ECE 201, Section 3
Lecture 18

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October 1, 2012

Ordinary Differential Equation Recap

- RL circuits obey:

$$V = IR + L\frac{dI}{dt}$$

- RC circuits can be written as:

$$V = \frac{1}{C}Q + R\frac{dQ}{dt}$$

- Overall solution is given by:

$$Y = Y_h + Y_i$$

- For RC and RL circuits, we obtain:

$$Y = Y_\infty + (Y_o - Y_\infty)e^{-(t-t_o)/\tau}$$

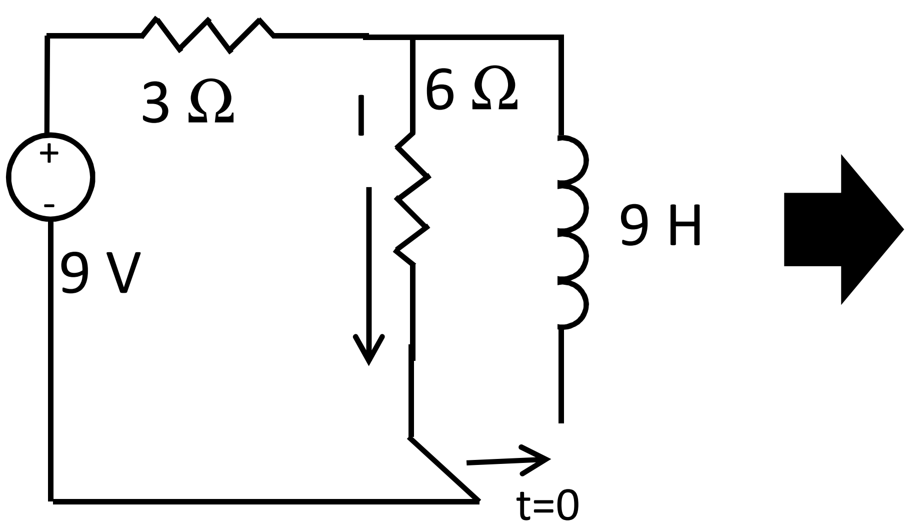
- Where the time constant τ is given by:
  - RL circuits: τ=L/R
  - RC circuits: τ=RC

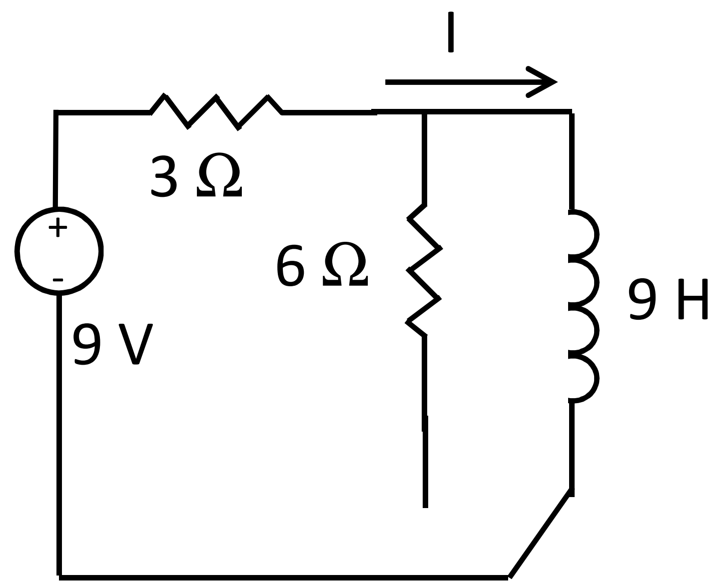
Solving ODEs

- In presence of stepped voltages, apply conservation laws
- Use values at end of previous interval as initial conditions for next interval
- Best illustrated through examples

