ECE 201, Section 3
Lecture 13

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September 19, 2012

Exam #1 Key Concepts

- Basic concepts: current, voltage, charge, Ohm's law, KCL, KVL, current & voltage division
- Combining resistors in series and parallel
- Resistor networks: node, supernode, loop analysis
- Source transformation

Formula Sheet for Exam 1

$I = dQ/dt$

$V = IR$ or $I = GV$

$R = 1/G$

In series:

$R_{eq} = \Sigma_k R_k$

$V_k = VR_k/R_{eq}$

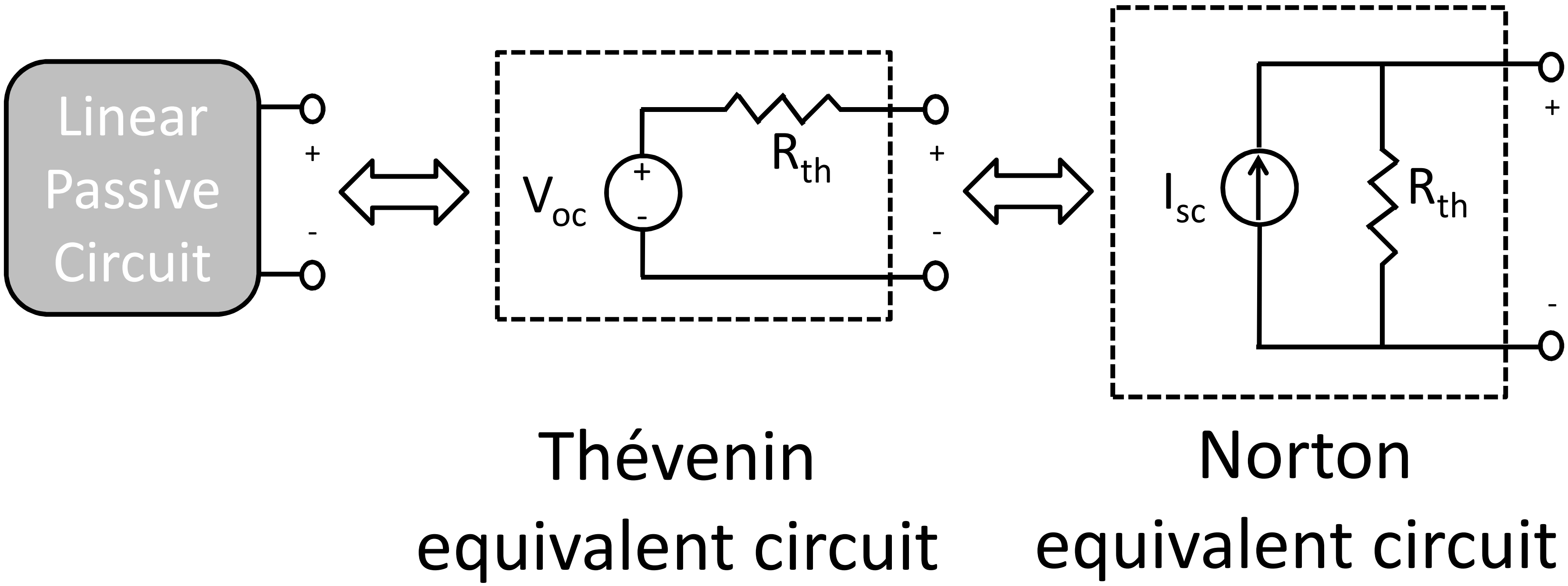
In parallel:

$G_{eq} = \Sigma_k G_k$

$I_k = V/R_k = IR_{eq}/R_k$

$P = IV$

Thevenin and Norton Circuits



Equivalent circuits related by:

$$V_{oc} = I_{sc} R_{th}$$

Example 1: Experimental Data

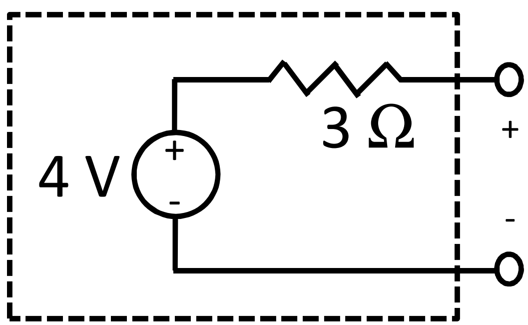
Given these currents and voltages observed at the outputs of a 2-terminal circuit, construct the Thévenin and Norton equivalent circuits.

