ECE 201, Section 3
Lecture 19

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Solving Ordinary Differential Equations

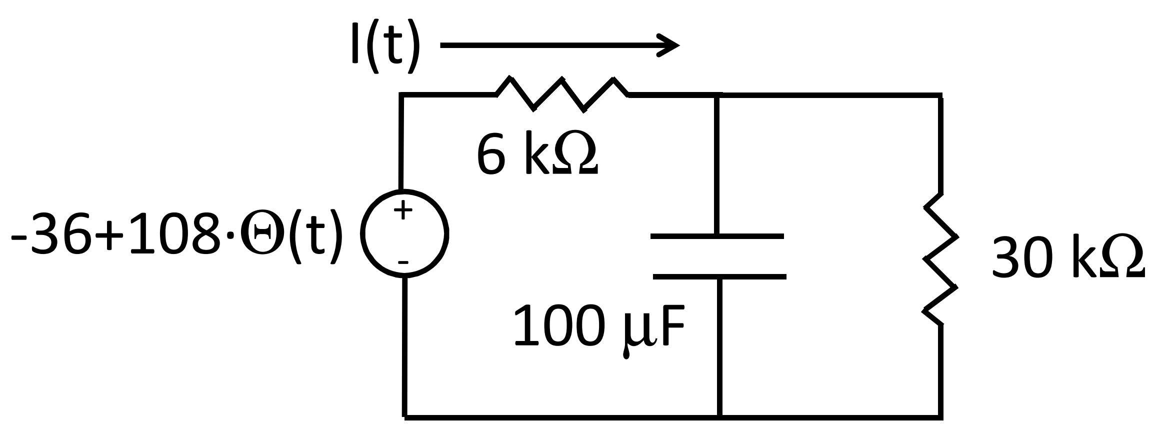
- For RC and RL circuits, we obtain:

$$X = X_\infty + (X_o - X_\infty)e^{-(t-t_o)/\tau}$$

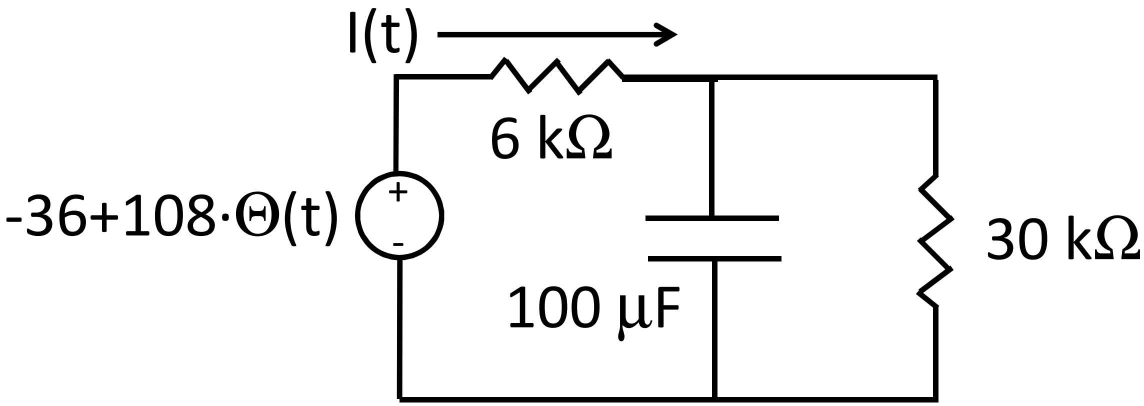
- Solution steps:
  - Choose X
    - RC circuits: X=Q or X=V
    - LR circuits: X=I
  - Find X_o (simplifying diagram)
  - Find X_∞ (simplifying diagram)
  - Find R_{th} (for inductor/capacitor)
  - Find time constant τ:
    - RL circuits: $\tau = L/R_{th}$
    - RC circuits: $\tau = R_{th}C$
  - If circuit changes at t_1, use $X(t_1)$ from prior solution for initial values

Example 1

- For this RC circuit, find the current flow I(t) at all times, including the discontinuity at t=0.



