indicate the standard deviation in the inversion result with six eigenvectors.

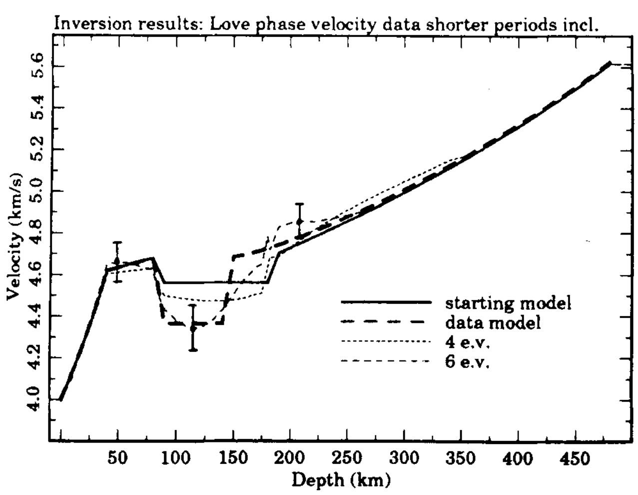

Figure 7(b). As Fig. 7(a), but with the data set extended to shorter periods.

are presented of inversions where the updated model was constructed with four or six eigenvectors. In these inversions only one linear inversion step is used and no iterations have been done. By iteration we mean a recomputation of the modes for the perturbed model and after that applying the linear inversion once again. The standard deviation in the solution with six eigenvectors, as calculated from the linearized mapping, is indicated by the error bars in Fig. 7(a). As mentioned above, these error bars indicate the band wherein, with about 66 per cent probability, the inversion result will lie when real data with errors of approximately 1 per cent are used. The error in the inversion result with four eigenvectors is about two-thirds of that indicated by the error bars. The inversion perturbs the initial model in the right direction, but the updated models differ significantly from the true model. Moreover, the standard deviation is large compared to the model update. When more eigenvectors are added the solution improves only slightly, but the variance increases rapidly. Therefore we can state that with this dispersion data set it is almost impossible to retrieve the true model in a statistically significant way and remove the non-uniqueness caused by a LVL.

The differences between the two dispersion curves for A1 and A2 are largest at periods below 20 s. These periods were not included in the realistic data set discussed above. In order to investigate whether it would be useful to try to measure phase velocities at these short periods we also performed inversions with a larger data set. We extended the dataset shown in Fig. 5, to periods as short as 10 s for the fundamental and second higher mode and 14 s for the first higher mode. It may be surprising that we do not include periods smaller than 14 s for the higher mode, since this mode exhibits obviously the largest sensitivity. There are two reasons to exclude shorter periods. First, phase velocities at periods smaller than 14 s are very hard to measure at the surface, because the modes become increasingly channelled in the LVL. Secondly, when the dispersion data at higher frequencies was added to our data set, the variance in the solution tended to increase. This appears to be due to the fact that the phase velocities of channelled modes like the first higher mode at these periods are not very well suited for linear-inversion techniques that rely on Rayleighs principle. Rayleigh's principle formulates the fact that the change in the dispersion curve does not depend to first order on the change of the modal structure. It is valid only when the modes are not degenerate. For the higher modes at periods below 10 s, the dispersion curves exhibit kissing points as can be seen in Fig. 5. At these points the modes are nearly degenerate and here Rayleigh's principle should be extended to the subspace of the nearly degenerate modes. A detailed analysis of the modal structure has shown that the modal structure for these modes is extremely sensitive to the details of the LVL, which signals a limited applicability of Rayleigh's principle for higher modes at shorter periods. This is consistent with the observation of Pěč & Novotný (1976) that the partial derivatives of the higher modes at higher frequencies are much more sensitive to an LVL than those of the fundamental mode.