I_A (A)	V_{AB} (V)
2	10
4	16
8	28

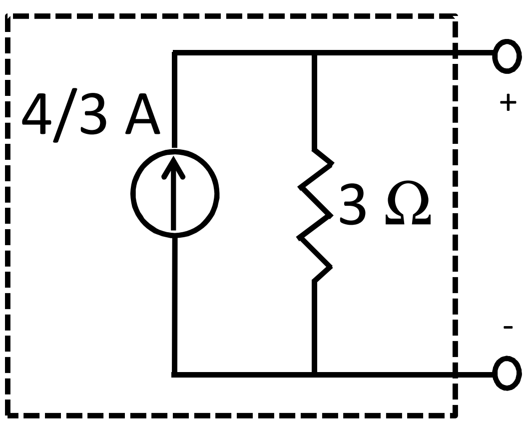
Example 1: Solution

I_A (A)	V_{AB} (V)
2	10
4	16
8	28

Fitting to $V_{AB} = \rho I_A + \nu$ yields ρ=3, ν=4

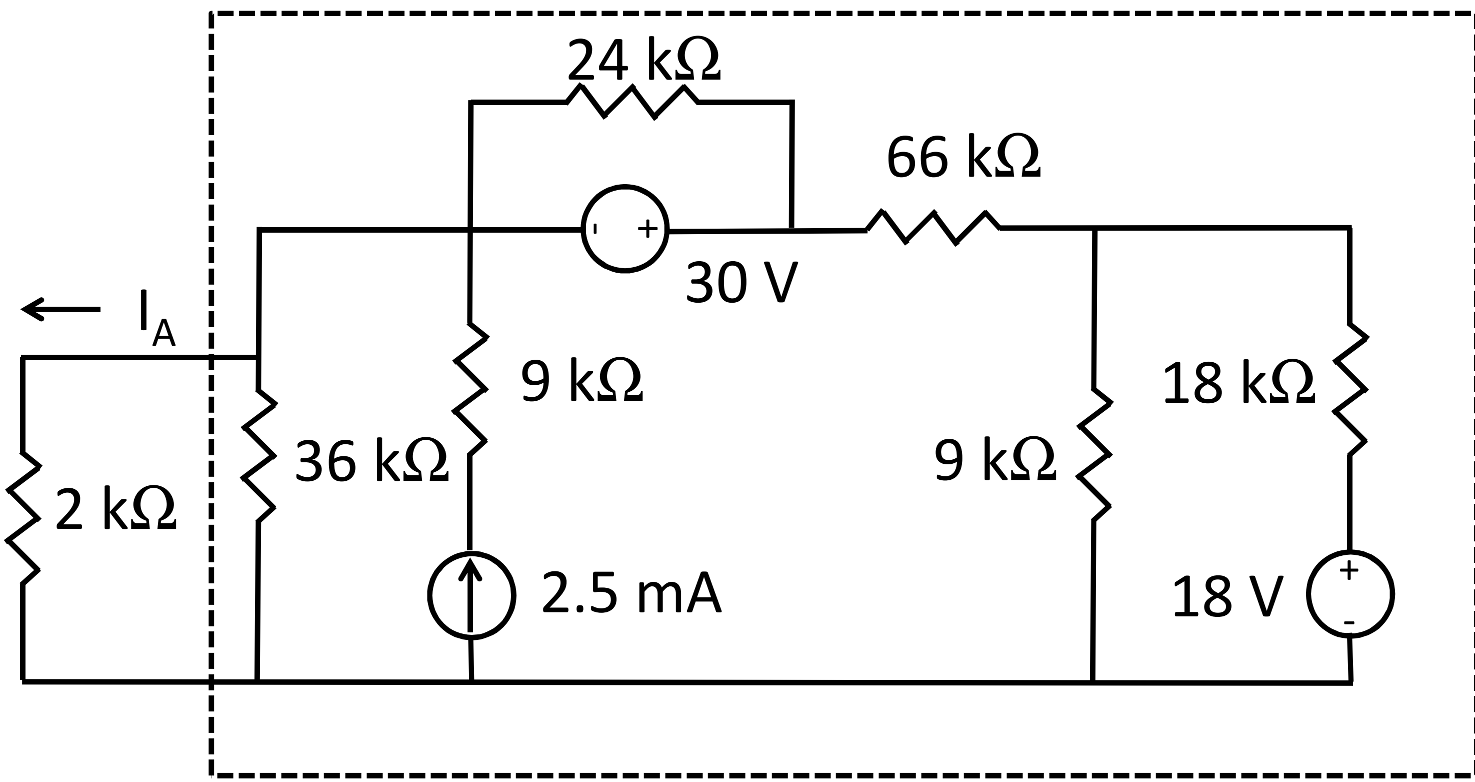