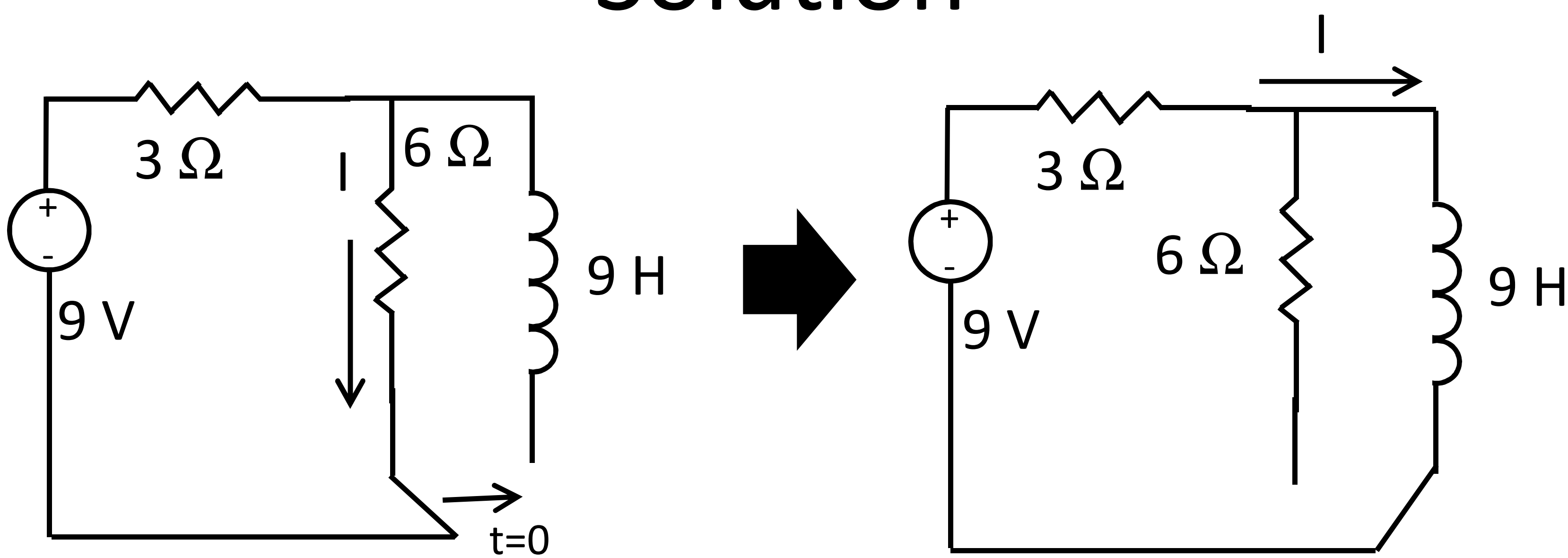
Example

What is the current flow in this circuit before and after the switch is flipped at time t=0?





Solution



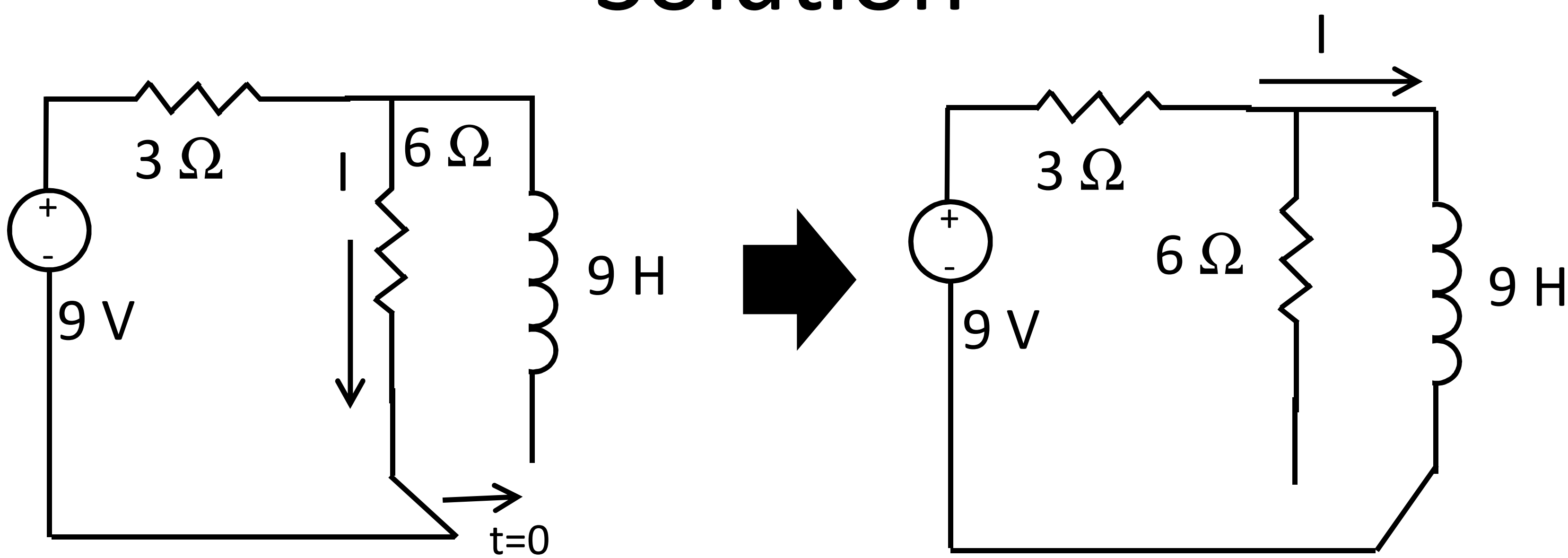
For t<0: $I = (9\ V) / (9\ \Omega) = 1\ A$

