ECE 201, Section 3
Lecture 10

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Linearity Theorem

- For linear resistive circuits, output voltages and currents are a linear combination of independent sources, i.e.:

$$V_A = \sum_{k=1}^{N} [\alpha_k V_k + \beta_k I_k]$$

$$I_A = \sum_{k=1}^{N} [\alpha_k V_k + \beta_k I_k]$$

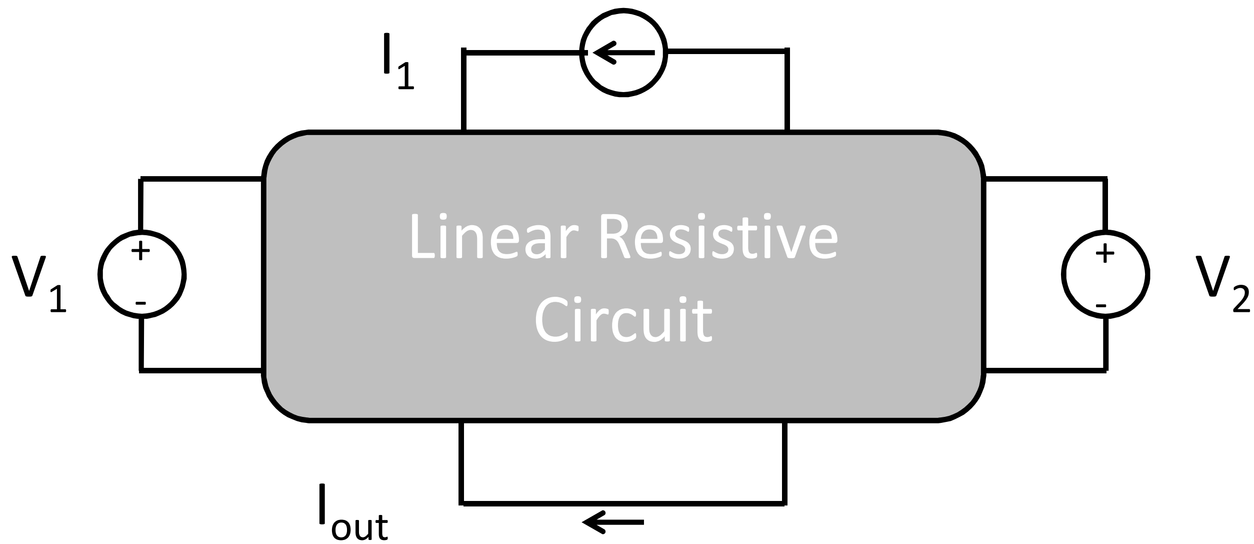
Superposition Property

- Superposition property: total output of linear circuit is sum of contributions from each independent source
- Special case of linearity property
- Applies to current and voltage but not power (in DC circuits)

Proportionality Property

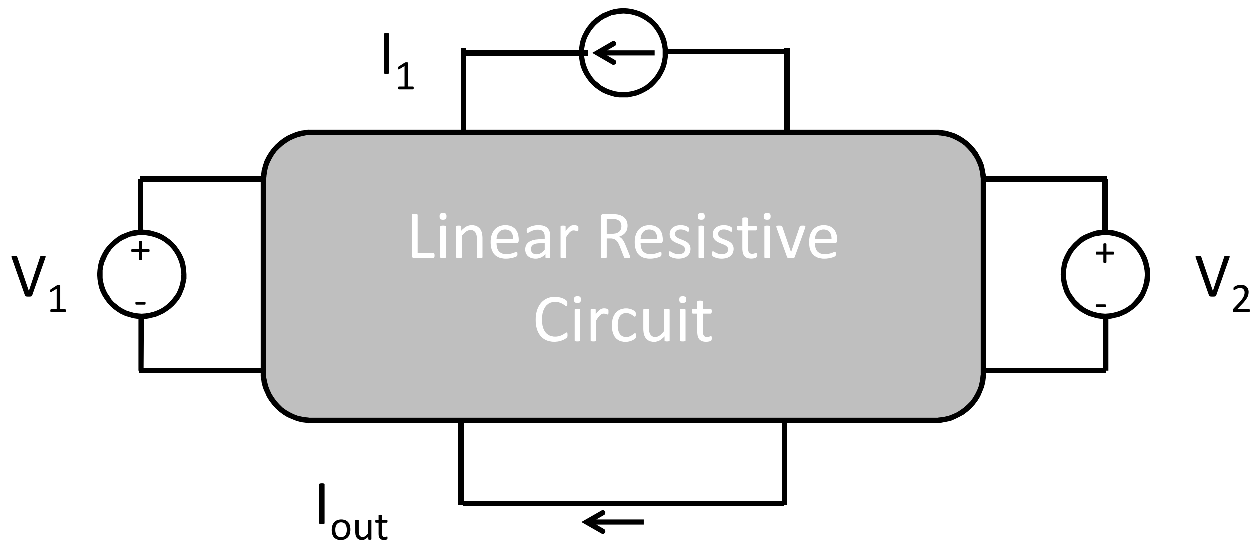
- Proportionality property: multiplying independent source amplitude by α changes corresponding output term by factor of α
- Again, applies to current and voltage but not power (in DC circuits)
- Will lead to some interesting tricks next time!