Fitting to $I_A = \gamma V_{AB} - \sigma$ yields γ=1/3, σ=4/3

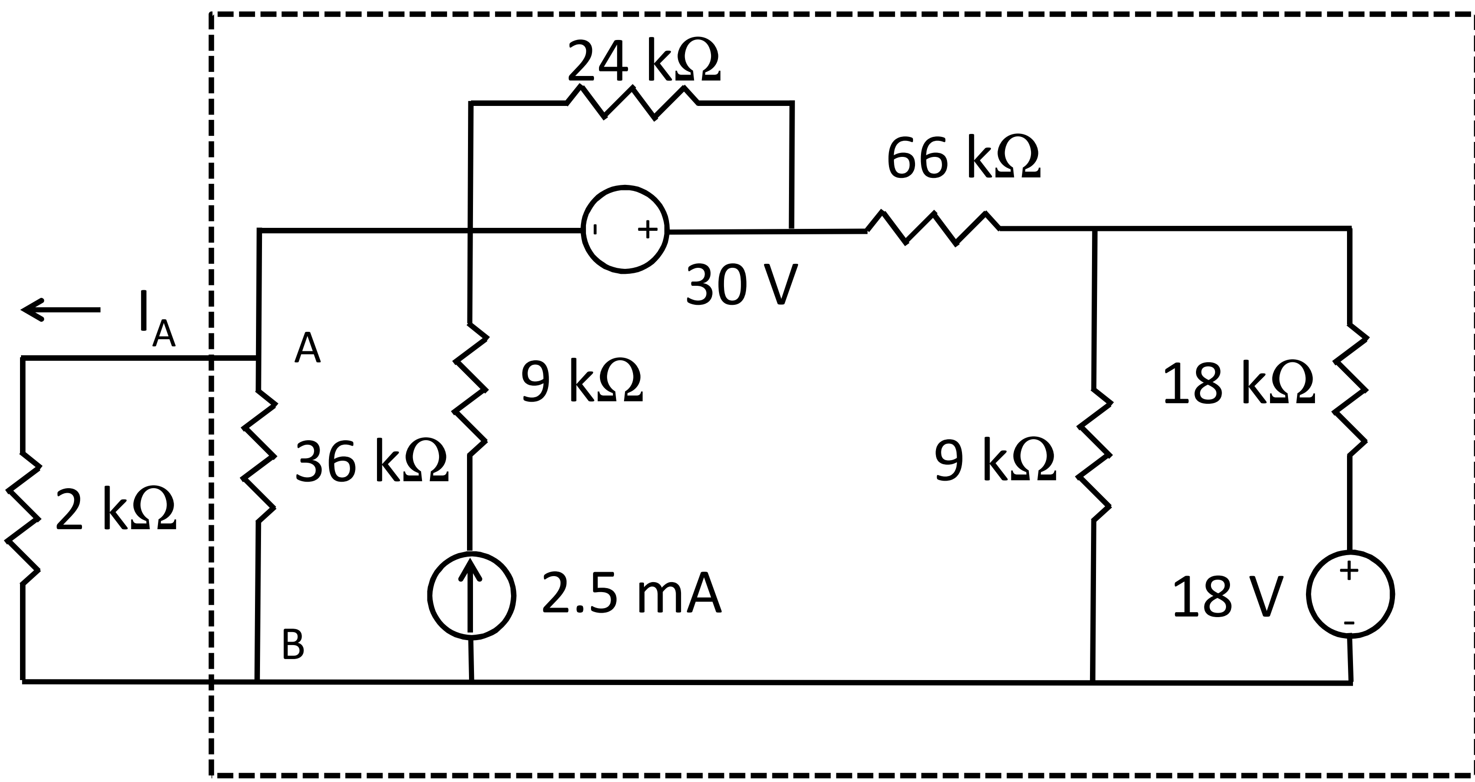


Example 2

(a) Find the Thévenin equivalent of the circuit below
(b) Find the output current and power at the load

