MA 265 Practice Test (Dr. Park)

Name ....

ID number ....

Test 1: Feb 23, 2017

INSTRUCTIONS in the Test

1. **Do not open this exam booklet until told to do so**.

2. **Show all your work - if you need more space, continue on the back of the page for that problem.**

3. **Show your final answer by enclosing it in a box or circle**.

5. The final page is a formula sheet. Please wait until you are instructed to start.

Problem 1. Let A and B be $n \times n$ matrices. Which of the following statements are always TRUE?

(a) $(-A)^T = -A^T$ (b) $(A - B)^T = A^T - B^T$

(c) $(A^T B)^T = AB^T$ (d) $(AB^{-1})^{-1} = A^{-1}B$

(e) $AB = BA$

Problem 2. Let A, B and C be arbitrary $n \times n$ matrices. What is the transpose of the matrix?

$$(A^T + 2017B)C^{-1}$$

Problem 3. Let A be a nonsingular matrix with its inverse

$$A^{-1} = \begin{pmatrix} 1 & 2 \\ 2 & 3 \end{pmatrix}$$

Which of the following statements is FALSE?

(a) For arbitrary 2×2 matrices B and C if $BA = BC$, then $A = C$.
(b) A^T is invertible.
(c) For arbitrary 2×2 matrices B and C if $AB = AC$, then $B = C$.
(d) $AA^{-1} = A^{-1}A$.
(e) A is symmetric.

Problem 4. Assume that the matrix

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 2 \\ 2 & a & 1 \end{bmatrix}$$

is invertible. What is the $(2,1)$-entry of the inverse of A?

Problem 5. Given constants a and b, consider the linear system

$$\begin{aligned} 9x + 2y &= a \\ 4x + y &= b. \end{aligned}$$

Find the solution of the linear system by using Cramer's rule.

Problem 6. For what values of h and k does the system $A\mathbf{x} = \mathbf{b}$ have infinitely many solutions?

$$A = \begin{bmatrix} 1 & 1 & 4 \\ -3 & -3 & h \\ 1 & 8 & 0 \end{bmatrix}, \qquad \mathbf{b} = \begin{bmatrix} -2 \\ k \\ 0 \end{bmatrix}.$$

Problem 7. Let A, B and C be invertible $n \times n$ matrices. If $A^{-1}B^{-1} = C^{-1}$, then what is A?

(a) $A = CB^{-1}$ (b) $A = C^{-1}B^{-1}$ (c) $A = BC^{-1}$ (d) $A = B^{-1}C$
(e) $A = BC$

Problem 8. Which of the following statements are true?

(i) A linear system of four equations in three unkowns is always inconsistent.
(ii) A linear system with fewer equations than unkowns must have infinitely many solutions.
(iii) If the system $A\mathbf{X} = \mathbf{b}$ has a unique solution, then A must be a square matrix.
(iv) For a square matrix A if $A\mathbf{X} = \mathbf{0}$ has a nonzero solution, then the homogeneous linear system has infinitely many solutions.

(a) all of them (b) (i) and (ii) (c) (ii) and (iii) (d) (iii) and (iv)
(e) (iv) only.

Problem 9. Find the solution (x_1, x_2, x_3) of the linear system of equations

$$\begin{aligned} x_1 + 3x_2 + x_3 &= 1 \\ 2x_1 + 4x_2 + 7x_3 &= 2 \\ 3x_1 + 10x_2 + 5x_3 &= 7 \end{aligned}$$

Problem 10. Given that

$$\det \begin{bmatrix} a_1 & a_2 & a_3 & a_4 \\ b_1 & b_2 & b_3 & b_4 \\ c_1 & c_2 & c_3 & c_4 \\ d_1 & d_2 & d_3 & d_4 \end{bmatrix} = 4,$$

what is the determinant of

$$\det \begin{bmatrix} a_4 & a_2 & a_3 & a_1 \\ b_4 & b_2 & b_3 & b_1 \\ 2c_4 + 3a_4 & 2c_2 + 3a_2 & 2c_3 + 3a_3 & 2c_1 + 3a_1 \\ d_4 & d_2 & d_3 & d_1 \end{bmatrix} ?$$

Problem 11. If A is a 3×3 matrix with $\det A = 5$ and $B = 2A$, then what is $\det\left(A^T B^{-1}\right)$?

Problem 12. Find the values of α for which A is singular:

$$A = \begin{bmatrix} 2 & 1 & 3\alpha & 4 \\ 0 & \alpha - 1 & 4 & 0 \\ 0 & 0 & 2 & 1 \\ 0 & 0 & \alpha & 4 \end{bmatrix}.$$

Problem 13. Let $\mathcal{P}_3$ be the vector space of all polynomials of degree ≤ 3. Which of the following set(s) is(are) subspace(s) of $\mathcal{P}_3$?

(a) $\{1 + t^2\}$

(b) $\{at + bt^2 + (a+b)t^3 \in \mathcal{P}_3 : \ a \text{ and } b \text{ are real numbers}\}$

(c) $\{a + bt + abt^2 \in \mathcal{P}_3 : \ a \text{ and } b \text{ are real numbers}\}$

(d) $\{p(t) \in \mathcal{P}_3 : \ p(2) = 0\}$

Problem 14. Let V be the set of all strictly positive real numbers, and let $\oplus$ and $\odot$ be defined by

$\mathbf{a} \oplus \mathbf{b} = ab$ for any $\mathbf{a}, \mathbf{b} \in V$ (that is, a, b are strickly positive real numbers)
$c \odot \mathbf{a} = a^c$ for any $\mathbf{a} \in V$ and real number c

Which of the following statement(s) is(are) FALSE?

(a) V is closed under the two operations $\oplus$ and $\odot$.

(b) for any $\mathbf{a}, \mathbf{b} \in V$ and any real number c, $c \odot (\mathbf{a} \oplus \mathbf{b}) \in V$

(c) Under the operation $\oplus$, the $\mathbf{0}$ element (the identity of $\oplus$) is the real number 1.

(d) There is at least one element $\mathbf{a}$ in V for which there is no element $-\mathbf{a}$ (the inverse of $\mathbf{a}$) such that $-\mathbf{a} \oplus \mathbf{a} = \mathbf{0}$.

(e) V is a vector space with the two operations.

Problem 15. Let A be an $n \times n$ matrix, and let $\mathbf{R}^n$ be the vector space of all the vectors in the n-dimensional space. Prove that the following subsets of $\mathbf{R}^n$ are its subspaces.

(a) $\mathbf{U} = \{\mathbf{X} \in \mathbf{R}^n : A\mathbf{X} = \mathbf{0}\}$ is the set of all the solutions of the homogeneous system $A\mathbf{X} = \mathbf{0}$.

(b) $\mathbf{W} = \{\mathbf{X} \in \mathbf{R}^n : \mathbf{X}$ is a linear combination of the columns of $A\}$ is the set of all the vectors that can be written as a linear combination of the columns of A.

Problem 16. Let V be the set of all real numbers. We define $\oplus$ by $\mathbf{u} \oplus \mathbf{v} = \mathbf{uv}$, and define $\odot$ by $c \odot \mathbf{u} = c + \mathbf{u}$. Is V a vector space? If it is, prove it. If it is not, show that which property out of the nine properties is not satisfied?