Example 1: Predicting Outputs



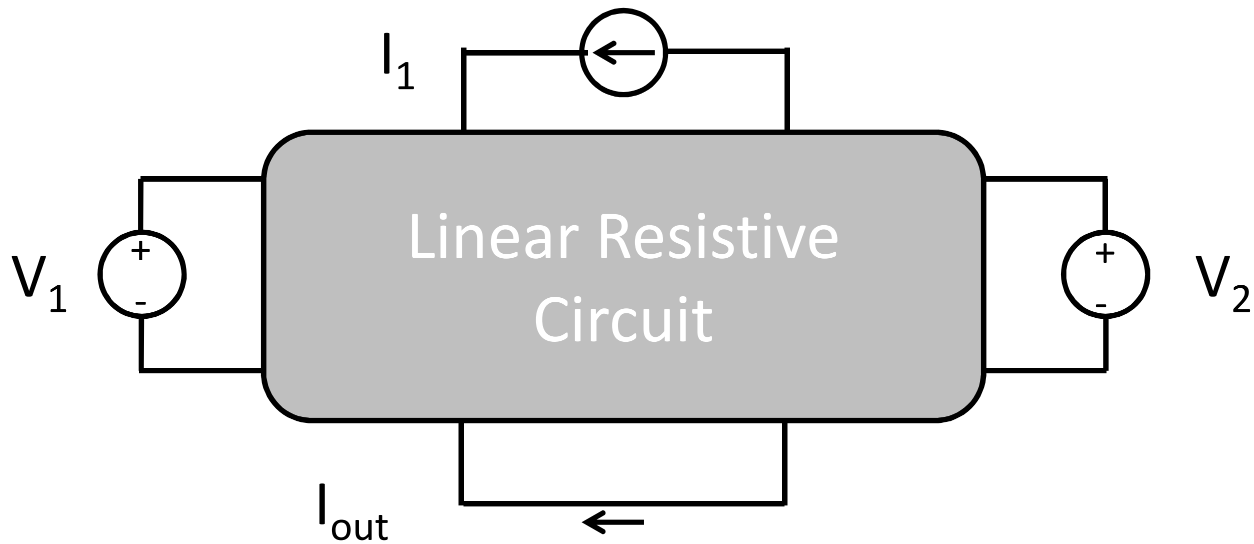
V_1 (V)	V_2 (V)	I_1 (A)	I_{out} (A)
8	0	0	4
0	6	0	12
0	0	3	21
4	2	6	??

Example 1: Solution



V_1 (V)	V_2 (V)	I_1 (A)	I_{out} (A)
8	0	0	4
0	6	0	12
0	0	3	21
4	2	6	4/2+12/3+2*21=48

Example 2: Predicting Outputs



V_1 (V)	V_2 (V)	I_1 (A)	I_{out} (A)
6	12	9	12
3	6	4	9
2	8	4	8
6	2	4	??