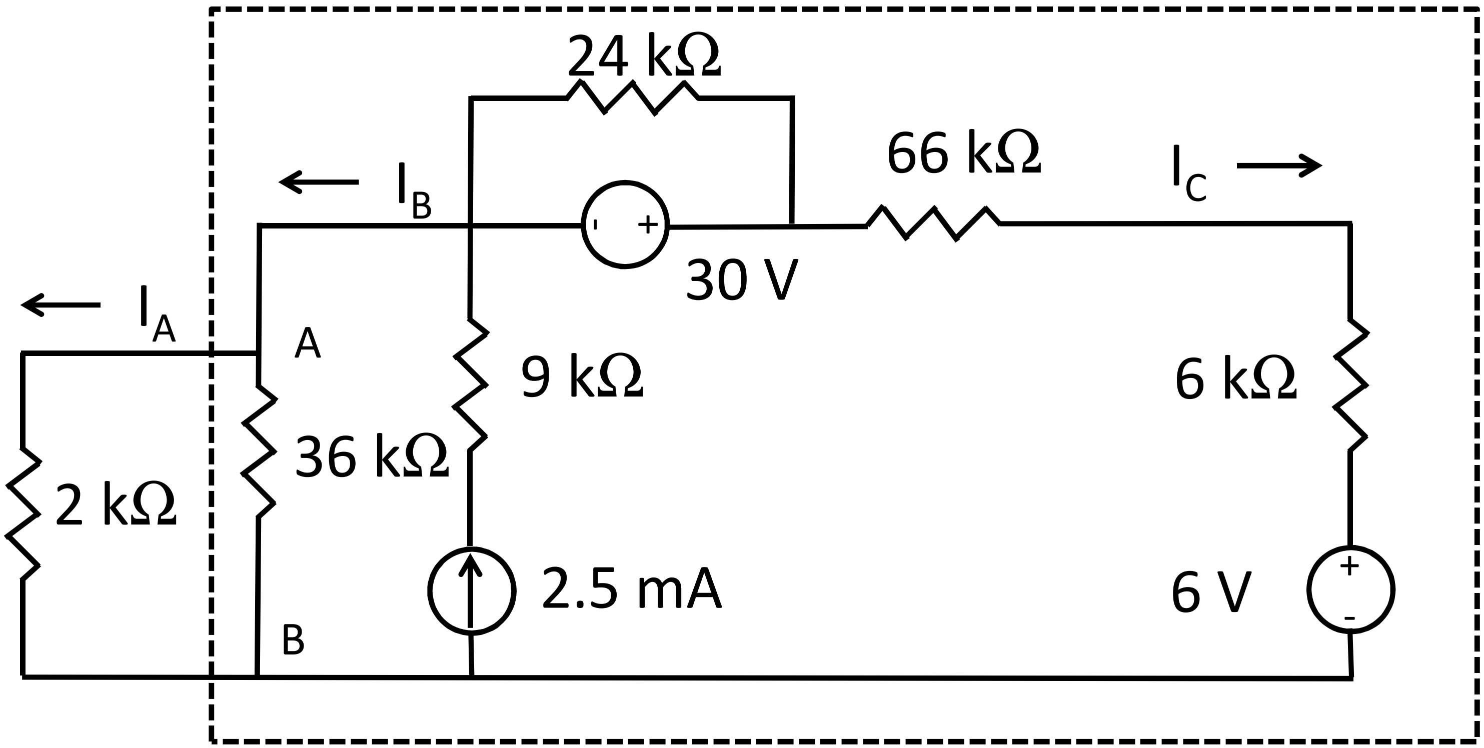


Solution



$$R_R = \left(\frac{1}{9} + \frac{1}{18}\right)^{-1} = 6\text{ k}\Omega;\ I = 1\text{ mA};\ \text{V=6 V}$$

