ECE 201, Section 3
Lecture 17

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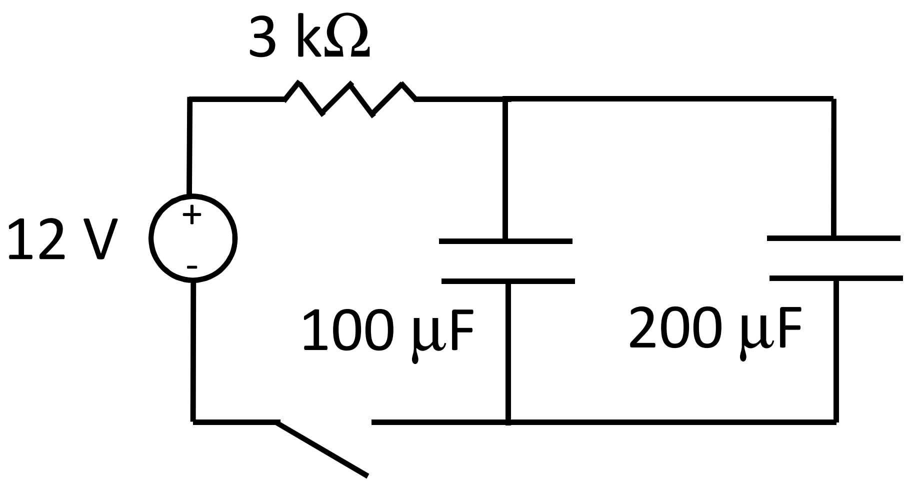
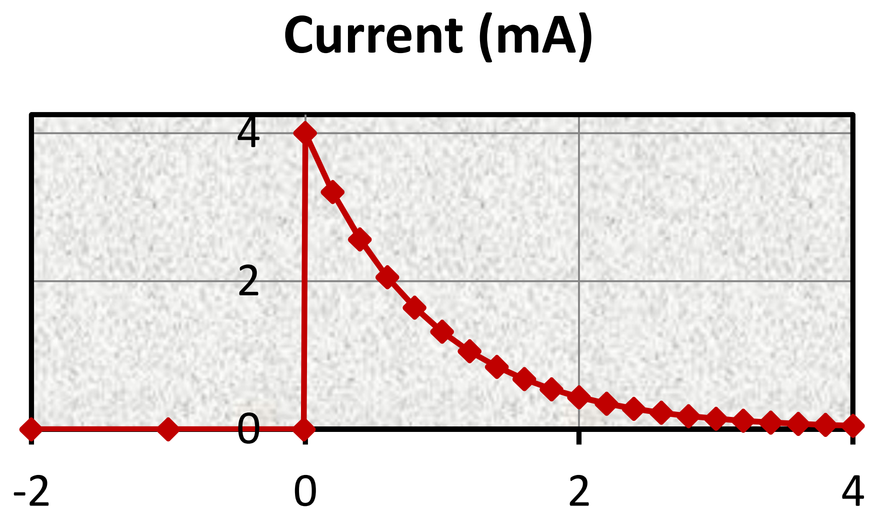
September 28, 2012