Hand Calculation Technique

- Can write down the following equations from the results above

$$3\alpha_1 + 6\alpha_2 + 4\beta_1 = 9$$
$$6\alpha_1 + 12\alpha_2 + 9\beta_1 = 12$$

If we multiply the first equation by -2 and add, we readily obtain β_1=-6. This yields:

$$\alpha_1 + 2\alpha_2 = 11$$
$$\alpha_1 + 4\alpha_2 = 16$$

This readily gives us α_2=2.5, α_1=6

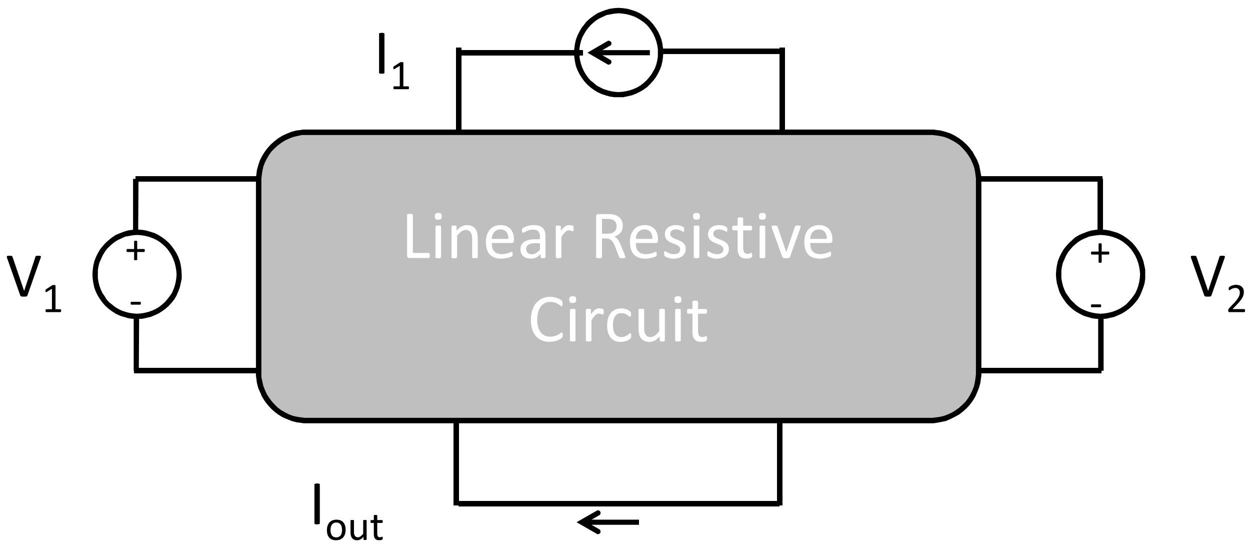
MATLAB Technique

```
>> M=[6 12 9; 3 6 4; 2 8 4]



M =

     6    12     9
     3     6     4
     2     8     4

>> A=[12 9 8]'

A =

    12
     9
     8

>> alpha=M\A

alpha =

    6.0000
    2.5000
   -6.0000
```

MATLAB code

```
>> B=[6 2 4]

B =

     6     2     4

>> B*alpha



ans =

    17
```