The results of the inversions with the extended data set are shown in Fig. 7(b). It is evident that adding shorter periods to the data set improves the resolving power considerably, when six eigenvectors are taken into account. In the inversion shown in Fig. 7(b), the number of eigenvectors is clearly the main limiting factor. The solution is strongly dependent on the number of eigenvectors that is used in the inversion. When six eigenvectors are used, model A2 is reasonably well resolved, whereas when four eigenvectors are used, the inversion result resembles the true model only little more than the starting model does. The error bars indicate, as in Fig. 7(a), one standard deviation in the solution with six eigenvectors. Unfortunately, the errors are about half of the model update so that the statistical significance of the resolved LVL is questionable. The inversions indicate that the information concerning the LVL is present in our data set, but that it is related to small eigenvalues in the inversion. When the eigenvectors corresponding to these eigenvalues are used in an inversion they magnify the observational errors by which

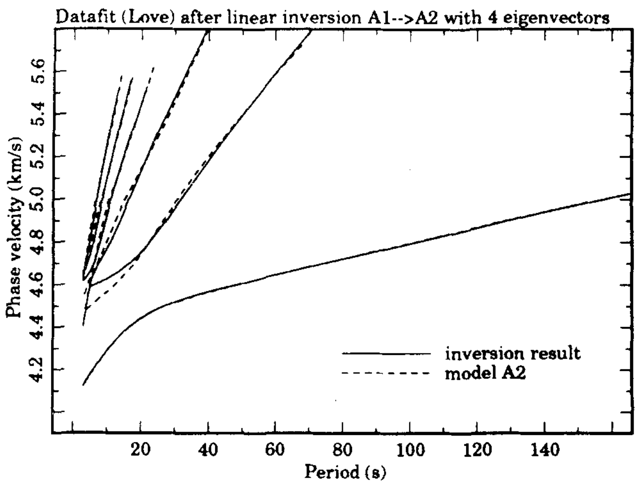

Figure 8. The data fit obtained from the inversion of the extended data set for the A models with four eigenvectors. The data fit is almost perfect for the modes and periods that are used in the inversion.

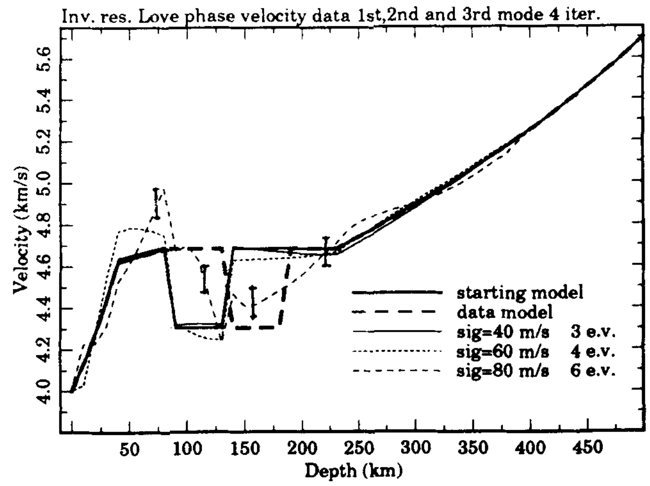

Figure 9. As Figs 7. Results of linear inversions for the B models. B1 is used as starting model and B2 as true model. The error bars indicate the standard deviation in the inversion with six eigenvectors.

the information that the small eigenvectors 'contain' is obscured. It is clear that if we want to resolve the structure of the LVL in a statistically significant way we need observational errors considerably smaller than 1 per cent. In Fig. 8 the data fit after the linear inversion of the extended data set with four eigenvectors is displayed. The data fit is almost perfect for the employed period range and it clear that if we want to reject this reconstructed model based on phase velocities we would need experimental errors considerably smaller than 1 per cent.

Using higher modes in an inversion increases the resolving power (Der *et al.* 1970; Nolet 1977). We performed inversions with only the fundamental mode, with the fundamental and the first higher mode, and with the fundamental mode, the first higher mode and the second higher mode. As expected, the resolving power is larger when all three modes are used. When only the fundamental mode or the fundamental and the first higher mode are used, the LVL is not well resolved, no matter how many eigenvectors are used; if too many eigenvectors are used, the solution is unstable and if less eigenvectors are used the LVL is not resolved. Therefore, we chose to use the fundamental and the first two higher modes. Adding more higher modes might improve the solution somewhat, but one has to bear in mind that the reliability of higher-mode data, if it is available at all, is considerably less than that of fundamental-mode observations.