Recap: RC Circuits

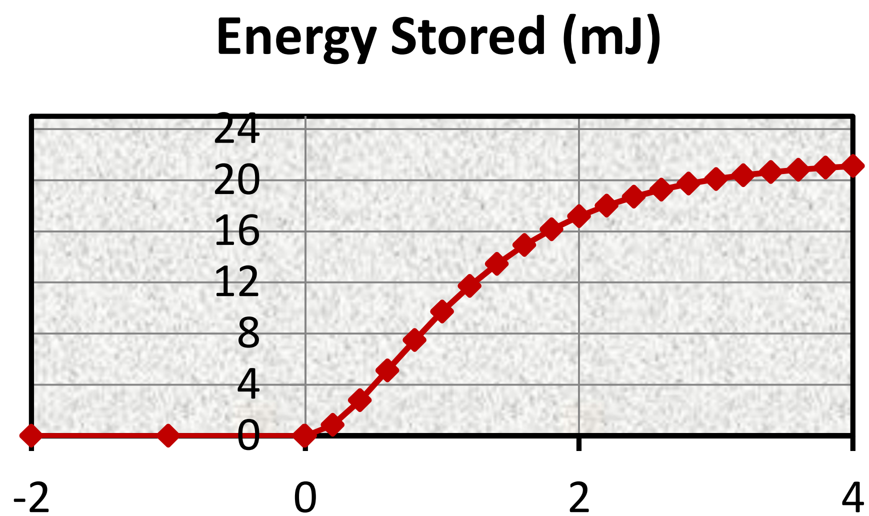




$$Q = Q_f\left[1 - e^{-t/\tau}\right]$$

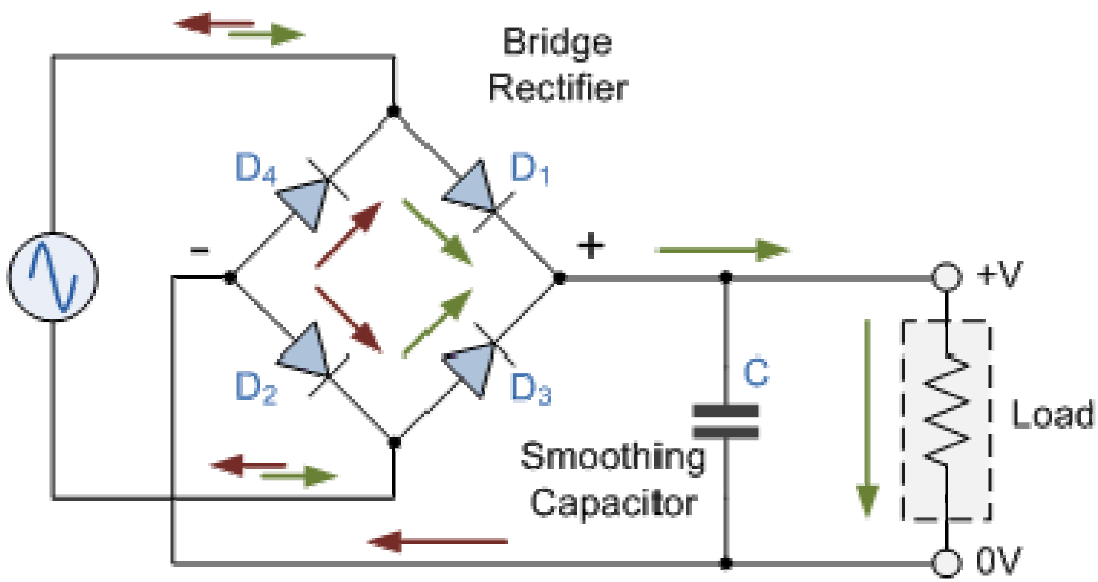
$$I = I_o e^{-t/\tau} \text{ (t>0)}$$

$$U = \frac{1}{2C}\left[Q_f\left(1 - e^{-t/\tau}\right)\right]^2$$



Example

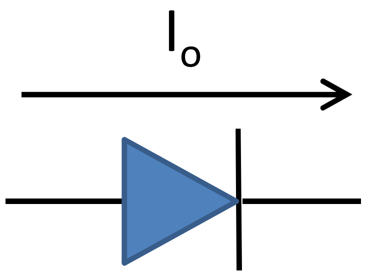
- How does this circuit modify a standard AC input voltage? What is its likely purpose?



Solution

- Diodes are a “one-way street” for electrons
- Ideal Diode:

$$I_{out}(t) = \begin{cases} I_{in}(t), & I_{in} > 0 \\ 0, & \text{otherwise} \end{cases}$$