Example 2: Solution

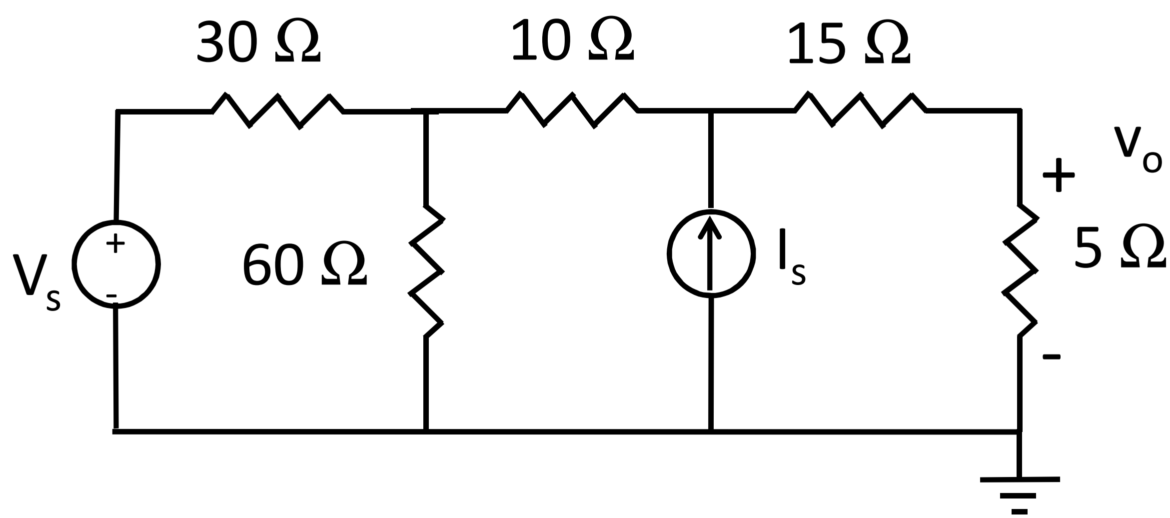


V_1 (V)	V_2 (V)	I_1 (A)	I_{out} (A)
6	12	9	12
3	6	4	9
2	8	4	8
6	2	4	17

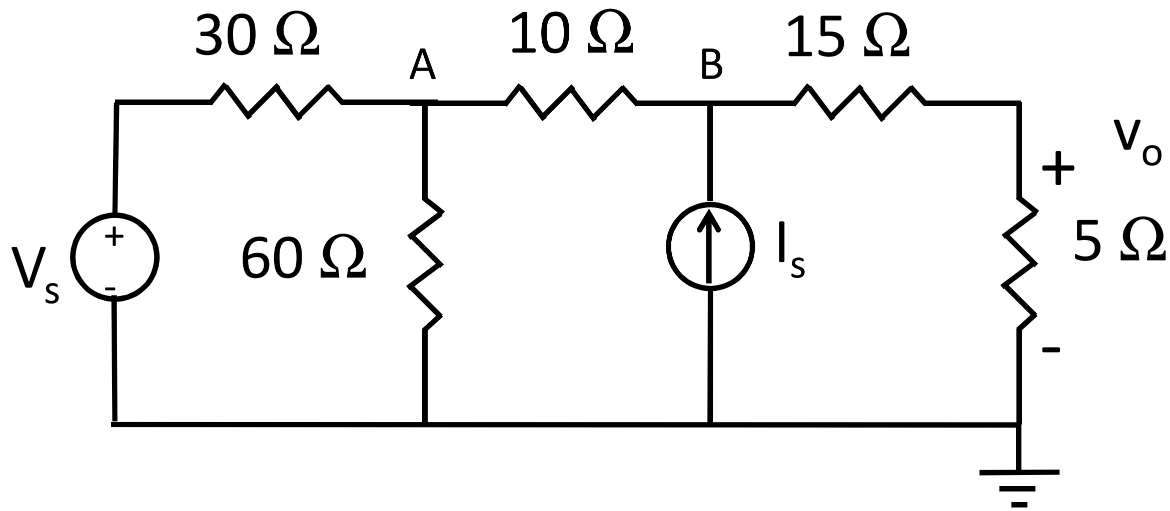
Example 3

- What are the coefficients α and β in:

$$v_o = \alpha V_s + \beta I_s$$



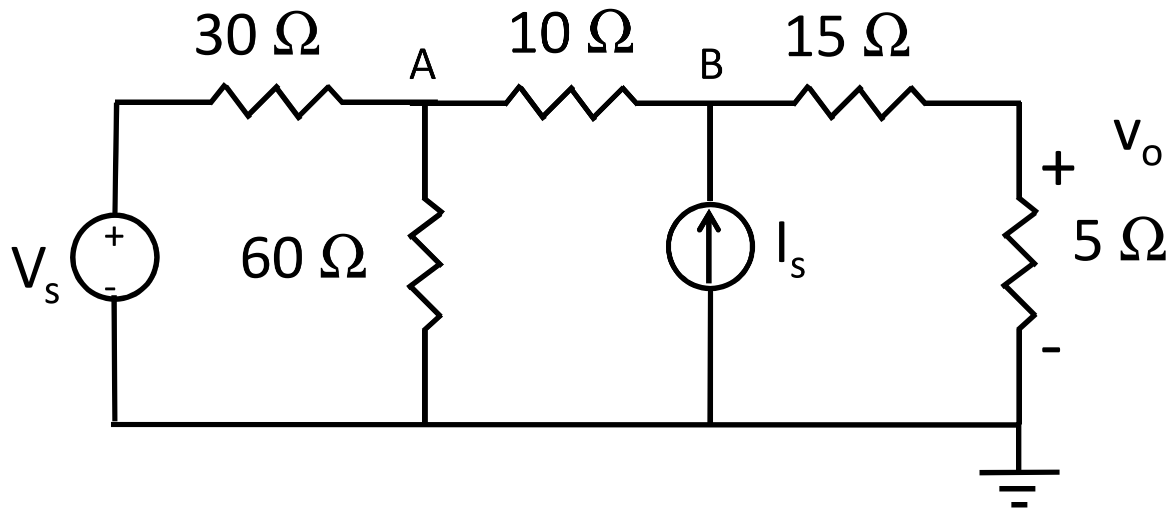
Example 3: Solution



$$\frac{V_s - V_a}{30} = \frac{V_a}{60} + \frac{V_a - V_b}{10}$$

$$I_s + \frac{V_a - V_b}{10} = \frac{V_b}{20}$$

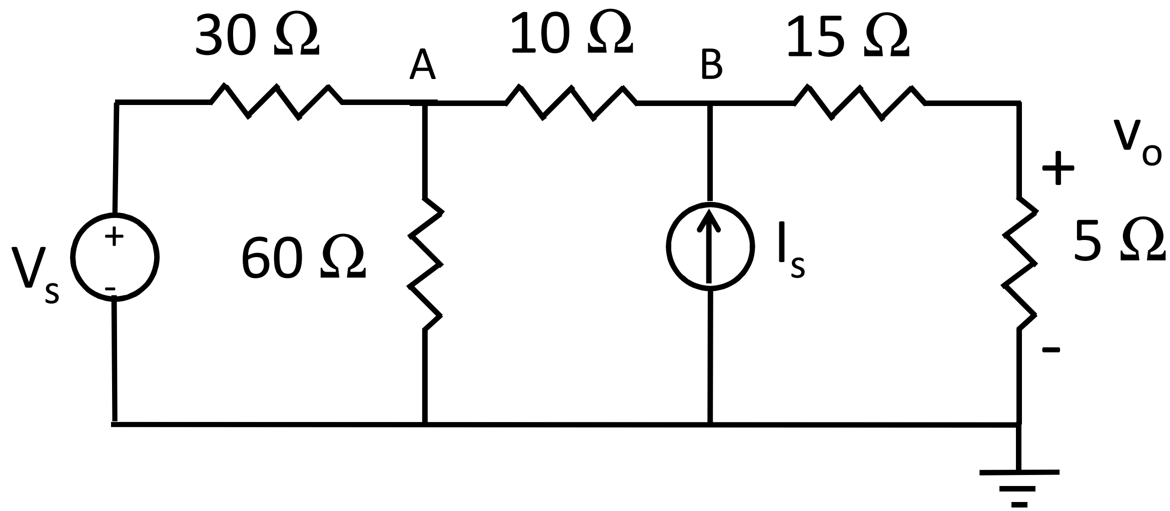
Example 3: Solution



$$\begin{bmatrix} 3/20 & -1/10 \\ -1/10 & 3/20 \end{bmatrix} \begin{bmatrix} V_a \\ V_b \end{bmatrix} = \begin{bmatrix} V_s/30 \\ I_s \end{bmatrix}$$

$$\begin{bmatrix} V_a \\ V_b \end{bmatrix} = \begin{bmatrix} 0.4V_s + 8I_s \\ 4V_s/15 - 12I_s \end{bmatrix}$$