We also performed inversions with the B models. In these inversions B1 was used as starting model and B2 as true model. The data set used in these inversions consists of phase velocities for periods down to 10 s, that are similarly distributed as the points in the extended data set used for the A models. In Fig. 9, the inversion results are shown with different numbers of eigenvectors. The approximate standard deviation in each solution is indicated in the legend. The error bars indicate one standard deviation in the solution obtained with six eigenvectors. The results are obtained by iterating the linear inversion four times. The data fit has improved considerably by the inversions; the inversion with six eigenvectors gives a variance reduction of more than 95 per cent, which is even better than the data fit shown in Fig. 8. In spite of this very good data fit, the model fit is relatively poor, even when six eigenvectors are used. Using more than six eigenvectors results in unacceptably large errors in the reconstructed model. A more detailed analysis indicates that the results for the B models are poor mainly because the employed linearization is inadequate and not because resolution is insufficient. When a realistic data set without the short periods is used, the reconstruction of the LVL is even worse than in the example of Fig. 9. It is clear that the inversions for both the A and B models give a very good fit to the data, but as shown in the example of Fig. 8, a good fit of phase velocities does not necessarily imply that the detailed structure of the LVL is resolved. Although the LVL itself is clearly present in all updated models, the error level of phase velocity measurements must be significantly less than 1 per cent in order to considerably reduce the range of models that fits the data within the errors and to resolve the details of the LVL. Finally it is evident that short-period data contribute considerably to the resolving power of an inversion.

5 GROUP VELOCITIES

Group velocities are in general more sensitive to the velocity structure than phase velocities. Group velocity curves of the A and B models are shown in Figs 10 and 11. The group velocities of the fundamental mode and the first higher mode are both presented, although in general it is only possible to measure the group velocity of the fundamental mode. Again, because Rayleigh and Love group velocities of the models exhibit similar behaviour, only one of two is shown for each model. With differences up to $100\,\mathrm{m\,s^{-1}}$ for the fundamental mode it is evident that the group velocities are indeed more sensitive to changes in the LVL than the phase velocities. Group velocities can be measured with approximately the same accuracy as phase velocities, which is indicated by the error bars of $40\,\mathrm{m\,s^{-1}}$, which corresponds

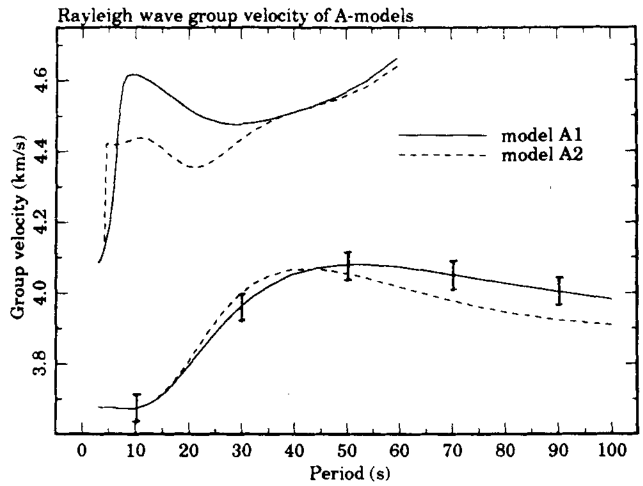

Figure 10. Rayleigh-wave group velocities of the A models. The error bar indicates a realistic experimental error.

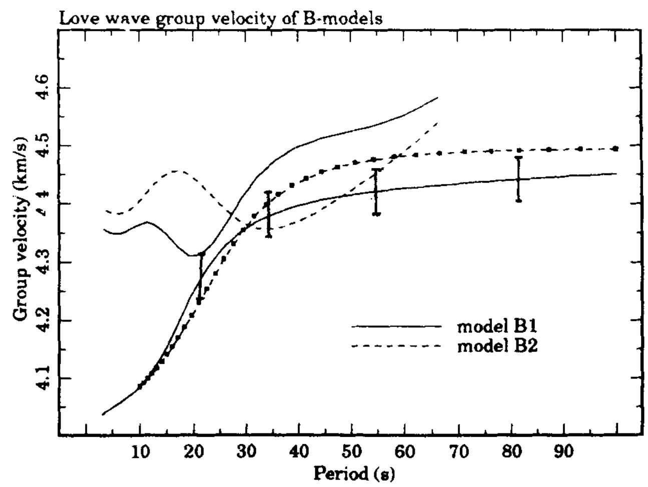

Figure 11. Love-wave group velocities of the B models. A realistic experimental error is indicated by the error bars. The points indicate the data set used in the linear inversion.