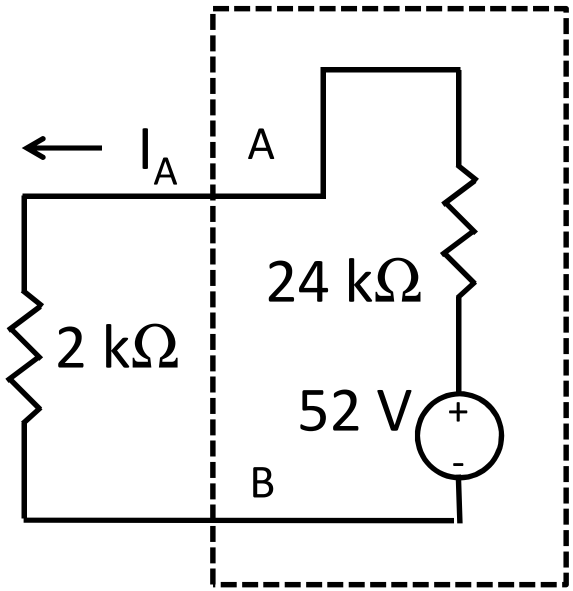
Solution



$$R_{th} = \left(\frac{1}{72} + \frac{1}{36}\right)^{-1} = 24\ \mathrm{k\Omega}$$

$$I_B + I_C = 2.5\ \mathrm{mA};\ 36 I_B = V';\ (66 + 6) I_C = 30 - 6 + V'$$

Solution

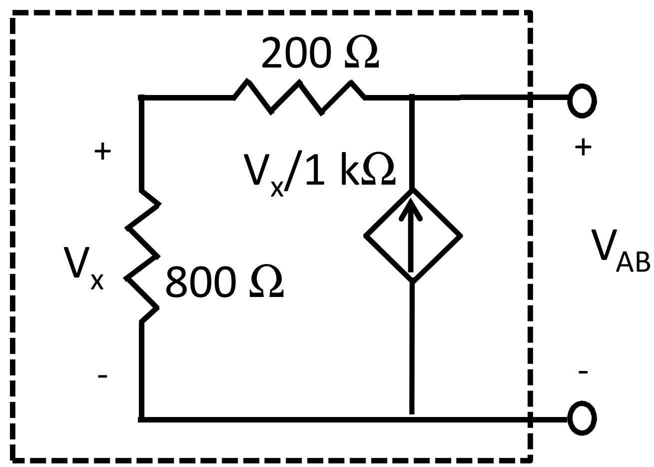


$$R_{th} = \left(\frac{1}{72} + \frac{1}{36}\right)^{-1} = 24\ \mathrm{k\Omega}$$

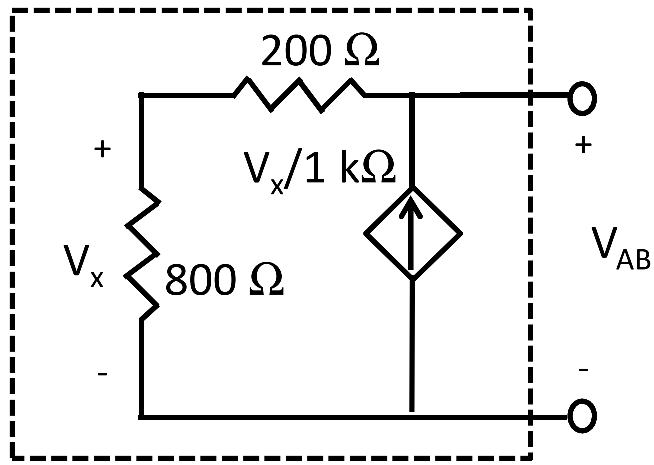
$$I_B = 13/9\ \mathrm{mA};\ V_{oc} = 52\ V;\ I_A = 2\ \mathrm{mA};\ P_A = 8\ \mathrm{mW}$$

Example 3: Active Circuits

- What are the Thévenin and Norton equivalents of this active circuit?



