ECE 201, Section 3
Lecture 14

Prof. Peter Bermel

September 21, 2012

Exam #1: Thursday, Sep. 20

- Posted a copy of the exam on Blackboard
- Will post scores and return copies of your scantron responses ASAP

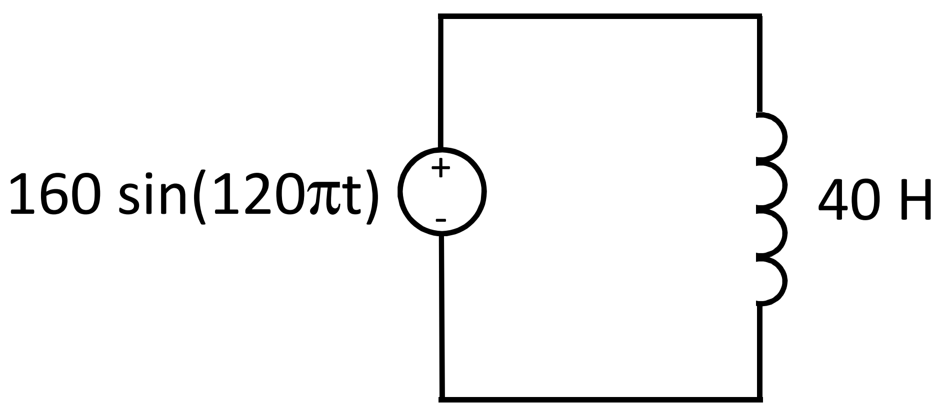
Inductors

$$V = L\, dI/dt$$

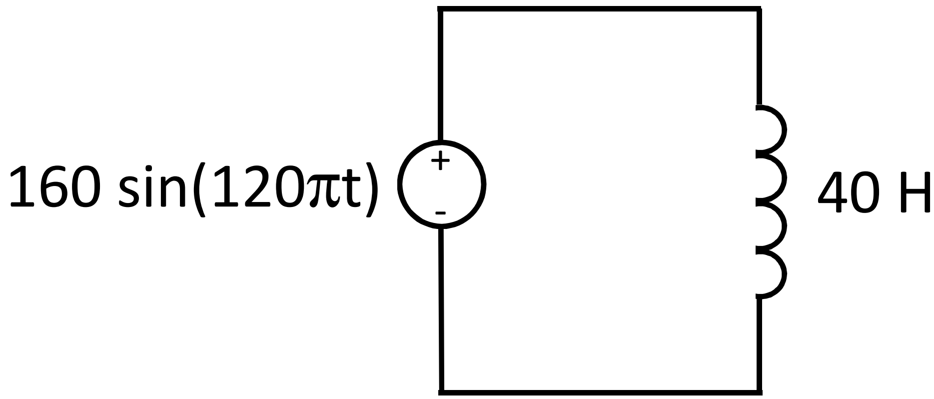
- Inductors resist changes in the flow of current
- Physically derived from:
  - Ampere's law: a flowing current creates a constant magnetic field
  - Faraday's law: a changing magnetic field induces an emf
- Mathematically, $V = L\, dI/dt$

Example 1

- With a wall AC voltage source of $V(t) = 160\sin(120\pi t)$, turned on at t=0, with an inductor L=40 H, what is the current as a function of time for this circuit?



Example 1: Solution



With the inductor:

$$0 = 160\sin(120\pi t) - 40\frac{dI}{dt}$$

$$I(t) = \int_0^t 4\sin(120\pi t)dt$$

$$I(t) = \frac{1}{30\pi}[1 - \cos(120\pi t)]$$

Calculating Inductance

- General formula for inductance:

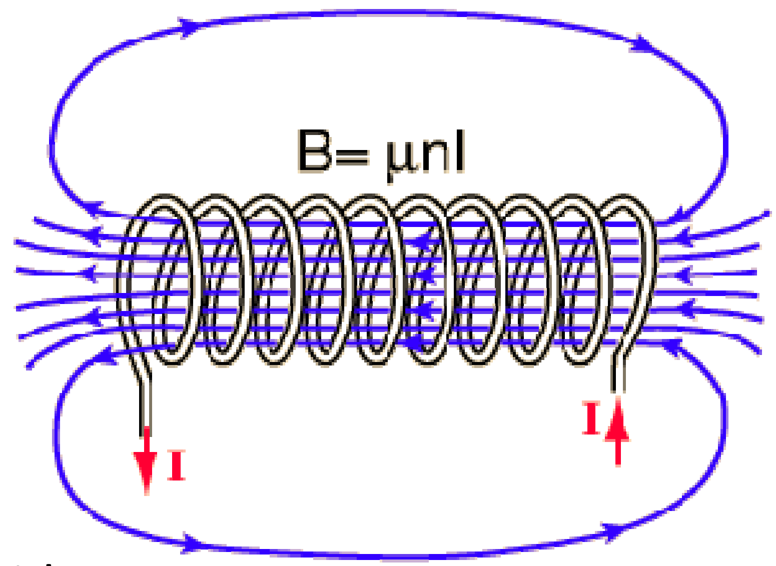
$$L = \frac{\mu}{4\pi} \oint_C \oint_{C'} \frac{ds \cdot ds'}{|s - s'|}$$

- For solenoid:

$$L = \mu N^2 A / l$$

where:

  - μ=magnetic constant (usu. $\mu_o = 4\pi x 10^{-7}$ H/m)
  - N=number of loops
  - A=cross-sectional area of solenoid
  - l=length of solenoid



Deriving Inductance

- Faraday's law states:

$$\mathcal{E} = N\frac{d\Phi}{dt} \equiv L\frac{dI}{dt}$$

From the definitions for L, Φ, and A:

$$L = \frac{N\Phi}{I} = \frac{N}{I}\int da \cdot B = \frac{N}{I}\oint_C ds \cdot A$$

From Ampère's law:

$$L = \frac{N}{I}\oint_C ds \cdot \oint_{C'} \frac{\mu I\, ds'}{4\pi|s - s'|}$$

Combining Inductors

- Define $L = \mu n N A$
- In series:

$$L_{eq} = \mu n (N_1 + N_2) A$$
$$L_{eq} = L_1 + L_2$$

Alternative proof:

$$L_1 \frac{dI_1}{dt} + L_2 \frac{dI_2}{dt} = L_{eq} \frac{dI}{dt}$$

By KCL, $I_1 = I_2 = I$, and $dI_1/dt = dI_2/dt = dI/dt$, so:

$$L_{eq} = L_1 + L_2$$

Combining Inductors

- In parallel:

$$L_1 \frac{dI_1}{dt} = L_2 \frac{dI_2}{dt} = L_{eq} \frac{dI}{dt}$$

Conservation of current ($I_1 + I_2 = I$) implies:

$$L_1 \left(\frac{dI}{dt} - \frac{dI_2}{dt} \right) = L_{eq} \frac{dI}{dt}$$

$$L_1 \left(\frac{dI}{dt} - \frac{L_{eq}}{L_2} \frac{dI}{dt} \right) = L_{eq} \frac{dI}{dt}$$

Combining Inductors

- Assuming dI/dt is non-zero:

$$L_1\left(1-\frac{L_{eq}}{L_2}\right)=L_{eq}$$

$$L_1=\left(1+\frac{L_1}{L_2}\right)L_{eq}$$

$$L_{eq}=\frac{L_1}{1+L_1/L_2}$$

$$L_{eq}=\frac{L_1L_2}{L_1+L_2}$$

Combining Inductors

- We can use induction to prove the following:
- Combining inductors in series:

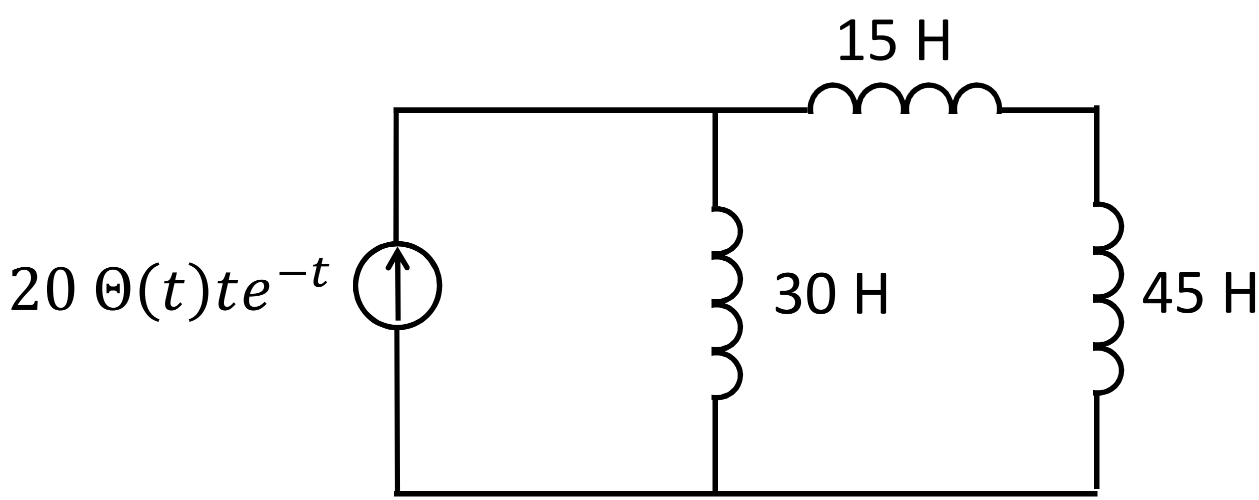
$$L_{eq} = \sum_{k=1}^{N} L_k$$

- Combining inductors in parallel:

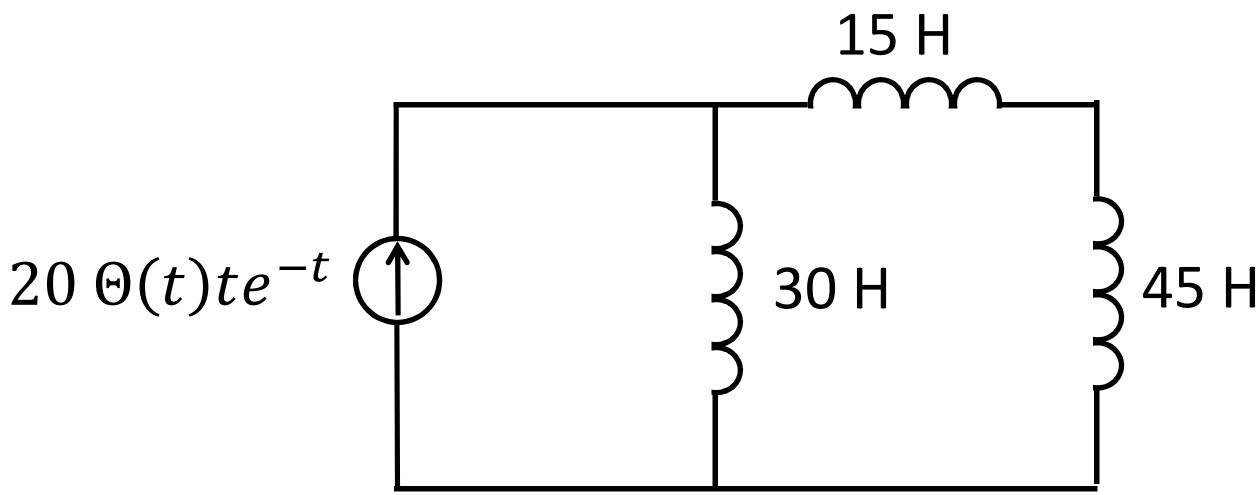
$$\frac{1}{L_{eq}} = \sum_{k=1}^{N} \frac{1}{L_k}$$

Example 2

- What is the equivalent inductance, current division, and power dissipated between the 2 branches of this circuit?