Solution

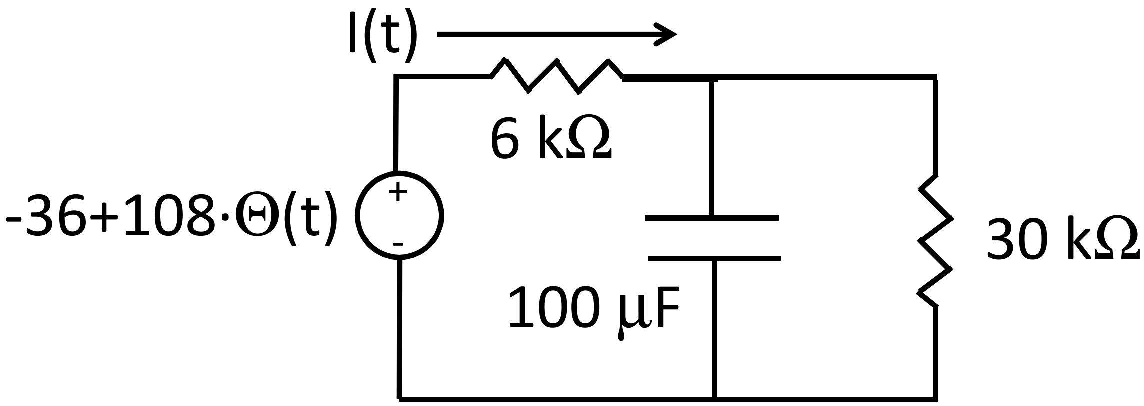


- For t<0: no capacitor current,

  $I=(-36\ V)/(30+6\ k\Omega)=-1\ mA$

  $V_C=(30/36)*(-36\ V)=-30\ V$
- At $t=0^+$, V_C is continuous, so $I=(72+30)/6=17\ mA$
- As $t \rightarrow \infty$, $I=2\ mA$
- Time constant $\tau=R_{th}C=(5\ k\Omega)*(100\ \mu F)=0.5\ s$