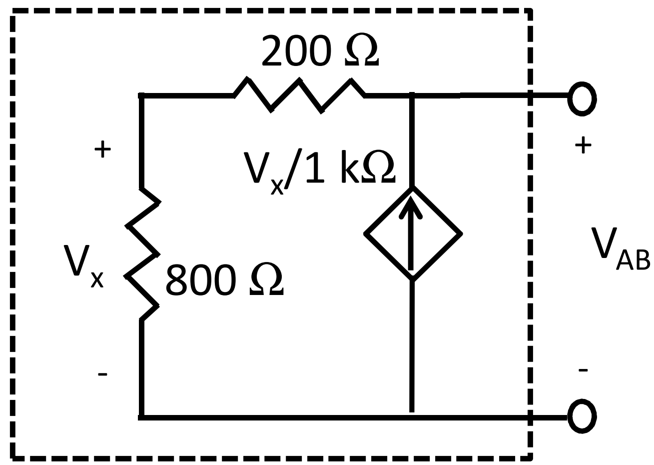
Example 3: Solution



- Use KCL and KVL to write:

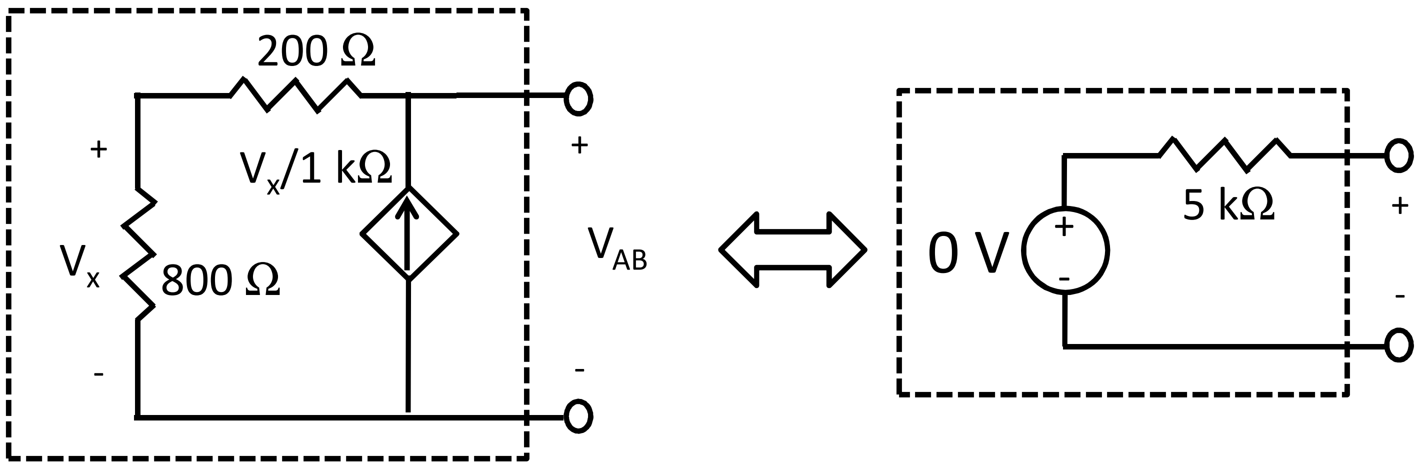
$$V_x \cdot (1\ \mathrm{mS}) = I_1 - I_A$$
$$V_{AB} = I_1 \cdot (1\ \mathrm{k\Omega})$$
$$V_x = I_1 \cdot (800\ \Omega)$$