For t>0: KVL yields $9 = (3\ \Omega)I + (9\ H)\ dI/dt$

Since L/R=3, assume solution takes the form:

$$I = 3 - Be^{-t/3}$$

Solution



For t>0: $9 = 3\left[3 - Be^{-\frac{t}{3}}\right] + 9\frac{B}{3}e^{-\frac{t}{3}}$

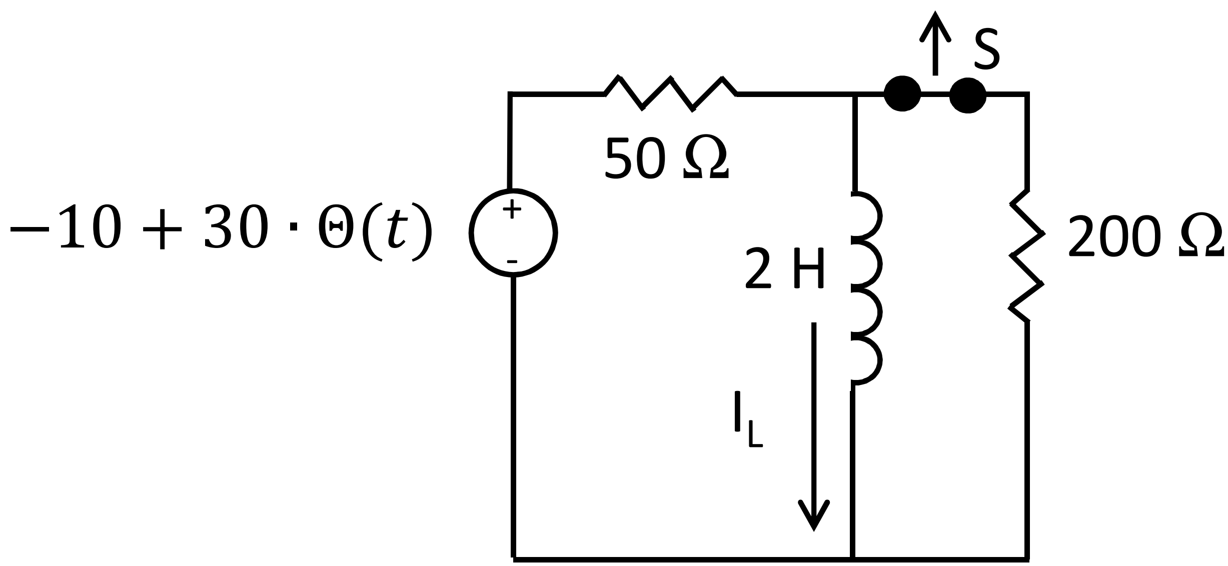
B is set by continuity condition for current:

$$I(0) = 0 = 3 - Be^{-t/3}$$

Thus, B=3, and $I(t) = 3\left[1 - e^{-t/3}\right], \quad t > 0$

Example

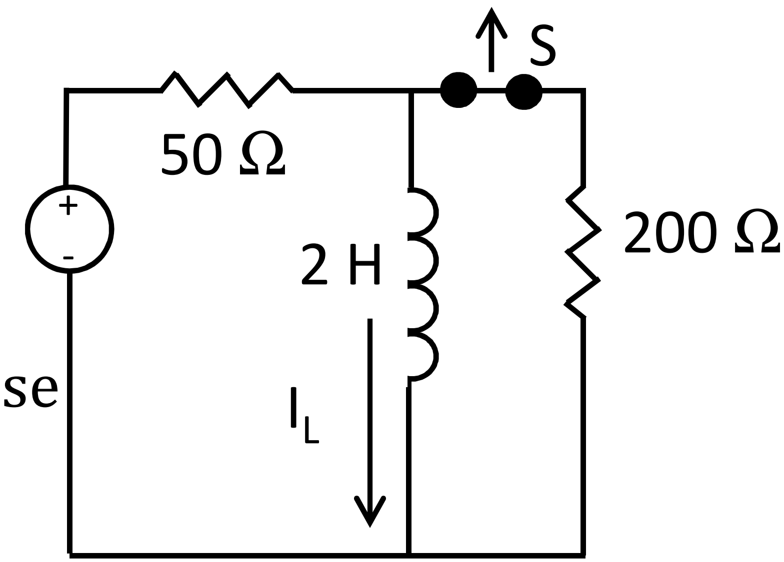
- What current flows through the inductor for t>0? How does that change if the switch S opens after 50 ms?