to a relative error of approximately 1 per cent. It is clear that an inversion of group-velocity data might resolve the LVL with a larger statistical significance than an inversion of phase velocities. Unfortunately the resolution in an inversion of group velocities is in general rather poor since it is normally impossible to measure group velocities of higher modes directly. This is mainly caused by the fact that the group velocities of the higher modes are very similar, which leads to interfering higher-mode arrivals that cannot be separated unless array techniques are used (Cara 1978; Dost 1990). To obtain a more quantitative view, we inverted synthetic Love-wave group velocities. Given the resemblance of the resolution kernels for Love and Rayleigh waves (Der *et al.* 1970), the results will be similar for inversions of Rayleigh-wave group velocities. In the inversions A1 is used as a starting model and A2 is used as a true model. The data points are equally distributed as those shown in Fig. 11 for the B models. The linear-inversion technique and the calculation of the variance in the inversion results are the same as used in the inversion of phase velocities.

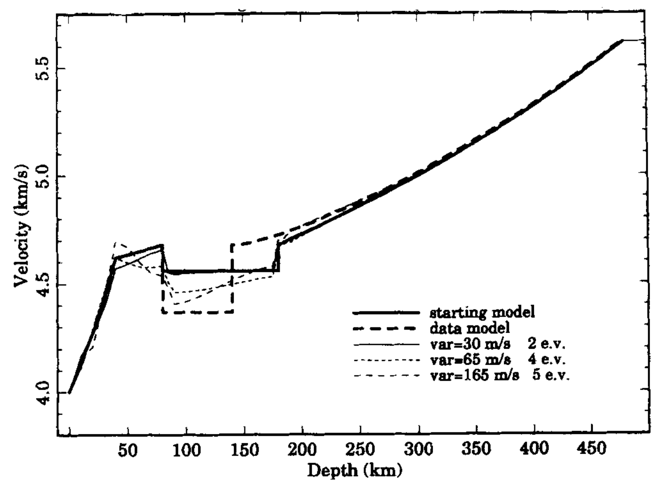

Figure 12. Results of inversions of fundamental-mode Love-group velocities for the A models. Results with different numbers of eigenvectors are shown. The standard deviation is indicated by 'var ='.

In Fig. 12, inversion results are shown for three different numbers of eigenvectors. The true model A2 is not well resolved, although the updated model resembles the true model somewhat more than the starting model A1 does. Note that when five eigenvectors are used, the standard deviation of the reconstructed model ($165\,\mathrm{m\,s^{-1}}$) is of the same order as the model update in the inversion. Using more eigenvectors in the inversion does not improve the solution and moreover the corresponding eigenvalues become so small that the variance increases dramatically, which makes the inversion unstable. Iterations do not improve the result. Therefore, it is not possible to constrain the detailed structure of the LVL with surface-wave group velocities in a satisfactory way. This is due to the fact that the resolving power of an inversion based solely on the fundamental mode is too small. Considering the data fit in Fig. 13, one can see that the fit after inversion is almost perfect, although the true model and the reconstructed

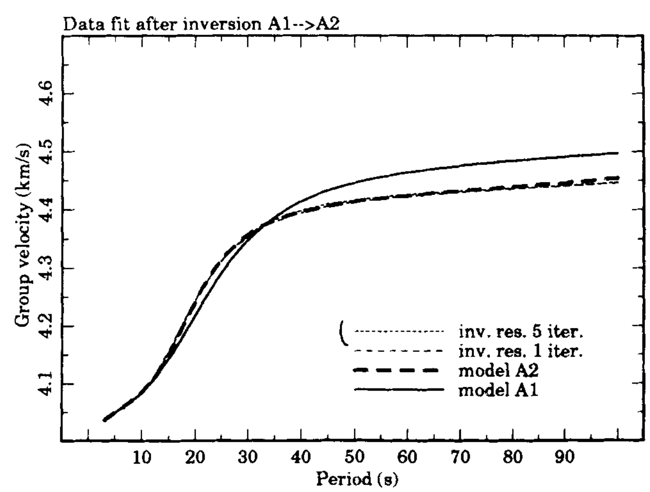

Figure 13. Data fit after the inversion with five eigenvectors for the A models.

