ECE 201, Section 3
Lecture 31

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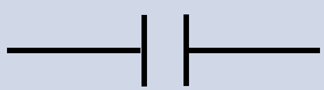
Phasor Review

- Shorthand for writing complex numbers:

$$\boldsymbol{V} = V_m \angle \phi = V_m e^{j(\omega t + \phi)}$$

- Ohm's law with phasors:

$$\boldsymbol{V} = Z(j\omega)\boldsymbol{I}$$

Circuit Element	Impedance	Admittance
	$Z(j\omega) = R$	$Y(j\omega) = \frac{1}{R}$
	$Z(j\omega) = j\omega L$	$Y(j\omega) = \frac{1}{j\omega L}$
	$Z(j\omega) = \frac{1}{j\omega C}$	$Y(j\omega) = j\omega C$

Impedance Properties

- For circuit elements in series, voltage division rule becomes:

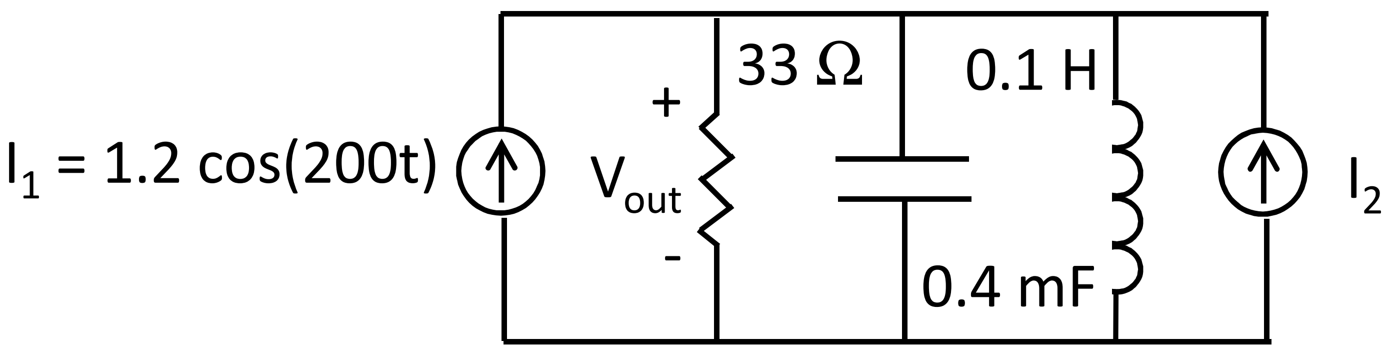
$$V_k = \frac{Z_k}{Z_{eq}} V_{tot}$$

- For circuit elements in parallel, current division rule becomes:

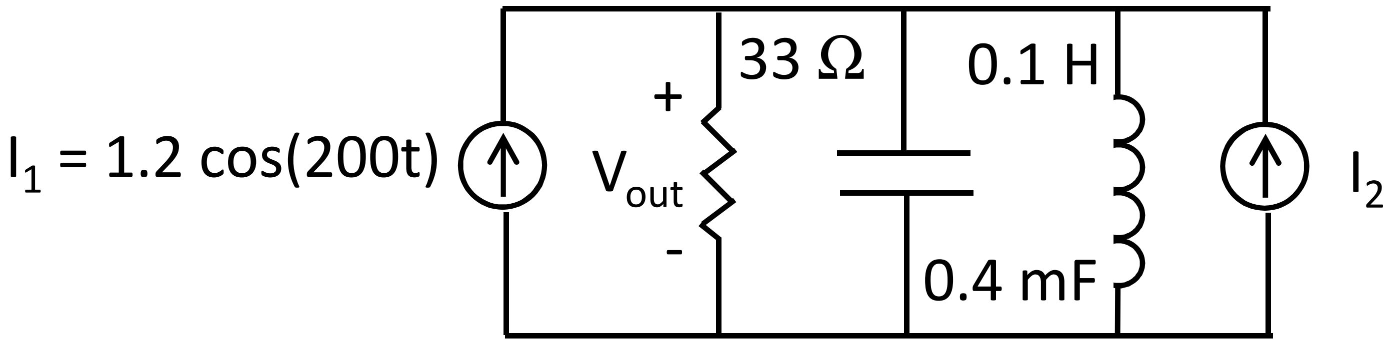
$$I_k = \frac{Y_k}{Y_{eq}} I_{tot}$$

Example #1

- If V_{out}=40 sin(200t), what are I_1 and V_{out}? What is the phasor I_2?