Solution

$$-10 + 30 \cdot \Theta(t)$$

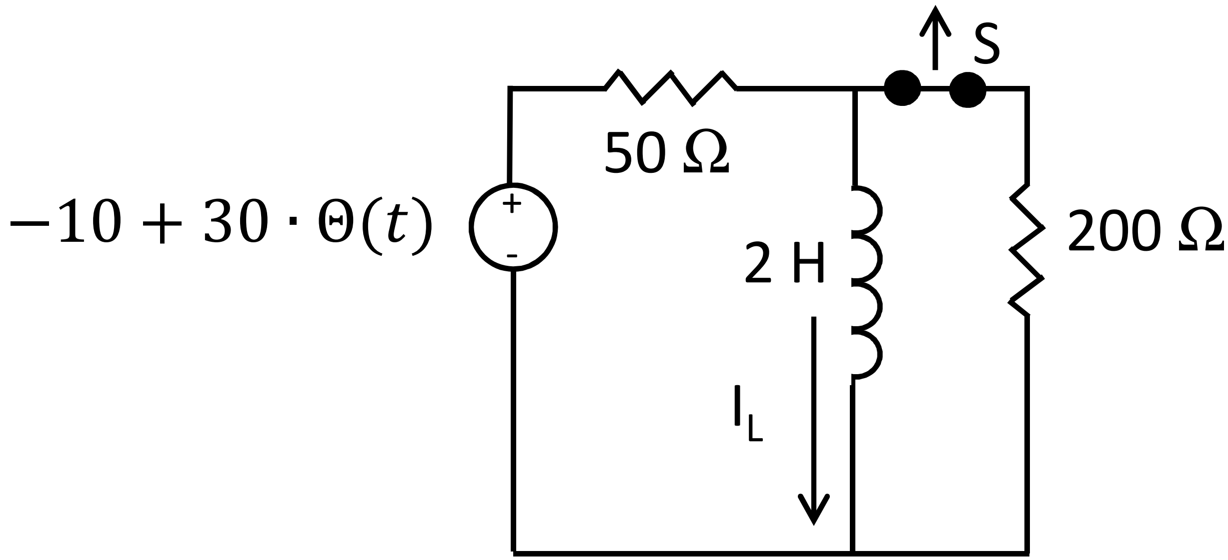
$$\Theta(t) = \begin{cases} 0, & t < 0 \\ 1, & \text{otherwise} \end{cases}$$



- Steady state response for t<0: I=-10/50=-0.2 A
- After t=0: R_{th}=40 Ω; τ=L/R_{th}=2/40=0.05 s; I_∞=20/50=0.4 A

$$I_L = 0.4 + (-0.2 - 0.4)e^{-20t}$$

Solution

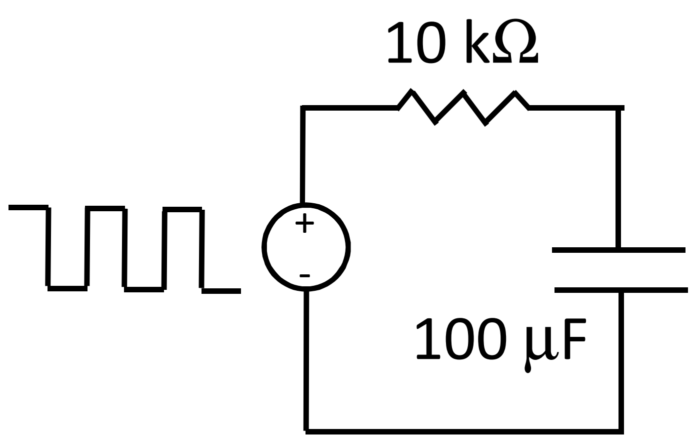


- If switch activates after 50 ms:

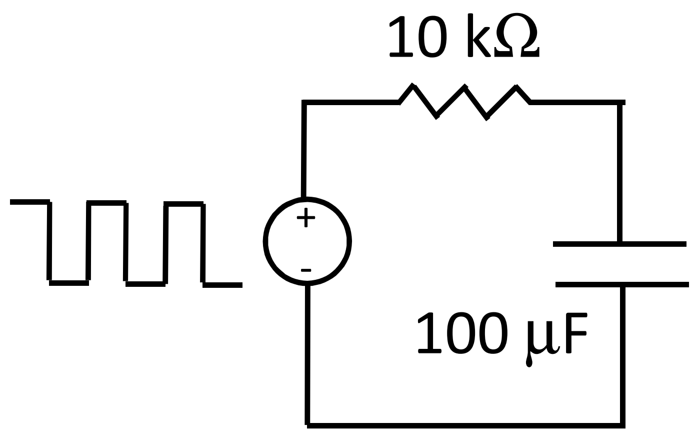
$$I_L = \begin{cases} 0.4 - 0.6e^{-20t}, & 0 \leq t < 0.05 \\ 0.4 - 0.222e^{-25t}, & 0.05 \leq t \end{cases}$$

Example

- What is the voltage as a function of time in this RC circuit with an AC square wave source at 1 Hz, and voltage varying from 0 to 1 V?



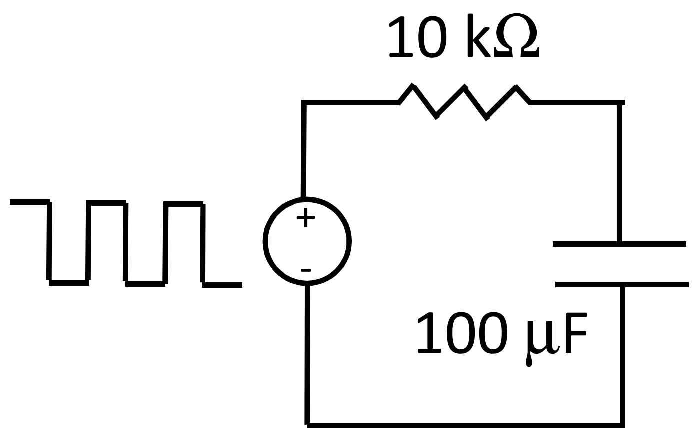
Solution



- In the presence of a periodic waveform, we must also have a periodic solution $V_c(t+mT) = V_c(t)$, for all integer m. The latter function is:

$$V_c(t) = \begin{cases} 1 + (v_o - 1)e^{-t}, & 0 \le t < 0.5 \\ v_{1/2}e^{-(t-1/2)}, & 0.5 \le t < 1 \end{cases}$$

Solution



- Our continuity conditions yield:

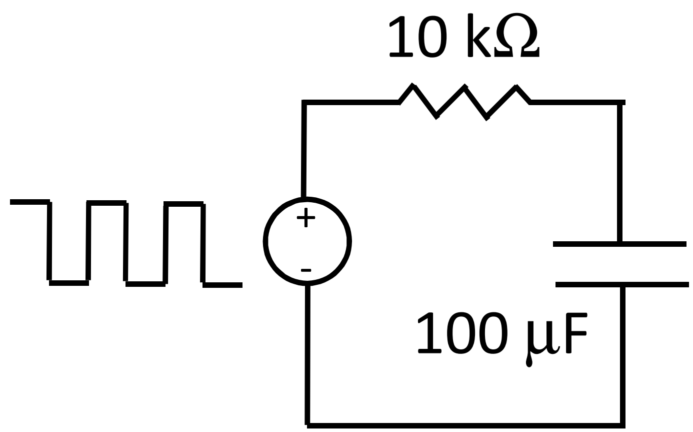
$$v_{1/2} = 1 + (v_o - 1)e^{-1/2}$$

$$v_o = v_{1/2}\, e^{-1/2}$$

- Combining terms:

$$v_o e^{1/2} = 1 + (v_o - 1)e^{-1/2}$$

Solution



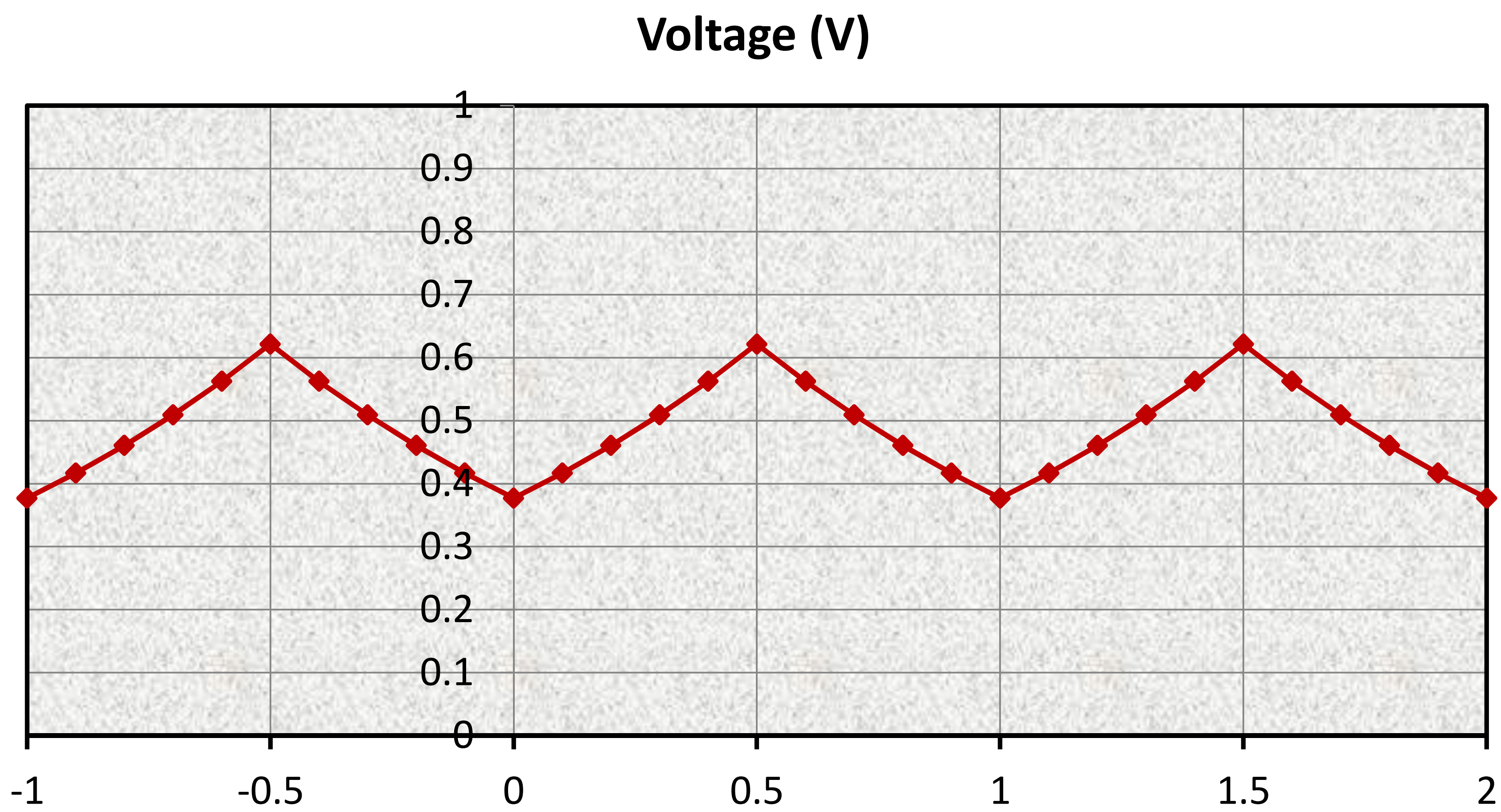
- Rearranging:

$$v_o\left(e^{1/2} - e^{-1/2}\right) = 1 - e^{-1/2}$$

- Numerically evaluating:

$$v_o = 0.377$$
$$v_{1/2} = 0.622$$

Solution

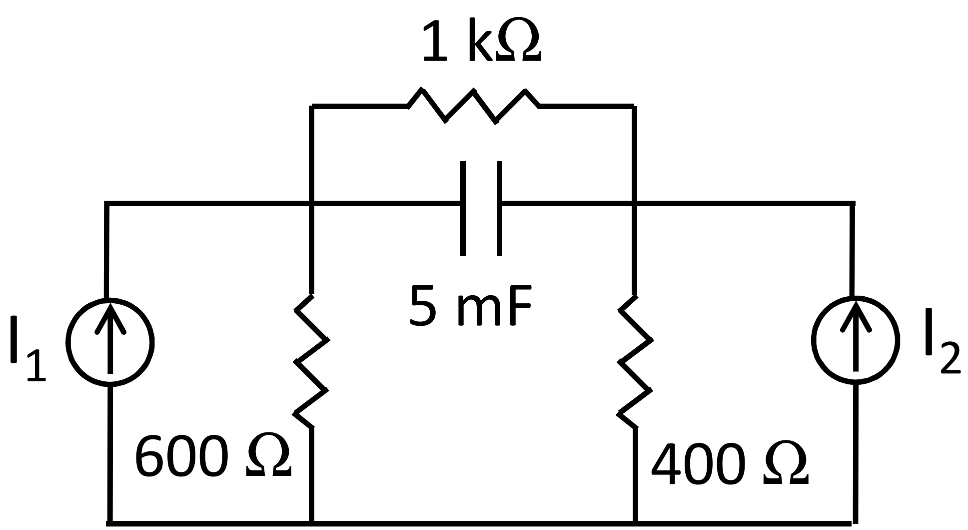


Linearity

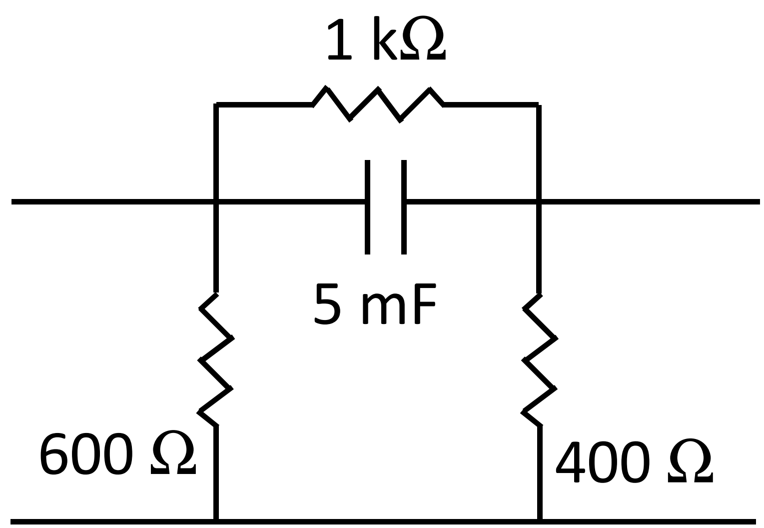
- Much like with networks of resistors and sources, networks with capacitors and resistors obey these principles:
  - Linearity
  - Superposition
  - Proportionality
- Initial conditions can be viewed as another superimposed source that shuts off at the beginning

Example

- Using superposition, calculate V_C for zero input (V_o=25 V), I_1 only (50 mA), I_2 only (25 mA), all combined, and when the sources are cut in half.



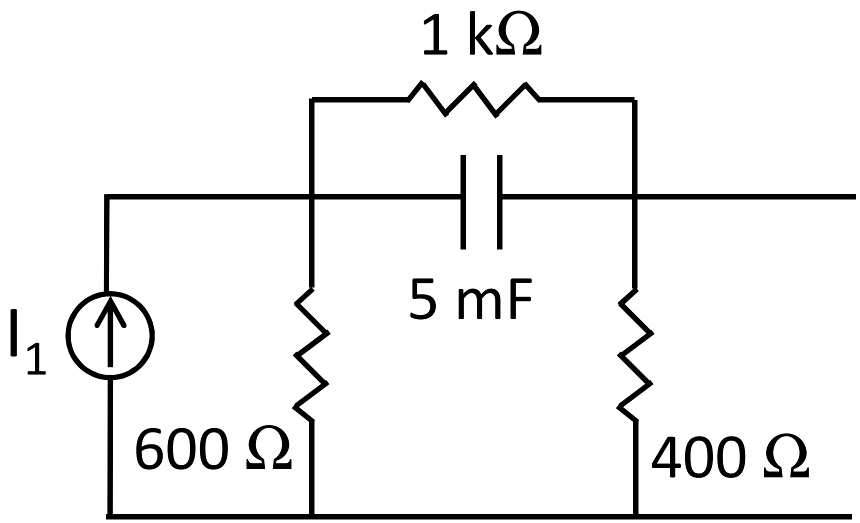
Solution



- V_C for zero input: capacitor sees two resistors of 1 kΩ; R_{th}=500 Ω:

$$V_C = 25\ e^{-0.4t}$$

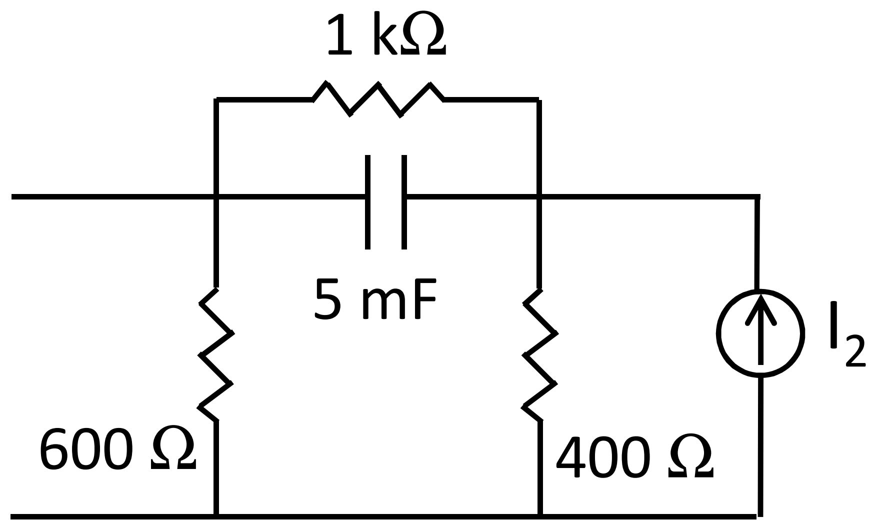
Solution



V_C from I_1 only: source transformation yields V_{oc}=30 V; combine series resistors; transform for I_{sc}=30 mA; R_{th}=500 Ω; transform again for V_{oc}=15 V. With τ=(5 mF)(500 Ω)=2.5 s, we obtain:

$$V = 15(1 - e^{-0.4t})$$

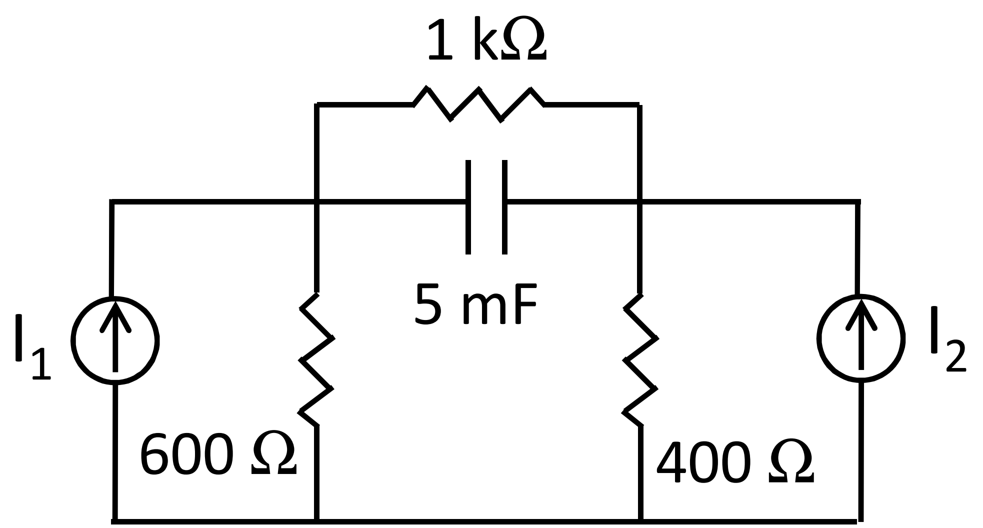
Solution



V_C from I_2 only: source transformation yields V_{oc}=-10 V; combine series resistors; transform for I_{sc}=-10 mA; R_{th}=500 Ω; transform again for V_{oc}=-5 V. With τ=(5 mF)(500 Ω)=2.5 s, we obtain:

$$V = -5(1 - e^{-0.4t})$$

Solution



Putting everything together, we obtain:

$$V = 10 + 15e^{-0.4t}$$

If the input sources are both cut in half, we get:

$$V = 5 + 20e^{-0.4t}$$

Homework

- HW #16 due today by 4:30 pm in EE 326B
- HW #17 due Wed.: DeCarlo & Lin, Chapter 7:
  - Problem 35
  - Problem 37
  - Problem 43
  - Problem 45
- HW #18 due Fri.: DeCarlo & Lin, Chapter 8:
  - Problem 1
  - Problem 5
  - Problem 6