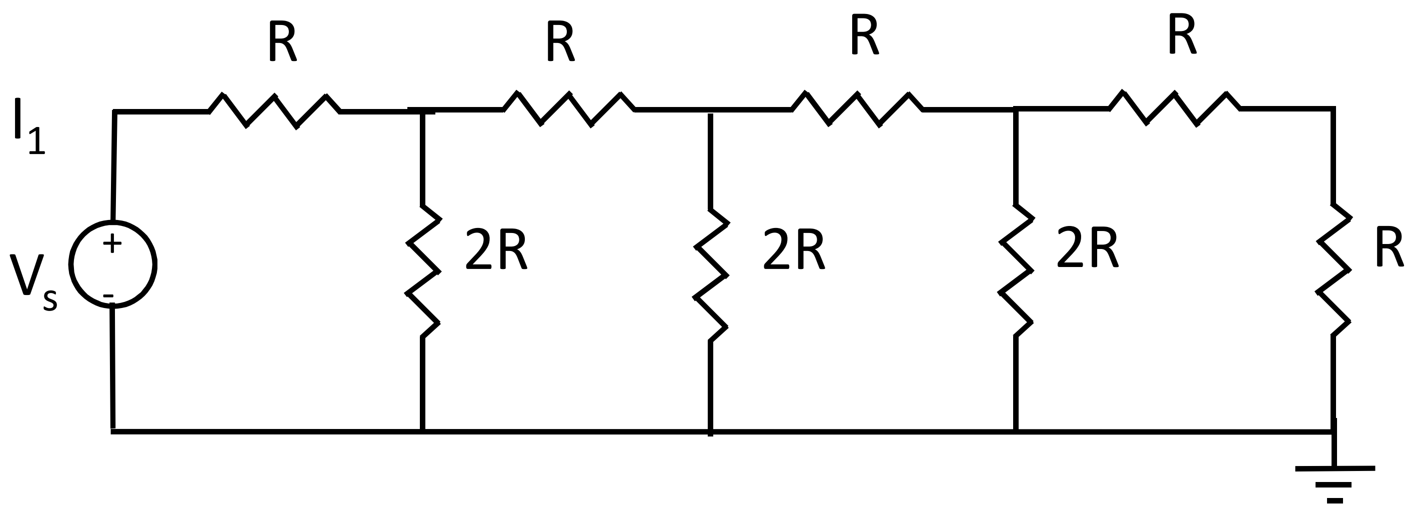
Example 3: Solution



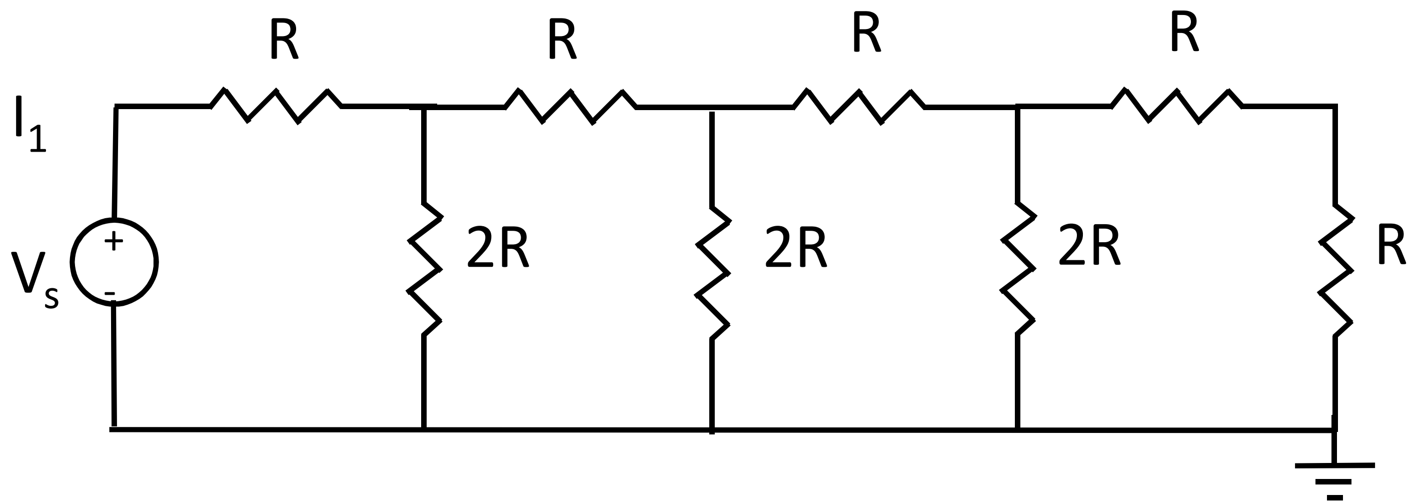
$$v_o = V_s/15 - 3I_s$$

Example 4: Ladder Networks

- Current and voltage everywhere for this 4-loop network? What about *N* loops?