Example 3: Solution



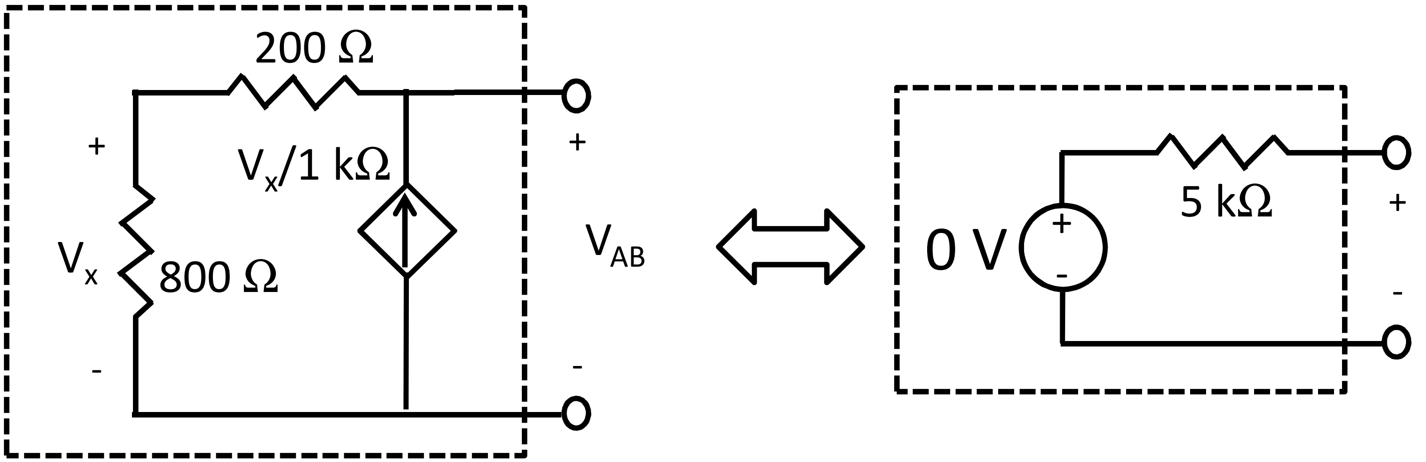
$$I_1 \cdot (800\ \Omega) \cdot (1\ \text{mS}) = I_1 - I_A$$
$$I_1 = 5I_A$$
$$V_{AB} = I_A \cdot (5\ \text{k}\Omega)$$

Example 3: Solution



- With no independent sources, can be reduced to a single resistor

Example 3: Solution



Power dissipated:

$$P = I_1 V_{AB} + \frac{4}{5} I_1 V_{AB}$$

$$P = \frac{9}{5} 5 I_A V_{AB}$$

Power dissipated:

$$P = I_A V_{AB}$$

Power dissipated in original network and equivalent circuits are not equal, in general

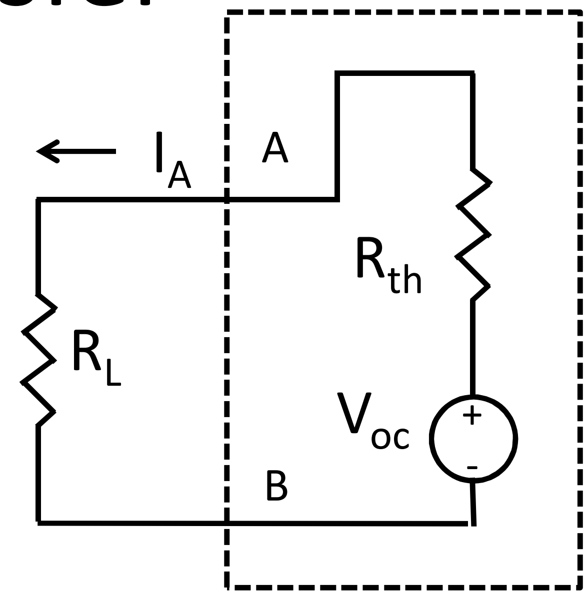
Max Power Transfer Theorem

- A 2-terminal linear network connected to a variable load R_L transfers max power when $R_L = R_{th}$, where R_{th} is the Thévenin equivalent resistance
- Max power given by:

$$P_{max} = \frac{{V_{oc}}^2}{4R_{th}}$$

Max Power Transfer

- Proof:



$$P_L = R_L \left(\frac{V_{oc}}{R_L + R_{th}} \right)^2$$

$$\frac{dP_L}{dR_L} = \left(\frac{V_{oc}}{R_L + R_{th}} \right)^2 - \frac{2R_L {V_{oc}}^2}{(R_L + R_{th})^3}$$

$$0 = R_L + R_{th} - 2R_L$$

$$R_L = R_{th}$$

Max Power Transfer

- Max power for a general load network:

$$P_L = IV = \left(\frac{V_{oc} - V}{R_{th}}\right)V$$

$$\frac{dP_L}{dV} = \frac{V_{oc} - 2V}{R_{th}}$$

$$0 = V_{oc} - 2V$$

$$V = V_{oc}/2$$

Homework

- HW #11 solution posted today (covering all topics on test)
- HW #12 due Friday by 4:30 pm in EE 326B (**note room change**)