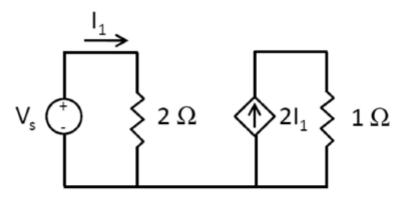
# ECE 201, Section 3 Lecture 15

Prof. Peter Bermel September 24, 2012

#### Solutions for Exam #1

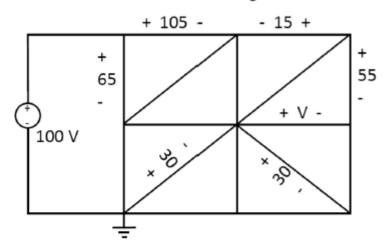
- Posted answer key for now
- Will post more comprehensive solutions soon
- Will review the following today:
  - Eight hard & medium difficulty problems: 3, 6, 8,10-13, 15
- Will not review seven problems in class: 1, 2,
  4, 5, 7, 9, 14

3. If an independent voltage source of 6V drives current  $I_1$  through a 2  $\Omega$  resistor, and a dependent current source generates a current of  $2I_1$  connected to a resistor of 1  $\Omega$ , what total power is dissipated by both resistors combined?



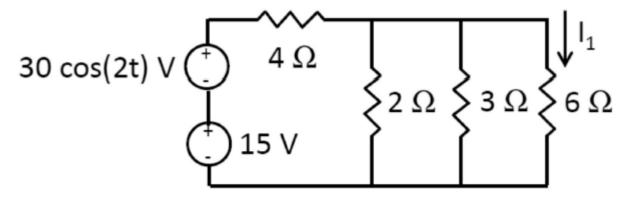
- (1) 6
- (2) 12
- (3) 18
- (4) 36
- (5) 54
- (6) 72
- (7) 108
- (8) None of the above

6. What is the voltage V in the indicated location of the diagram below?



- (1) 5
- (2) 10
- (3) 15
- (4) 20
- (5) 25
- (6) 30
- (7) 35
- (8) None of the above

8. What is the current  $I_1$  (in Amperes) flowing through the 6  $\Omega$  resistor on the right-hand side of the diagram below?



- (1) 0.5+cos(2t)
- (2) 1+cos(2t)
- (3) 1+2cos(2t)
- (4) 1.5+1.5cos(2t)
- (5) 1.5+3cos(2t)
- (6) 2.5
- (7) 5 cos(2t)
- (8) None of the above

10. Write down the matrix equation for the currents in the circuit below using loop analysis, in terms of the given resistances and voltages (Note: R<sub>124</sub>=R<sub>1</sub>+R<sub>2</sub>+R<sub>4</sub>).

Answers:

$$(1) \begin{bmatrix} R_{4} & 0 & R_{4} + R_{5} \\ -R_{2} & R_{2} + R_{3} & 0 \\ R_{124} & -R_{2} & R_{4} \end{bmatrix} \begin{bmatrix} I_{1} \\ I_{2} \\ I_{3} \end{bmatrix} = \begin{bmatrix} V_{s} \\ V_{r} \\ V_{s} \end{bmatrix}$$

$$(2) \begin{bmatrix} R_{4} & 0 & R_{4} + R_{5} \\ -R_{2} & R_{3} & 0 \\ R_{124} & R_{2} & -R_{4} \end{bmatrix} \begin{bmatrix} I_{1} \\ I_{2} \\ I_{3} \end{bmatrix} = \begin{bmatrix} V_{r} \\ V_{r} \\ V_{s} \end{bmatrix}$$

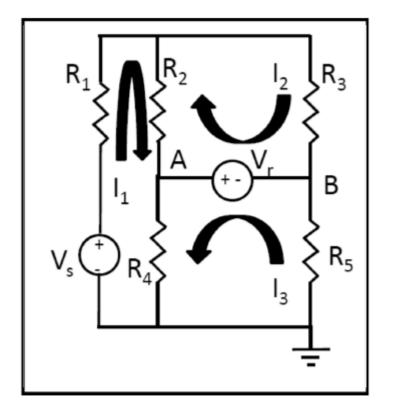
$$(3) \begin{bmatrix} R_{1} & R_{2} + R_{3} & R_{4} + R_{5} \\ -R_{2} & R_{1} + R_{4} & 0 \\ R_{124} & -R_{2} & R_{4} \end{bmatrix} \begin{bmatrix} I_{1} \\ I_{2} \\ I_{3} \end{bmatrix} = \begin{bmatrix} V_{r} \\ V_{s} \\ I_{3} \end{bmatrix} = \begin{bmatrix} V_{r} \\ V_{s} \\ V_{r} \end{bmatrix}$$