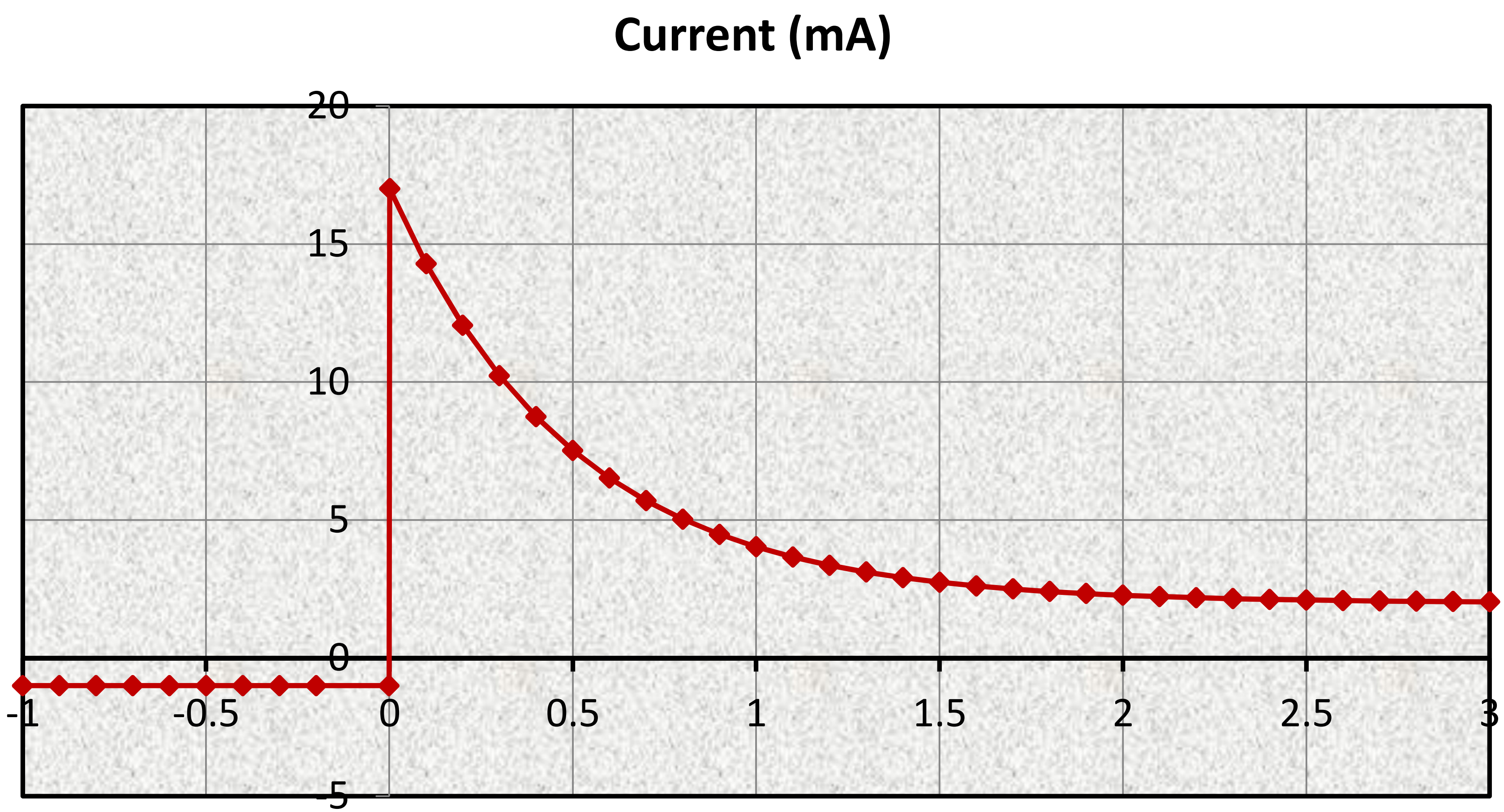
Solution



- Matching our solution with the boundary conditions yields:

$$I = \begin{cases} -1 \text{ mA}, & t < 0 \\ 2 + 15e^{-2t}, & t > 0 \end{cases}$$