Solution



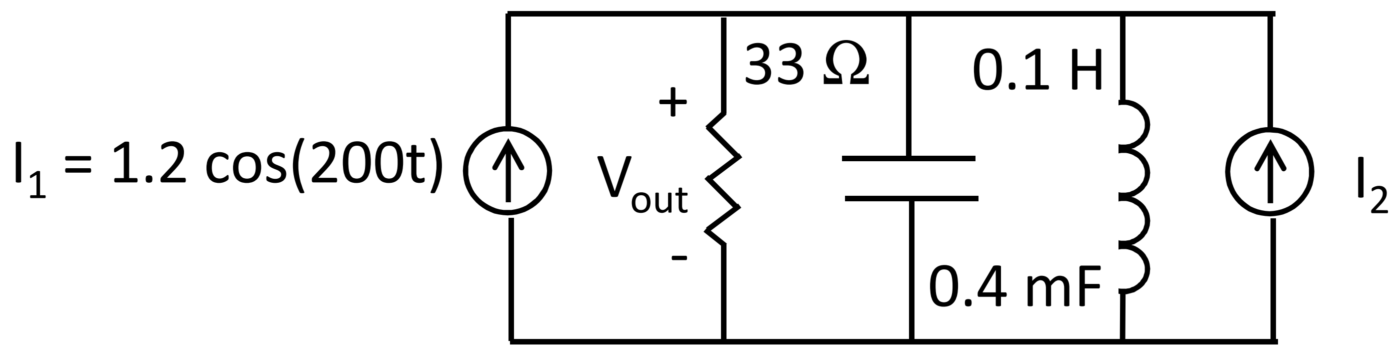
- Phasors given by:

$$I_1 = 1.2\angle 0°$$
$$V_{out} = 40\angle -90°$$

- Calculate admittance:

$$Y = \frac{1}{33} + 0.0004 \cdot 200j + \frac{1}{0.1 \cdot 200j} \approx 0.03 - 0.03j$$

Solution



- Total current given by:

$$\boldsymbol{I} = Y\boldsymbol{V} = 0.0424e^{-j45°} \cdot 40e^{j(200t-90°)}$$

$$\boldsymbol{I} = Y\boldsymbol{V} = 1.696e^{j(200t-135°)} = 1.696\angle-135°$$

- Current on the right is given by:

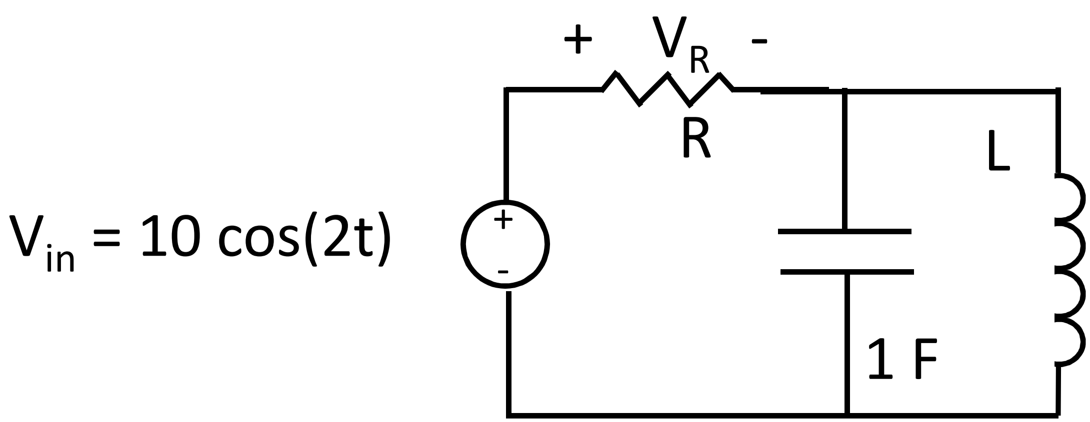
$$\boldsymbol{I}_2 = \boldsymbol{I} + \boldsymbol{I}_1$$