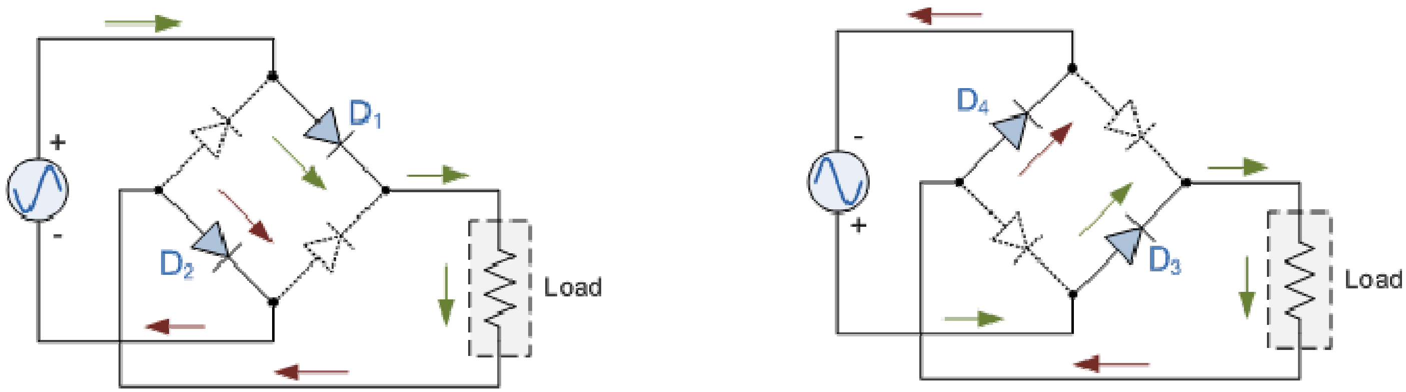


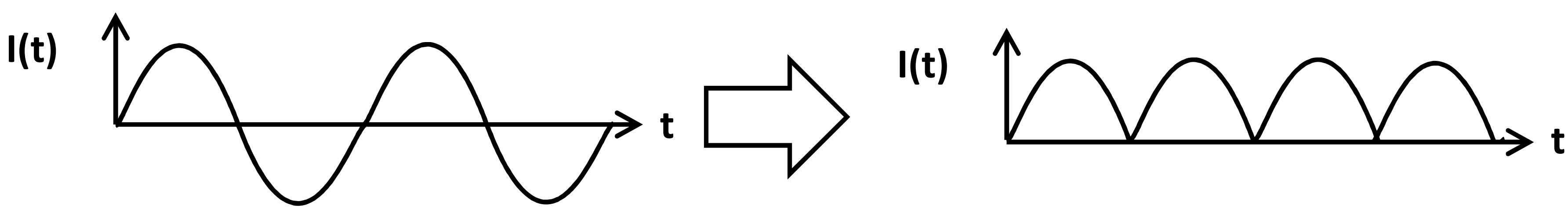
- Realistic (Shockley) Diode:

$$I_{out}(t) = I_s\left(e^{qV/nkT} - 1\right)$$

Solution

- Analyzing the diode bridge:





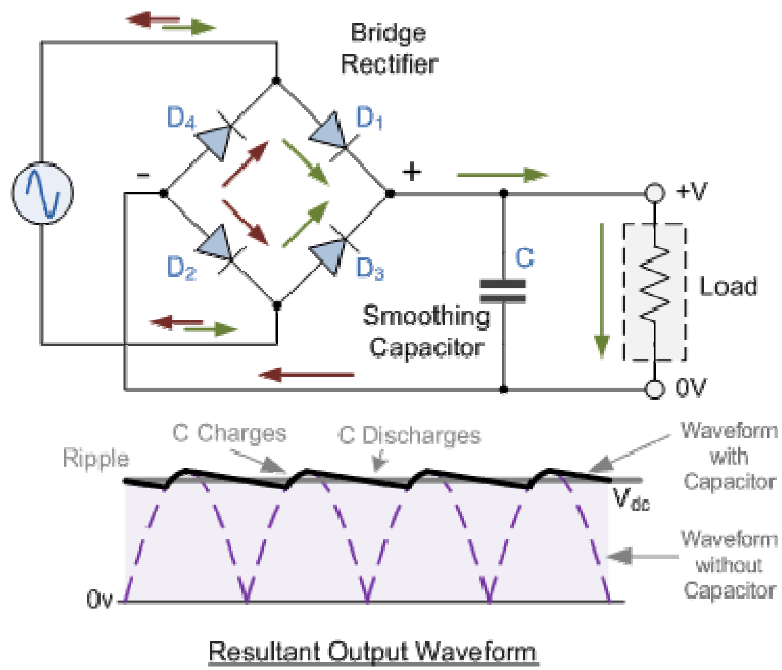
Solution

- Combining the diode bridge output with the capacitor yields:

$$Q = CV(t) = \int_0^t dt'(|I| - I_L)$$

$$V_{rip} \approx I/fC$$

- For AC wall plug
  - R_L=1 kΩ, I=160 mA, V_r=1 V
  - $C > \frac{I}{f(1\,V)} = 2.7$ mF



Solving Differential Equations

- Using KVL, RL circuits obey:

$$V = IR + L\frac{dI}{dt}$$

- RC circuits can be written as:

$$V = \frac{1}{C}Q + R\frac{dQ}{dt}$$

- General form decomposes into homogeneous and inhomogeneous equations:

$$0 = Y_h + \tau\frac{dY_h}{dt}$$

$$f = Y_i + \tau\frac{dY_i}{dt}$$

Solving Differential Equations

- Can solve each part separately
  - Homogeneous part:

$$Y_h = -\tau \frac{dY_h}{dt}$$

$$\frac{dY_h}{Y_h} = -\frac{dt}{\tau}$$

  - Inhomogeneous part:

$$f = Y_i + \tau \frac{dY_i}{dt}$$

$$Y_i = f, \qquad f \text{ constant}$$

Solving Differential Equations

- Use separation of variables and the following rules:

$$\int_{t_o}^{t_1} t^n dt = \frac{{t_1}^{n+1} - {t_o}^{n+1}}{n+1}$$

$$\int_{u_o}^{u_1} \frac{du}{u} = \ln u_1 - \ln u_o$$

$$\ln a - \ln b = \ln \frac{a}{b}$$

$$a \ln b = \ln b^a$$

$$e^{a \ln b} = b^a$$

Solving Differential Equations

- We can integrate our homogeneous ODE:

$$\frac{dY_h}{Y_h} = -\frac{dt}{\tau}$$

- To obtain:

$$\int_{Y_i}^{Y_f} \frac{dY_h}{Y_h} = -\int_{t_i}^{t_f} \frac{dt}{\tau}$$

$$\ln \frac{Y_f}{Y_i} = -\left(\frac{t_f - t_i}{\tau}\right)$$

$$Y_f = Y_i e^{-(t_f - t_i)/\tau}$$

Solving Differential Equations

- Overall solution is given by:

$$Y = Y_h + Y_i$$

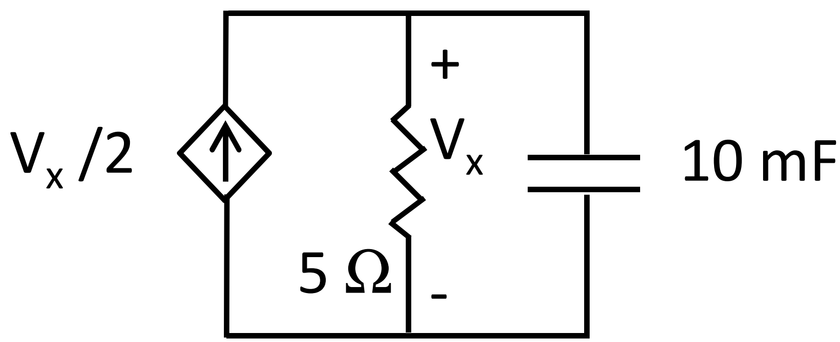
- For RC or RL circuits, we obtain:

$$Y = Y_i e^{-(t-t_i)/\tau} + f, \qquad f \text{ constant}$$

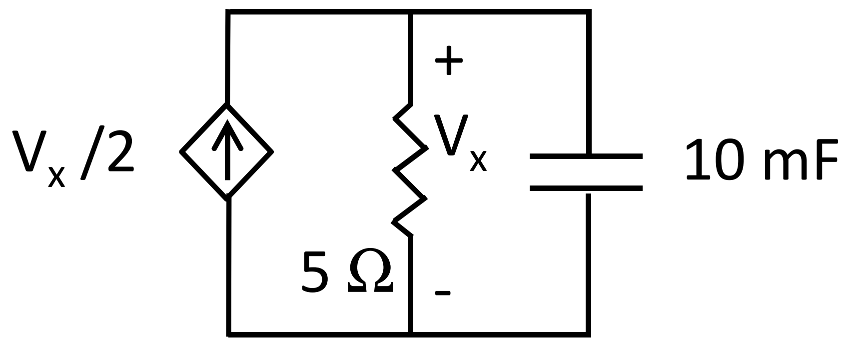
- Where the time constant τ is given by:
  - RL circuits: τ=L/R
  - RC circuits: τ=RC

Example 1

- What is the voltage response of this circuit as a function of time if V(0)=1 mV?



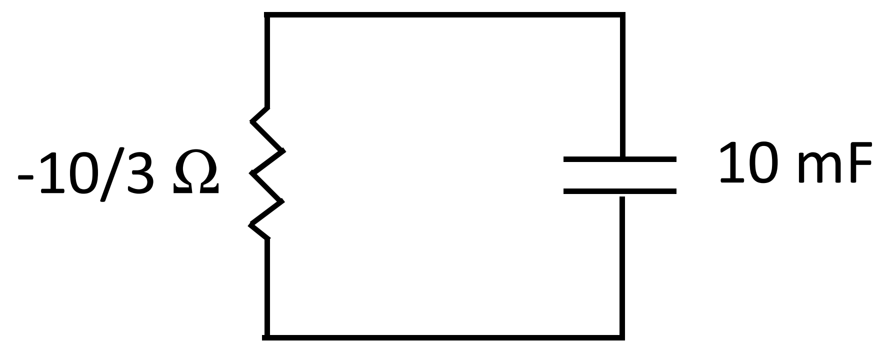
Solution



- Calculate Thèvenin equivalent for left-hand side:
  - No independent sources, so V_{oc}=0
  - Thèvenin resistance obtained from KCL:

$$I_A + V_{ab}/2 = V_{ab}/5$$
$$I_A = -3V_{ab}/10$$
$$R_{th} = -10/3$$

Solution



- Applying homogeneous solution from before:

$$Q = Q_o e^{-t/\tau}$$

Since τ=RC=-(1/30) s, V_o=1 mV, Q_o=10 μC:

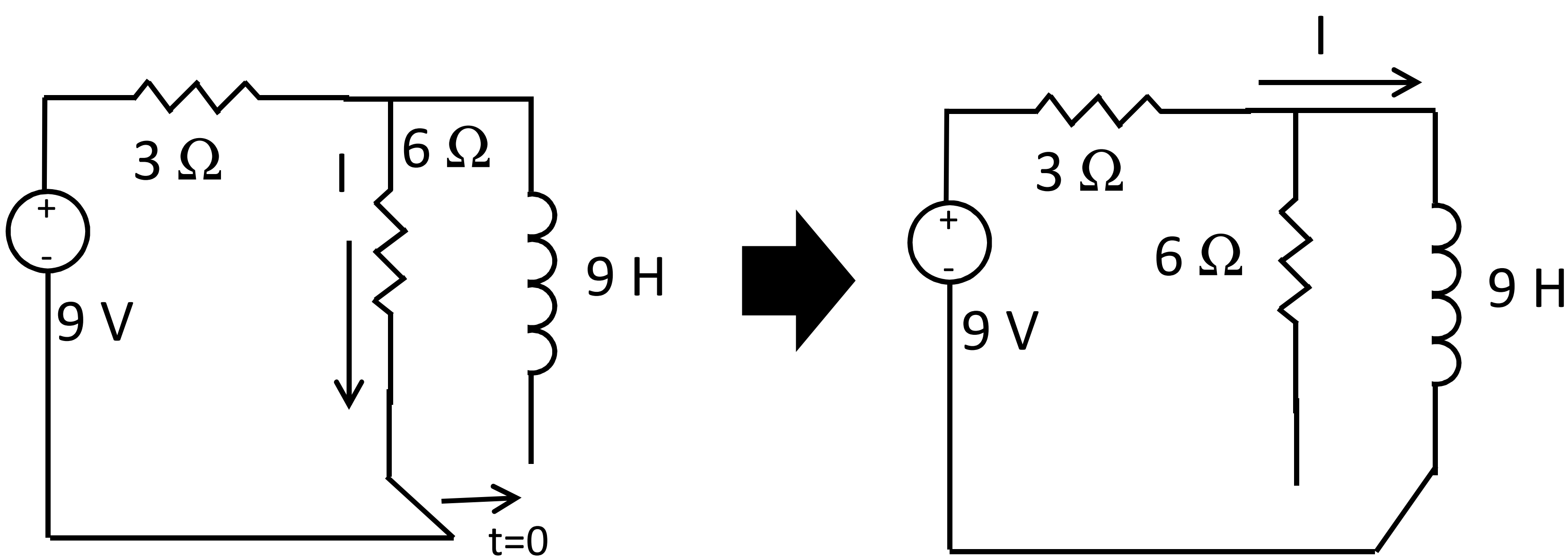
$$Q = (10\ \mu C)e^{30t}$$

Charge exceeds that of known universe in:

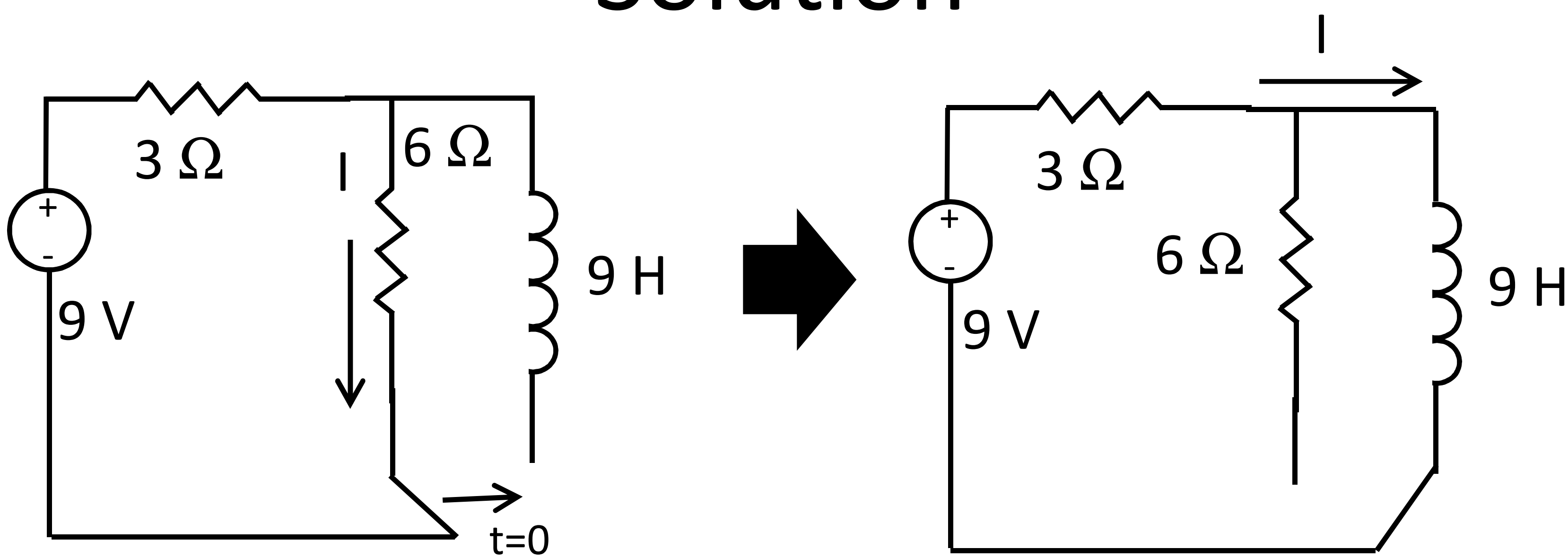
$$t = \frac{1}{30}\ln\left(\frac{10^{80}\cdot 10^{-19}}{10^{-5}}\right) = 5.06\ \text{s!}$$

Example 2

What is the current flow in this circuit before and after the switch is flipped at time t=0?