$$(4) \begin{bmatrix} R_{1} & 0 & R_{4} + R_{5} \\ -R_{2} & R_{2} & 0 \\ R_{124} & -R_{2} & R_{4} \end{bmatrix} \begin{bmatrix} I_{1} \\ I_{2} \\ I_{3} \end{bmatrix} = \begin{bmatrix} V_{r} \\ V_{r} \\ V_{s} \end{bmatrix}$$

$$(5) \begin{bmatrix} R_{4} & 0 & R_{4} + R_{5} \\ R_{2} & R_{2} - R_{3} & R_{1} \\ R_{124} & -R_{2} & R_{4} \end{bmatrix} \begin{bmatrix} I_{1} \\ I_{2} \\ I_{3} \end{bmatrix} = \begin{bmatrix} V_{r} \\ V_{r} \\ V_{s} \end{bmatrix}$$

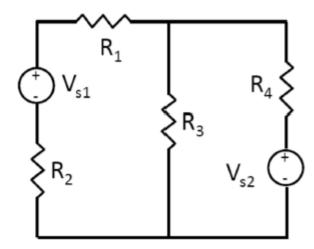
$$(6) \begin{bmatrix} R_{4} + R_{5} & 0 & 0 \\ 0 & R_{2} + R_{3} & 0 \\ 0 & 0 & R_{124} \end{bmatrix} \begin{bmatrix} I_{1} \\ I_{2} \\ I_{3} \end{bmatrix} = \begin{bmatrix} V_{r} \\ V_{r} \\ V_{s} \end{bmatrix}$$

$$(7) \begin{bmatrix} R_{4} & 0 & R_{4} + R_{5} \\ -R_{2} & R_{2} + R_{3} & 0 \\ R_{2} & R_{2} + R_{3} & 0 \\ R_{2} & R_{2} + R_{3} & 0 \end{bmatrix} \begin{bmatrix} I_{1} \\ I_{2} \\ I_{3} \end{bmatrix} = \begin{bmatrix} V_{r} \\ V_{r} \\ V_{r} \end{bmatrix}$$



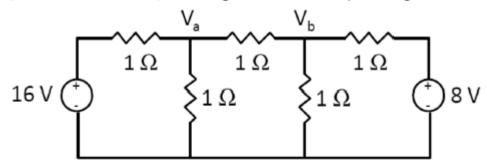
(8) None of the above

11. Find the voltage drop across R3, if R1=R2=R3=R4=1  $\Omega$  and Vs1=Vs2=1 V:



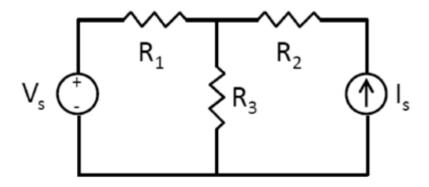
- (1) 0.1 V
- (2) 0.2 V
- (3) 0.3 V
- (4) 0.4 V
- (5) 0.5 V
- (6) 0.6 V
- (7) 0.7 V
- (8) None of the above

12. Find V<sub>a</sub> and V<sub>b</sub> for the circuit below, assuming that the bottom plane is grounded:



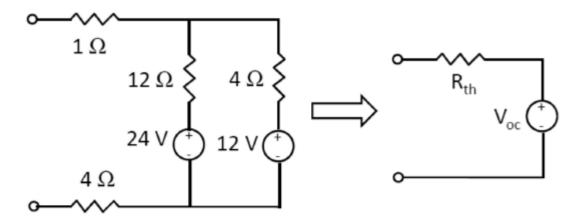
- (1)  $V_a = 2 V; V_b = 1 V$
- (2)  $V_a = 5 V$ ;  $V_b = 5 V$
- (3)  $V_a = 7 V$ ;  $V_b = 4 V$
- (4)  $V_a = 7 V$ ;  $V_b = 5 V$
- (5)  $V_a = 9 V$ ;  $V_b = 7 V$
- (6)  $V_a = 16 \text{ V}; V_b = 8 \text{ V}$
- (7)  $V_a = 16 \text{ V}; V_b = 16 \text{ V}$
- (8) None of the above

13. Find the voltage across  $R_3$  as a function of the independent sources  $V_s$  and  $I_s$  for the circuit below, if  $R_1$ = $R_2$ =3  $\Omega$  and  $R_3$ =6  $\Omega$  (Hint – try simplifying the diagram and using superposition):



- (1)  $V_s/3 + I_s$
- (2)  $2V_s/3 + I_s$
- $(3) 2V_s/3 + 6I_s$
- (4)  $V_s/2 + 6I_s$
- (5)  $V_s + 6I_s$
- (6)  $4V_s/3 + 4I_s$
- $(7) 2V_s/3 + 2I_s$
- (8) None of the above

15. Using source transformation, find the voltage source voltage and resistance for the circuit on the right-hand side needed to make it equivalent to the circuit on the left-hand side:



Answers:

(1) 
$$R_{th} = 5 \Omega$$
;  $V_{oc} = 12 V$ 

(2) 
$$R_{th} = 5 \Omega$$
;  $V_{oc} = 24 V$ 

(3) 
$$R_{th} = 8 \Omega$$
;  $V_{oc} = 8 V$ 

(4) 
$$R_{th} = 8 \Omega$$
;  $V_{oc} = 15 V$ 

(5) 
$$R_{th} = 9 \Omega$$
;  $V_{oc} = 15 V$ 

(6) 
$$R_{th} = 9 \Omega$$
;  $V_{oc} = 24 V$ 

(7) 
$$R_{th} = 21 \Omega$$
;  $V_{oc} = 36 V$ 

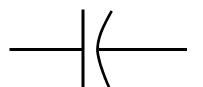
(8) None of the above

### Introduction to Capacitors

 Capacitors are circuit elements capable of storing charge



 Voltage drop proportional to stored charge: V=Q/C, or Q=CV



- Capacitance C in units of C/V or F
- Current flow given by:

$$I = \frac{dQ}{dt} = C\frac{dV}{dt}$$

## Calculating Capacitance

- Capacitance determined by geometry and materials present
- Calculated using Gauss' Law:

$$Q_{\rm enc} = \epsilon \Phi_E = \epsilon \oint_S E \cdot dA$$

• For parallel plate capacitor, E=Q/εA=V/d, thus:

$$Q = \left(\frac{\epsilon A}{d}\right)V = CV$$

• In vacuum,  $\epsilon = \epsilon_o = 8.854 \cdot 10^{-12}$  F/m

### Calculating Capacitance

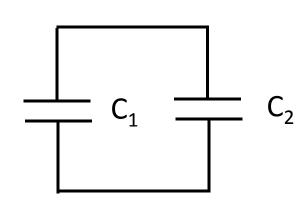
For two capacitors in parallel:

$$V = \frac{Q_1}{C_1} = \frac{Q_2}{C_2} = \frac{Q_{\text{tot}}}{C_{eq}}$$

$$C_{eq}V = Q_{\text{tot}} = Q_1 + Q_2$$

$$= C_1V + C_2V$$

$$C_{eq} = C_1 + C_2$$



• For *N* capacitors in parallel:

$$C_{eq} = \sum_{k=1}^{N} C_k$$

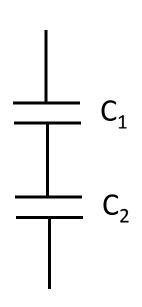
## Calculating Capacitance

• For two capacitors *in series*:

$$I = C_{1} \frac{dV_{1}}{dt} = C_{2} \frac{dV_{2}}{dt} = C_{eq} \frac{dV_{\text{tot}}}{dt}$$

$$\frac{I}{C_{eq}} = \frac{dV_{1}}{dt} + \frac{dV_{2}}{dt} = \frac{I}{C_{1}} + \frac{I}{C_{2}}$$

$$\frac{1}{C_{eq}} = \frac{1}{C_{1}} + \frac{1}{C_{2}}$$

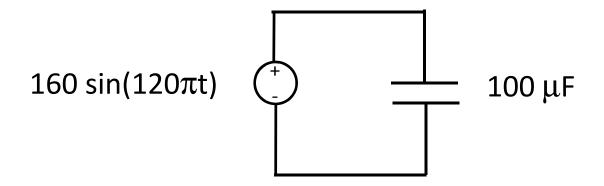


For N capacitors in parallel:

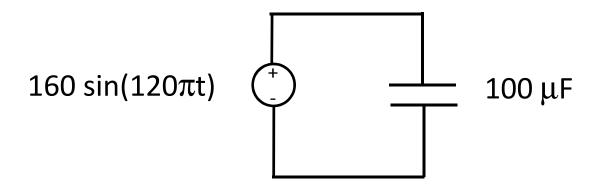
$$\frac{1}{C_{eq}} = \sum_{k=1}^{N} \frac{1}{C_k}$$

#### Example 1

• What is the current produced by plugging a  $100~\mu F$  capacitor into the wall? How much power is dissipated by it?



#### Solution



$$I = C \frac{dV}{dt} = (100 \,\mu\text{F}) \cdot 160 \cdot 120\pi \cos(120\pi t)$$
$$I = 1.92 \cos(120\pi t)$$

$$P = IV = 1.92 \cos(120\pi t) \cdot 160 \sin(120\pi t)$$
$$P = 153.6 \sin(240\pi t)$$

#### Homework

- HW #13 due today by 4:30 pm in EE 326B
- HW #14 due Wed.: DeCarlo & Lin, Chapter 6:
  - Problem 38
  - Problem 50
  - Problem 53 [Correction: The independent source on the left side of Figure P6.53 is a current source. Change this symbol to an upward arrow.]