Solution

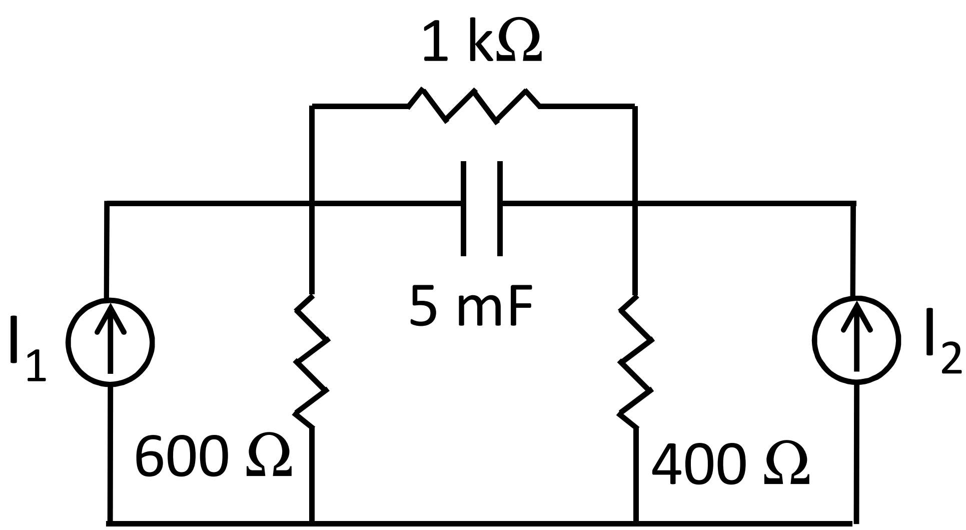


Linearity

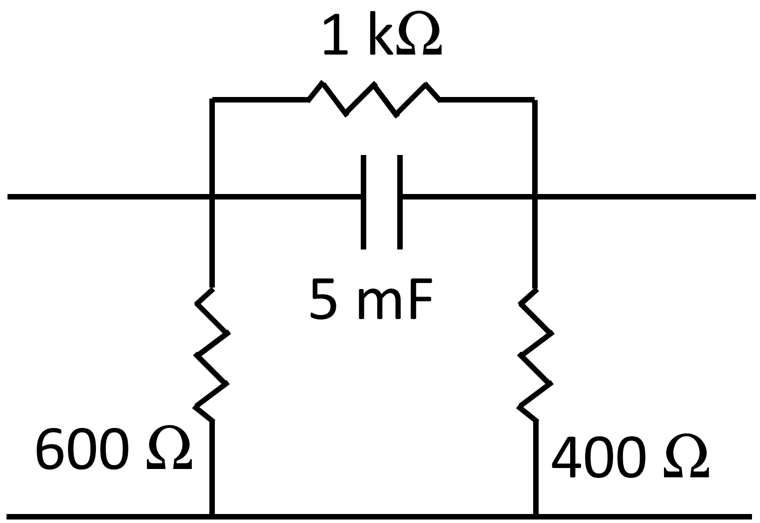
- Much like with networks of resistors and sources, networks with capacitors and resistors obey these principles:
  - Linearity
  - Superposition
  - Proportionality
- Initial conditions can be viewed as another superimposed source that shuts off at the beginning

Example 2

- Using superposition, calculate V_C for zero input (V_o=25 V), I_1 only (50 mA), I_2 only (25 mA), all combined, and when the sources are cut in half.



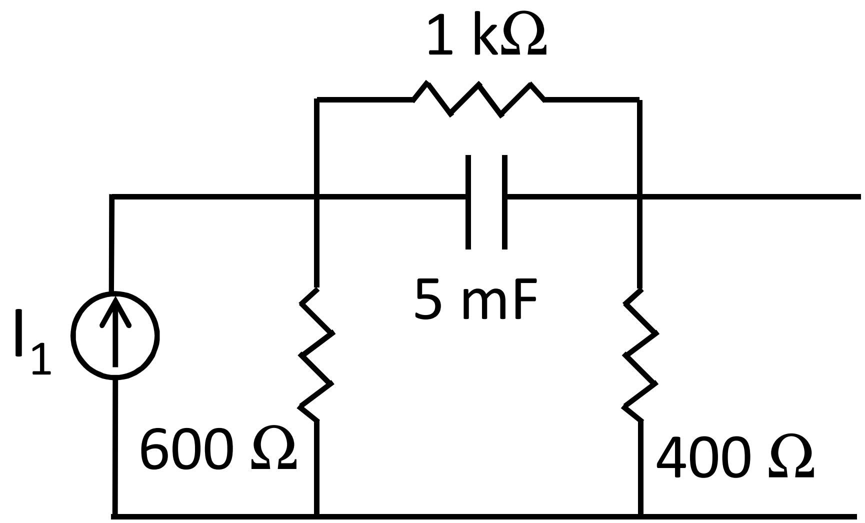
Solution



- V_C for zero input: capacitor sees two resistors of 1 kΩ; R_{th}=500 Ω:

$$V_C = 25\ e^{-0.4t}$$

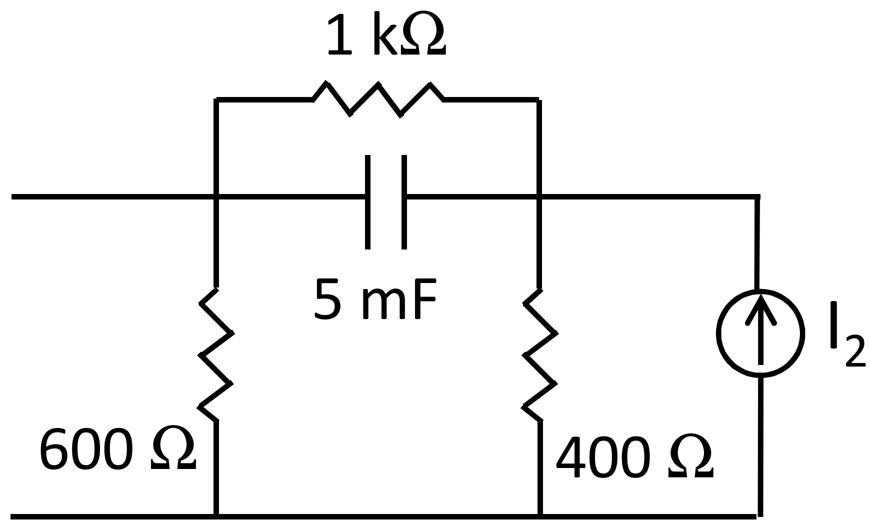
Solution



V_C from I_1 only: source transformation yields V_{oc}=30 V; combine series resistors; transform for I_{sc}=30 mA; R_{th}=500 Ω; transform again for V_{oc}=15 V. With τ=(5 mF)(500 Ω)=2.5 s, we obtain:

$$V = 15(1 - e^{-0.4t})$$

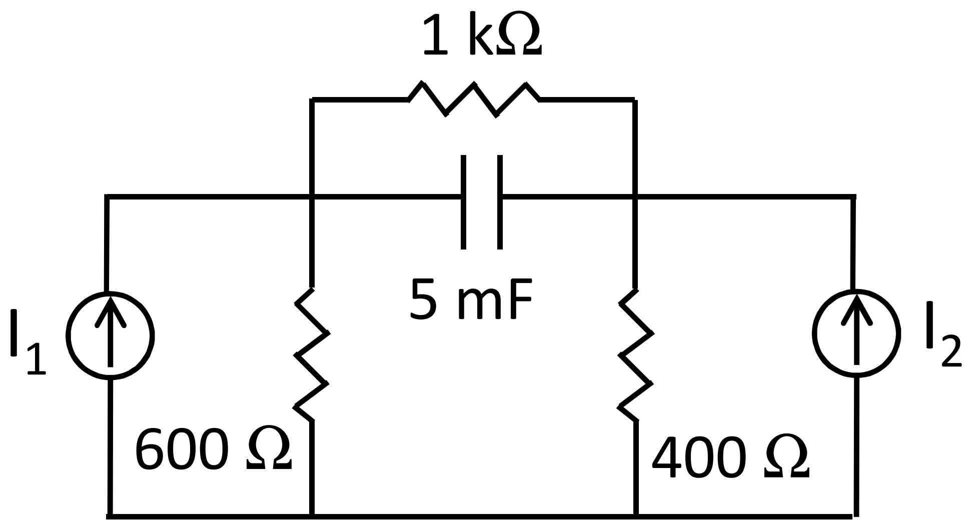
Solution



V_C from I_2 only: source transformation yields V_{oc}=-10 V; combine series resistors; transform for I_{sc}=-10 mA; R_{th}=500 Ω; transform again for V_{oc}=-5 V. With τ=(5 mF)(500 Ω)=2.5 s, we obtain:

$$V = -5(1 - e^{-0.4t})$$

Solution



Putting everything together, we obtain:

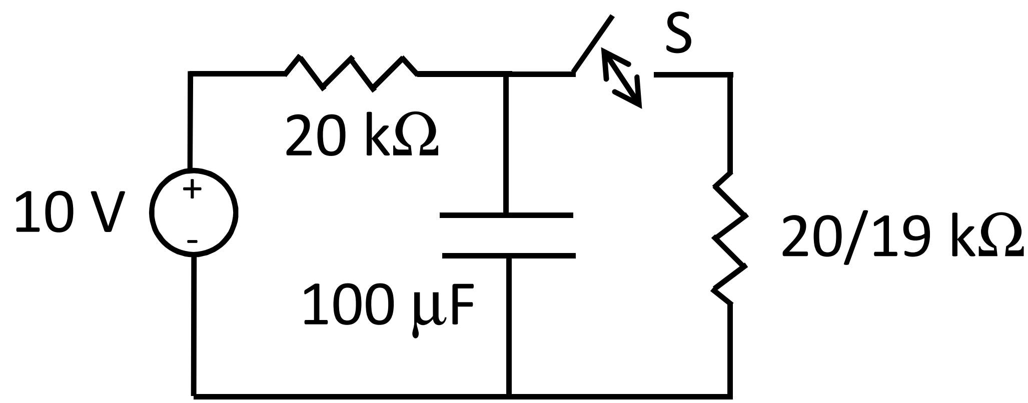
$$V = 10 + 15e^{-0.4t}$$

If the input sources are both cut in half, we get:

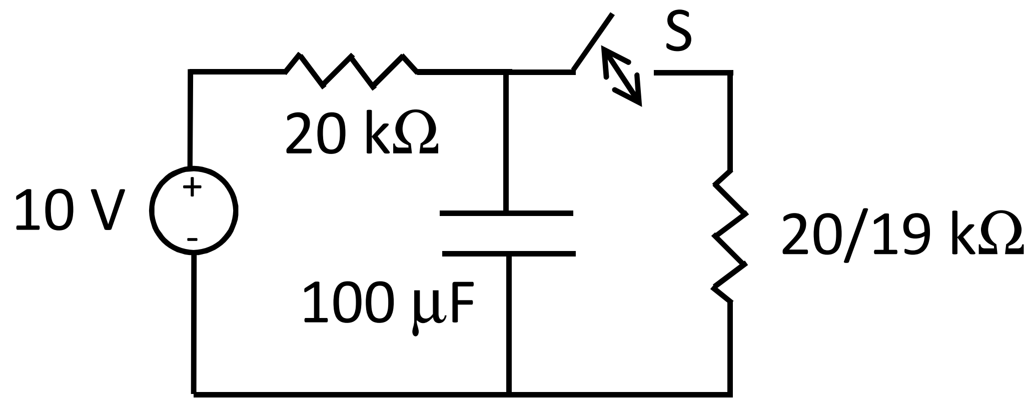
$$V = 5 + 20e^{-0.4t}$$

Example 3

- What waveform is produced by this circuit with a mechanical switch S that flips on for 1 s, then off for 1 s, and so on?

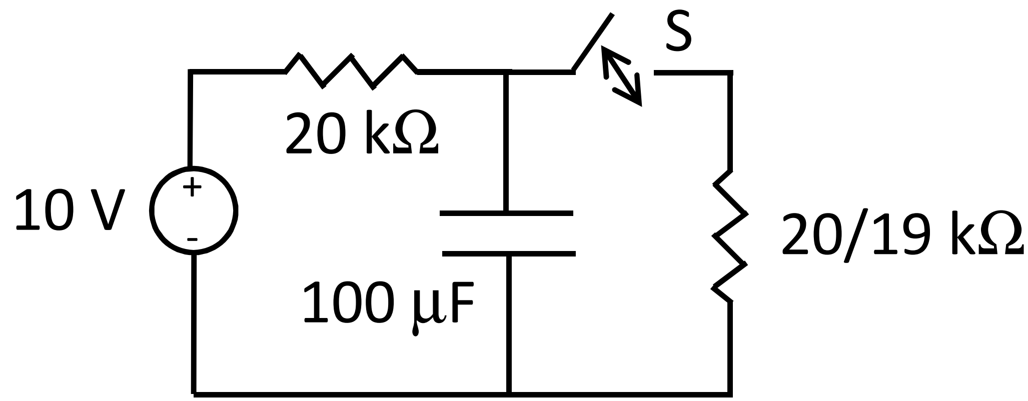


Solution



- Consider 2 cases separately:
  - Switch open: RC=(20 kΩ)(100 μF)=2 s, $V_\infty = 10\text{ V}$
  - Switch closed: RC=(1 kΩ)(100 μF)=0.1 s, $V_\infty = 0.5\text{ V}$
- After closing switch, will quickly reach final value