Solution



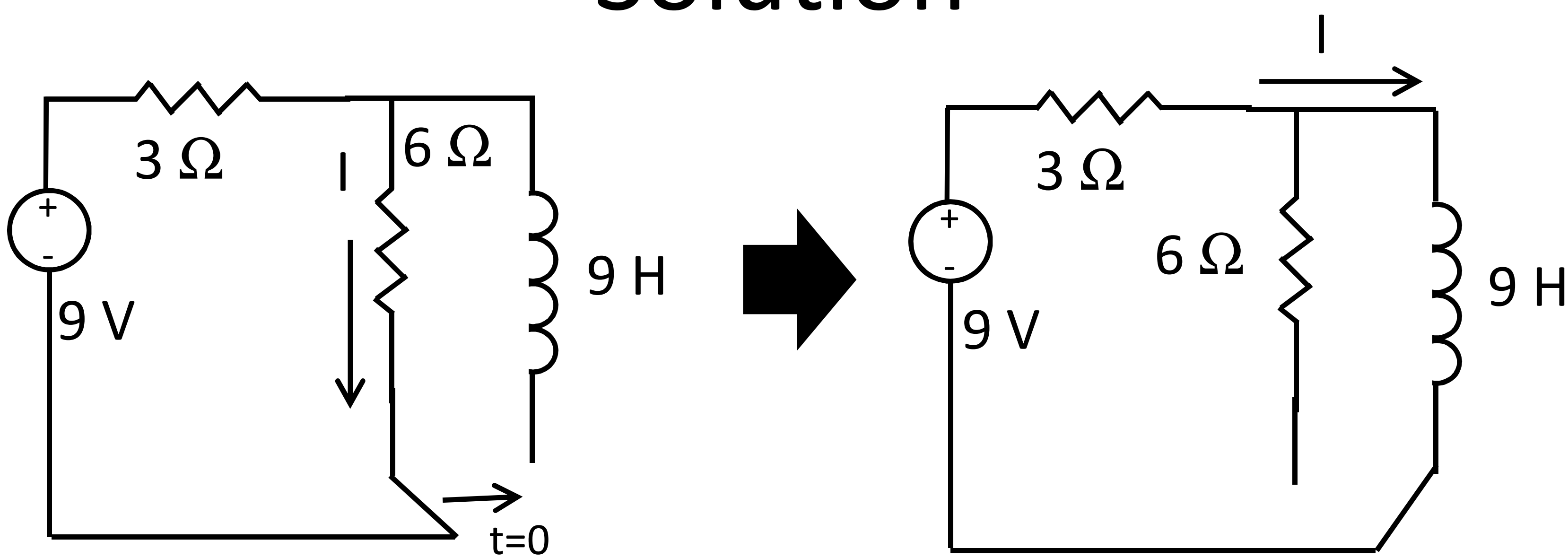
For t<0: $I = (9\text{ V}) / (9\ \Omega) = 1\text{ A}$

For t>0: KVL yields $9 = (3\ \Omega)I + (9\text{ H})\, dI/dt$

Since L/R=3, assume solution takes the form:

$$I = 3 - Be^{-t/3}$$

Solution



For t>0: $9 = 3\left[3 - Be^{-\frac{t}{3}}\right] + 9\frac{B}{3}e^{-\frac{t}{3}}$

B is set by continuity condition for current:

$$I(0) = 0 = 3 - Be^{-t/3}$$

Thus, B=3, and $I(t) = 3\left[1 - e^{-t/3}\right], \quad t > 0$

Homework

- HW #15 due today by 4:30 pm in EE 326B
- HW #16 due Mon.: DeCarlo & Lin, Chapter 7:
  - Problem 11
  - Problem 17 [Corrections: In (a), omit "Is the output … C_1? Why?" In (c), replace "[0, t]" with "[0, 10 ms]".]