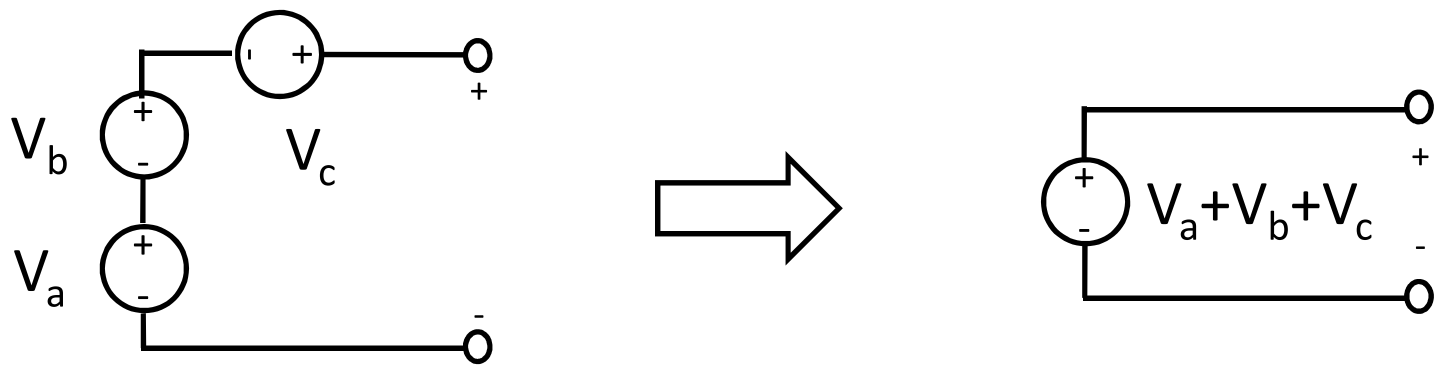
Example 4: Solution



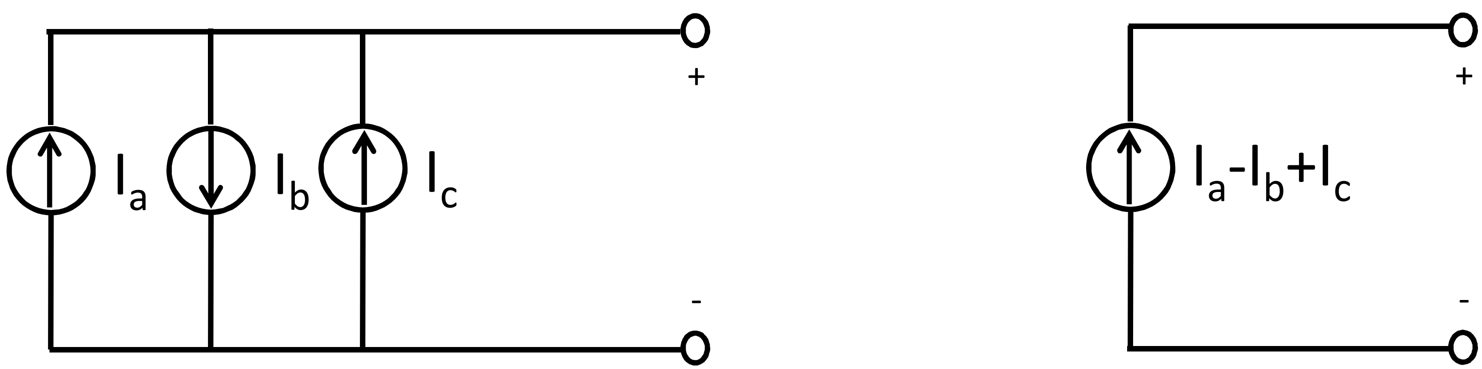
- Assume 1V across resistor at end
- 2V across first loop
- 4V across second loop
- 16 V across fourth loop
- 2^N V with N loops
- Actual voltages: V_s*2^{M-N}