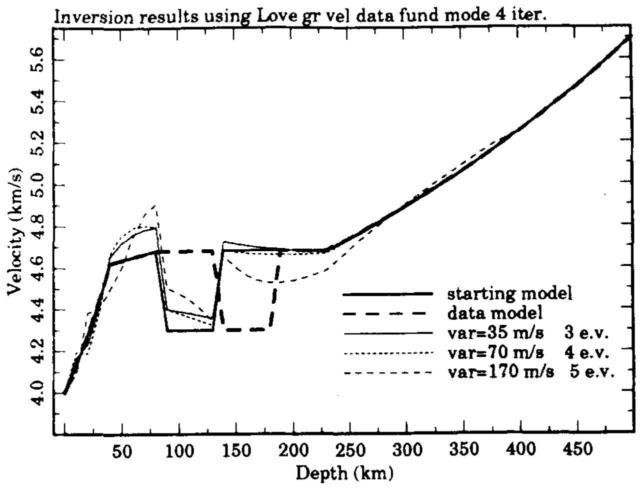

Figure 14. As Fig. 12, but for the B models. The solutions were obtained after four iterations. The standard deviation is indicated by 'var ='.

model are quite different. This shows a practical non-uniqueness that, given the fact that group velocities are a derivative property of phase velocities may well be related to the theoretical non-uniqueness Gerver & Kazhdan (1972) found for fundamental-mode phase velocities.

For the B models the results of inversions of group velocities with the data set indicated in Fig. 11 is shown in Fig. 14. The results, obtained after four iterations, are even worse than that for the A models; the data fit is almost perfect and comparable to that in Fig. 13, but the updated model does not resemble the true model much more than the starting model. To conclude, we can state that it is very difficult to resolve the ambiguity in the structure of the LVL using fundamental-mode group velocities for both A and B models. The fact that the models obtained in the inversions give such a good data fit indicates that for practical purposes a wide range of models with different LVLs will provide an equally acceptable data fit.

6 DISCUSSION

The first conclusion one can draw from the synthetic examples is that surface waves do not suffer from the same non-uniqueness for variations in a LVL as body waves. Waveforms, phase velocities and group velocities of models that result in the same traveltimes are different, but the differences are not very large. Waveforms show only small differences for shallow sources. Although the differences are larger for deeper sources it is questionable if a waveform inversion, which is also influenced by lateral heterogeneity and an inaccurate knowledge of the seismic source, can resolve the exact shape of an LVL from data that are contaminated with noise. For the realistic data set shown in Fig. 5, the differences between the phase velocities of the models that give the same traveltime curves are so small that with the present-day accuracy of measurement we can hardly resolve the different structures of the LVL as in the A and B models. In the case of the A models, the linear inversion of phase velocities does solve the structure of the LVL rather well when short-period higher-mode phase velocity measurements are added to the realistic data set. However, this result is statistically not very significant when observational errors are taken into account. We doubt whether other inversion techniques will provide a better resolution of the LVL, because the data fit that is obtained with the linear inversion is very good and falls well within realistic experimental errors. The linear inversion of phase velocities does not work well for the B models. The data fit is improved considerably, but the true model is not well resolved. When phase velocities are used, a practical non-uniqueness therefore exists that is caused by the experimental errors and the fact that it impossible to measure phase velocities of all overtones at all periods. Group-velocity inversions suffer from a lack of resolving power, since only the group velocities of the fundamental mode can be measured. In the numerical examples, the group velocities of the fundamental mode are almost perfectly fitted. Even if it were possible to increase the accuracy in our measurements enormously, there will be a wide range of models that give an equally good data fit. Therefore, it is clear that inversion of fundamental-mode group velocities alone will not help very much to remove the non-uniqueness of traveltime inversions, since this data type also suffers from practical non-uniqueness. Finally, it is important to realize that the starting model in our inversions is very good and that it was assumed that the density was exactly known. This is generally not true for inversions of real data, which further complicates the determination of the structure of the LVL. Although our experiments indicate that the existence of an LVL can be established, it is clear that the ambiguity in the structure of a LVL that is present for traveltimes of body waves cannot be entirely removed in a statistically significant way with a realistic set of surface-wave dispersion data. This implies that the inverted LVL is quantitatively not very reliable, which hampers a direct use of obtained seismological models of an LVL in geodynamic studies.

ACKNOWLEDGMENTS

We would like to thank Bernard Dost for many useful discussions. This research was supported by the Netherlands Organization for Scientific Research (NWO) through the Pionier project PGS 76–144.

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