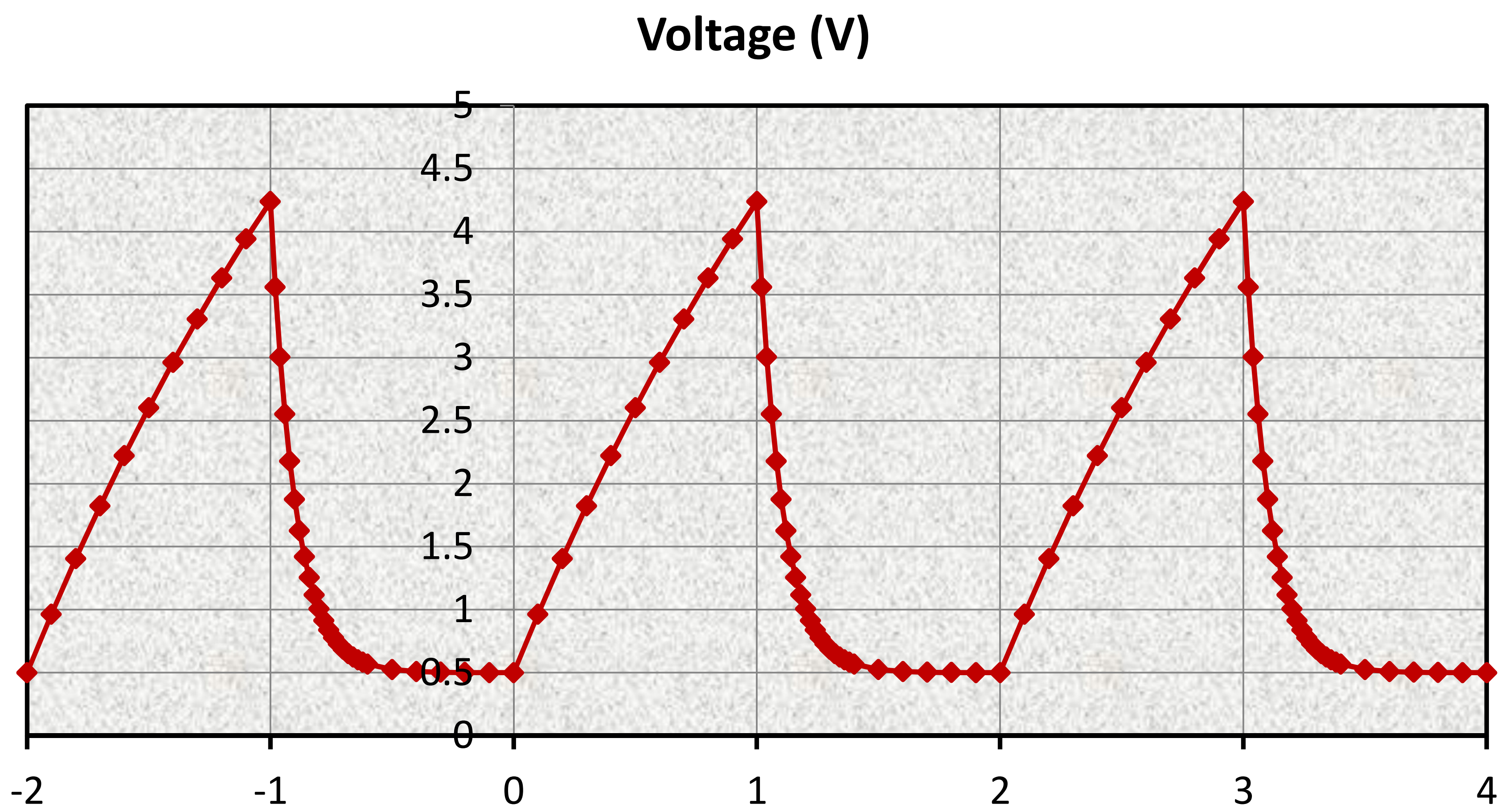
Solution



- Thus, to an excellent approximation:

$$V_C(t) = \begin{cases} 10 - 9.5e^{-t/2}, & 0 \le t < 1 \\ 0.5 + 3.74e^{-10(t-1)}, & 1 \le t < 2 \end{cases}$$

Solution



Homework

- HW #18 due today by 4:30 pm in EE 326B
- HW #19 due Wed.: DeCarlo & Lin, Chapter 8:
  - Problem 18(a),(b)
  - Problem 21(a),(b)
  - Problem 22