Source Transformations

Combining voltage sources:

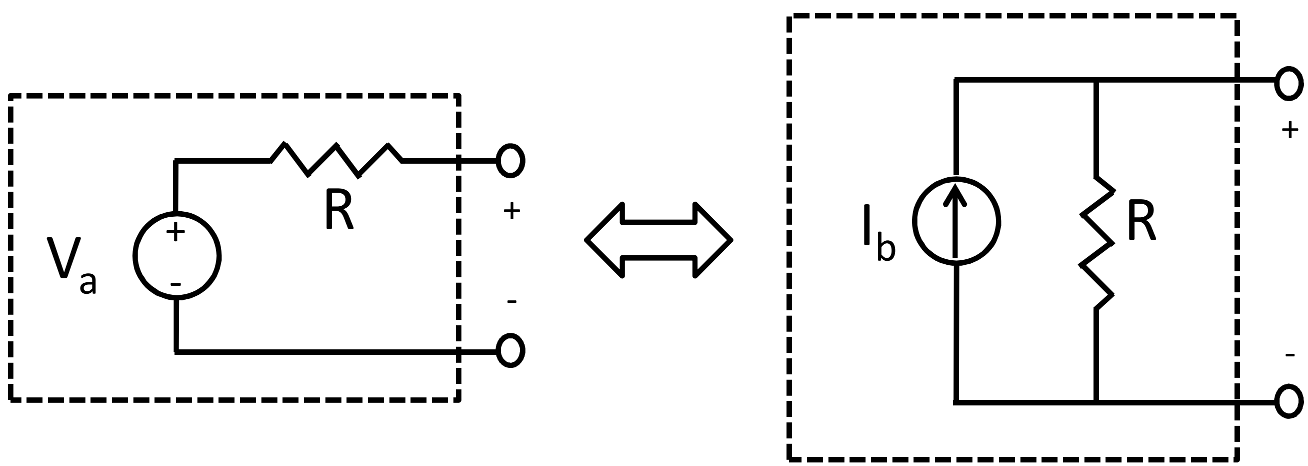


Combining current sources:

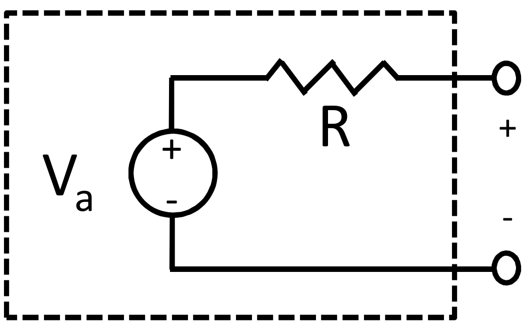


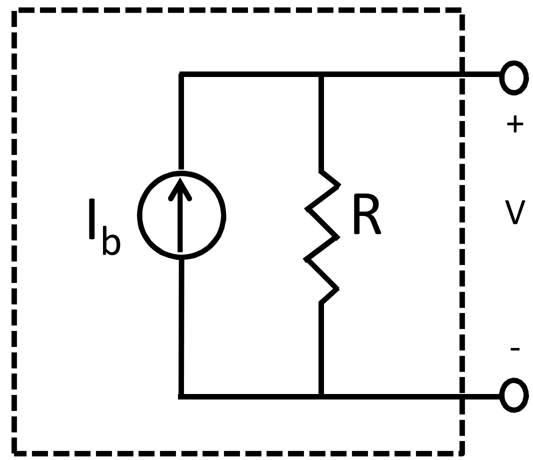
Source Transformation Theorem

- The following 2-terminal networks are equivalent:
  - Voltage source V_a in series with resistor R
  - Current source I_b=V_a/R in parallel with resistor R



Source Transformation





$$V = V_a + IR$$

If $V_a = RI_b, V = R(I_b + I)$

$$I = V/R - I_b$$

$$V = R(I_b + I)$$

Homework

- HW #9 due today by 4:30 pm at EE 325B
- HW #10 due Friday: DeCarlo & Lin, Chapter 5:
  - Problem 3
  - Problem 8(a)
  - Problem 22(a)