Example 2: Solution

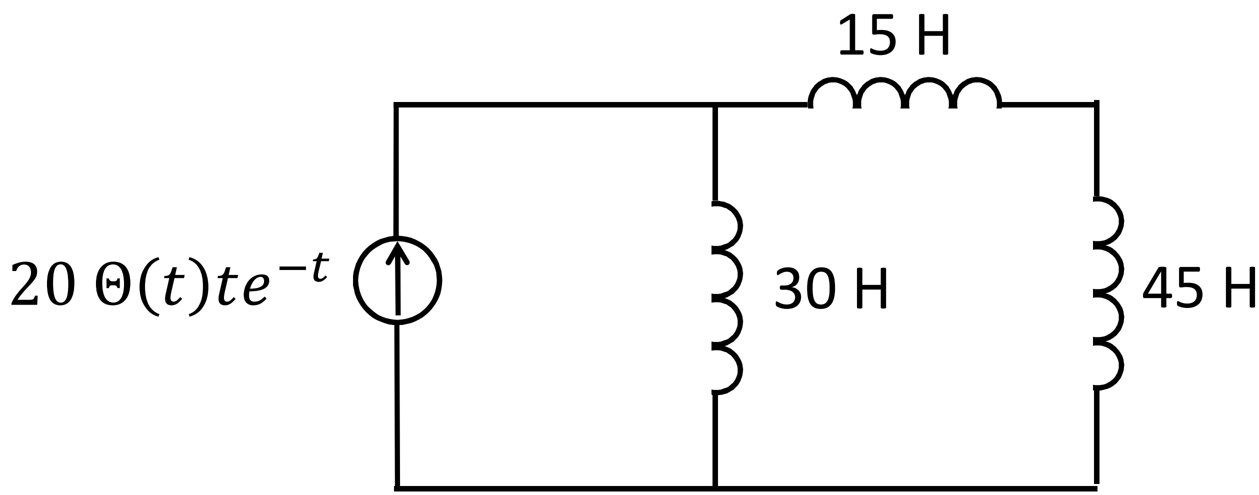


$$L_{eq} = \left(\frac{1}{30} + \frac{1}{15+45}\right)^{-1} = 20 \text{ H}$$

$$30\ dI_1/dt = 20dI/dt$$

$$I_1 = (2/3)I;\ \ I_2 = (1/3)I$$

Example 2: Solution



$$\frac{dI}{dt} = 20[\delta(t)te^{-t} + \Theta(t)e^{-t}(1-t)]\ \text{A/s}$$

$$P = I_1 \cdot 30\, dI_1/dt + I_2 \cdot 60\, dI_2/dt$$

$$P = 20I \cdot dI/dt$$

$$P = 400\,\Theta(t)e^{-2t}\,t(1-t)$$

Power in Inductors

- Since the instantaneous power in an inductor:

$$P = IV = IL\frac{dI}{dt}$$

- The energy stored between t_o and t_1 is:

$$U = \int_{t_o}^{t_1} LI\frac{dI}{dt}dt = \int_{I_o}^{I_1} LI\,dI = \frac{1}{2}L\left[{I_1}^2 - {I_o}^2\right]$$

- For ac wave with period T=1/f:

$$U = 0, \text{ if } t_1 = t_o + mT, \text{ for integer } m$$

Homework

- HW #12 due today by 4:30 pm in EE 326B
- HW #13 due Mon.: DeCarlo & Lin, Chapter 6:
  - Problem 14
  - Problem 16
  - Problem 21
  - Problem 24