Example: Ladder Networks

- Current and voltage everywhere for this 4-loop network? What about $N$ loops?
Example Solution

- Assume 1V across resistor at end
- 2V across first loop
- 4V across second loop
- 16 V across fourth loop
- \(2^N\) V with N loops
- Actual voltages: \(V_s \cdot 2^{M-N}\)
Source Transformations

Combining voltage sources:

\[ V_a + V_b + V_c \]

Combining current sources:

\[ I_a - I_b + I_c \]
Source Transformation Theorem

• These 2-terminal networks are equivalent:
  – Voltage source \( V_a \) in series with resistor \( R \)
  – Current source \( I_b = V_a / R \) in parallel with resistor \( R \)

\[
V = V_a + IR
\]

If \( V_a = R I_b \),
\[
V = R(I_b + I)
\]

\[
I = \frac{V}{R} - I_b
\]

\[
V = R(I_b + I)
\]
Example 1

• What current flows through the central 3 \( \Omega \) resistor?
Example 1: Solution

Rewrite each of the resistor/voltage source pairs as parallel resistor/current source pairs with currents of $36/12=3\,\text{A}$; $30/6=5\,\text{A}$; $40/40=1\,\text{A}$; $48/24=2\,\text{A}$.
Example 1: Solution

Combine parallel current sources and resistors.

\[ I_L = 3 + 5 = 8 \, \text{A}; \quad I_R = 1 + 2 = 3 \, \text{A} \]

\[ R_L = \left( \frac{1}{6} + \frac{1}{12} \right)^{-1} = 4 \, \Omega; \quad R_R = \left( \frac{1}{24} + \frac{1}{40} \right)^{-1} = 15 \, \Omega \]
Example 1: Solution

Transform back into V-R pairs:
\[ V_L = 32 \text{ V}; \quad V_R = 45 \text{ V} \]
Example 1: Solution

Combine in series:

\[ V = 13 \text{ V}; \ R = 22 \ \Omega \]

\[ I = \frac{V}{R} = 0.59 \ \text{A} \]
Equivalent Networks

• Equivalent networks: distinct 2-terminal circuits which exhibit the same current-voltage relationship

• Examples:

\[ V = 8 + 4I \]

\[ I = \frac{V}{4} - 2 \]
Equivalent Network Examples

\[ I = \frac{V}{4} - 2 \]

\[ V = 8 + 4I \]
Car Battery Problem

What is the minimum value of $V_0$ required to achieve a 50 A starter voltage?
• Source transformation yields

\[ I = \frac{(12 + V_o)}{0.02} \]
\[ R_{eq} = 0.01 \text{ W} \]

\[ V = 6 + 0.5V_o \]

\[ I = 50 = \frac{(6 + 0.5V_o)}{0.21} \Rightarrow V_o = 9 \text{ V} \]
Last Lecture on Exam #1

Monday begins Exam #2 material
Thévenin’s Theorem

• An arbitrary 2-terminal network of independent sources and resistors can be represented by its Thévenin equivalent circuit.
Norton’s Theorem

• An arbitrary 2-terminal network of independent sources and resistors can be represented by its Norton equivalent circuit.
Thevenin and Norton Circuits

• A bare voltage source doesn’t have a Norton equivalent
• A bare current source doesn’t have a Thévenin equivalent
• When both exist, \( V_{oc} = R_{th} I_{sc} \)
• Can also deduce Thévenin resistance from:

\[
R_{th} = \frac{V_{oc}(t)}{I_{sc}(t)}
\]
Homework

• HW #10 due today by 4:30 pm in EE 325B (if no one’s there, leave it in an envelope on the door)
• HW #11 due Monday: DeCarlo & Lin, Chapter 5:
  – Problem 40
  – Problem 42
  – Problem 45