$$\boldsymbol{I}_2 = 1.696\cos(-135°) + j1.696\sin(-135°) + 1.2$$

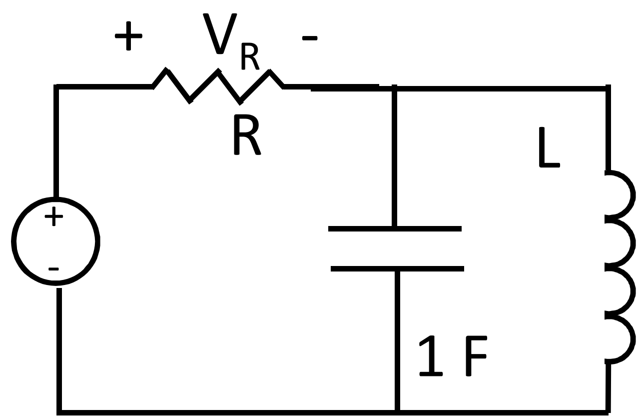
$$\boldsymbol{I}_2 = -1.2j = 1.2\angle-90°$$

Example #2

- If Z(2j)=4+2j, what are R and L? What is the voltage phasor across the resistor?



Solution



V_{in} = 10 cos(2t)

- Impedance here is given by:

$$Z(j\omega) = R + \left[j\omega C + \frac{1}{j\omega L}\right]^{-1}$$

- Setting equal to our target impedance at ω=2:

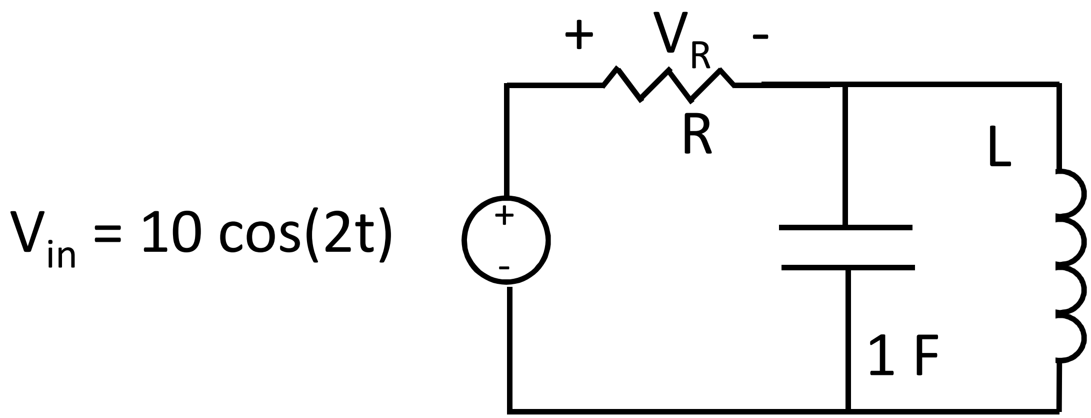
$$Z(2j) = R + \left[2j + \frac{1}{2jL}\right]^{-1} = 4 + 2j$$

- Equating real and imaginary parts yields R=4 Ω and:

$$\frac{1}{2j} = 2j + \frac{1}{2jL}$$

$$2jL = \frac{1}{\frac{1}{2j} - 2j} = \frac{2j}{5}$$; thus, L=0.2 H

Solution



- Using the voltage division rules:

$$V_R = \frac{R}{Z(2j)} V_{in}$$

- Substituting the values found before:

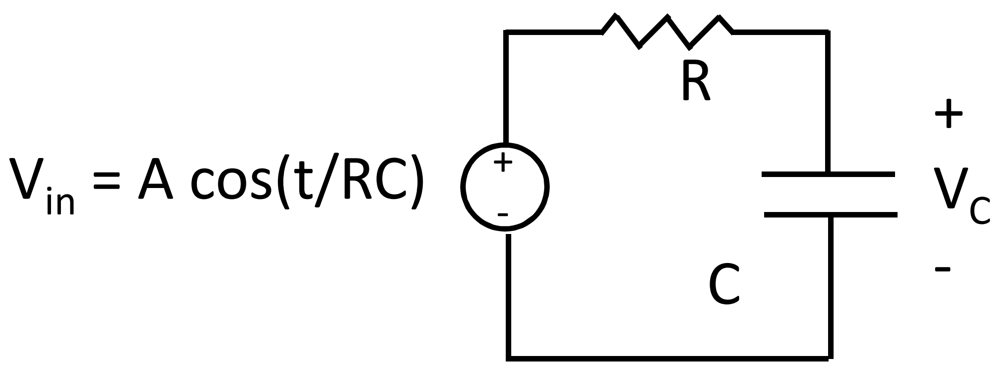
$$V_R = \frac{4}{4+2j} 10e^{j2t}$$

$$V_R = \frac{40e^{j2t}}{\sqrt{4^2+2^2}e^{j\tan^{-1}2/4}}$$

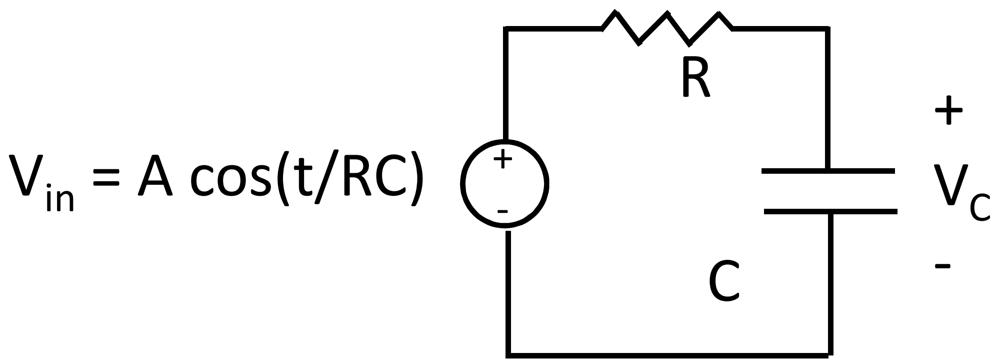
$$V_R = 8.94e^{j(2t-26.6°)}$$

Example #3

- What is the voltage across the capacitor as a function of time, with a source A cos(t/RC)?



Solution



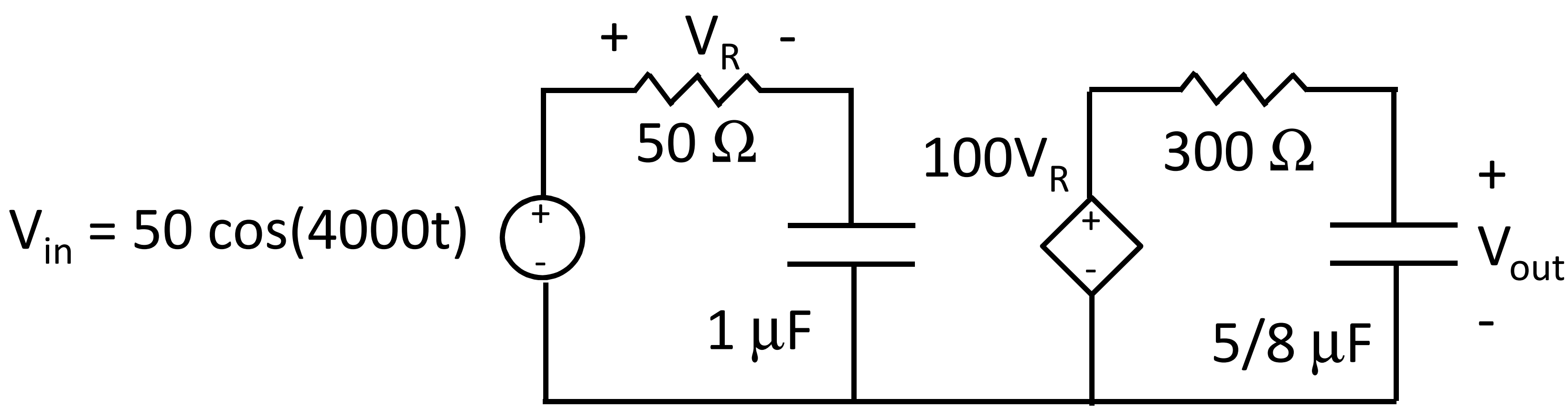
- From the voltage division rule:

$$V_C = \frac{-jRC/C}{R - jRC/C} A e^{jt/RC}$$

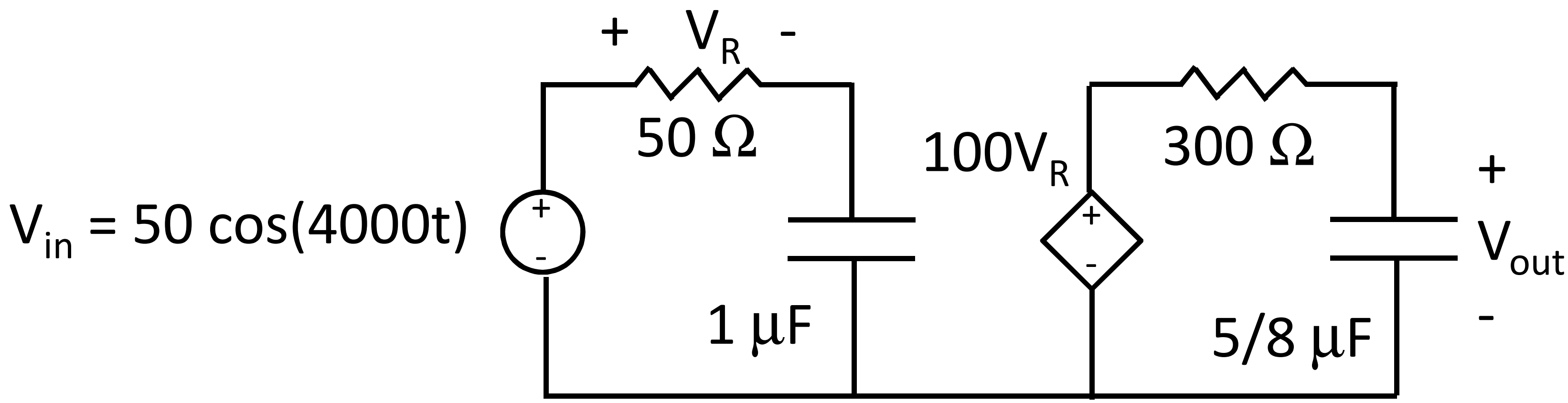
$$V_C = \frac{A}{\sqrt{2}} e^{j\left(\frac{t}{RC} - 90° + 45°\right)}$$

Example #4

- What is the voltage across the right capacitor V_{out} as a function of time?



Solution



- On the left side, voltage division yields:

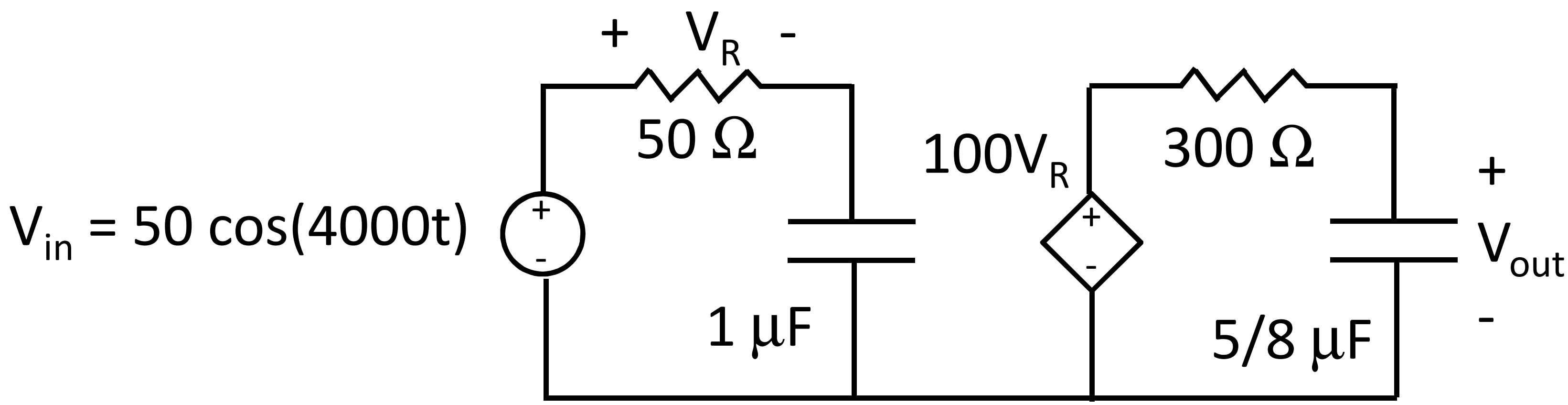
$$V_R = \frac{50 \cdot 50 e^{j4000t}}{50 + 1/(10^{-6} \cdot 4000j)}$$

$$V_R = 9.806 e^{j(4000t + 78.69°)}$$

- Thus, our VCVS has an output of:

$$V_s = 980.6 e^{j(4000t + 78.69°)}$$

Solution



- Since our VCVS voltage is $980.6e^{j(4000t+78.69°)}$
- Voltage division on the right-hand side yields:

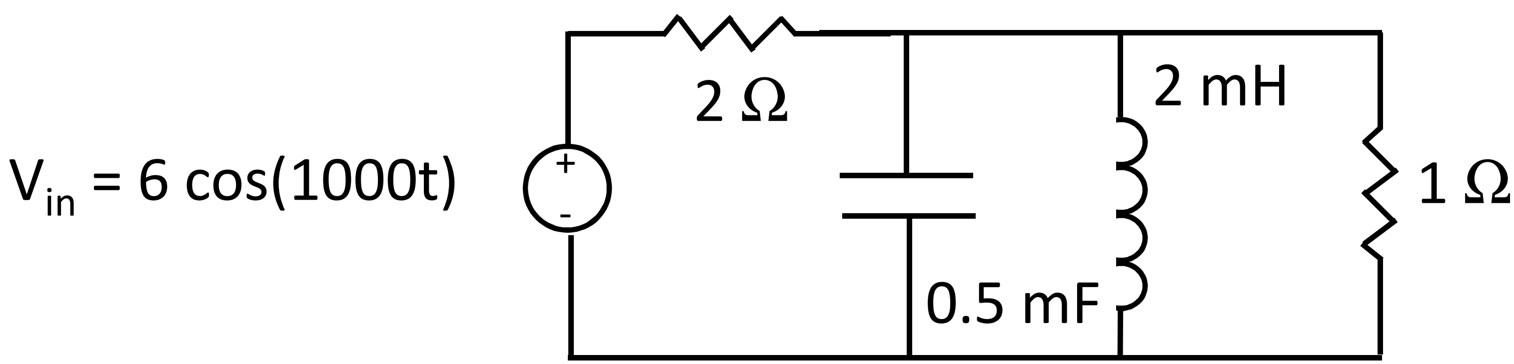
$$V_{out} = \frac{980.6e^{j(4000t+78.69°)}/(j4000 \cdot 5/8 \cdot 10^{-6})}{300 + 1/(j4000 \cdot 5/8 \cdot 10^{-6})}$$

$$V_{out} = 784.5e^{j(4000t+78.69°-90°+53.13°)}$$

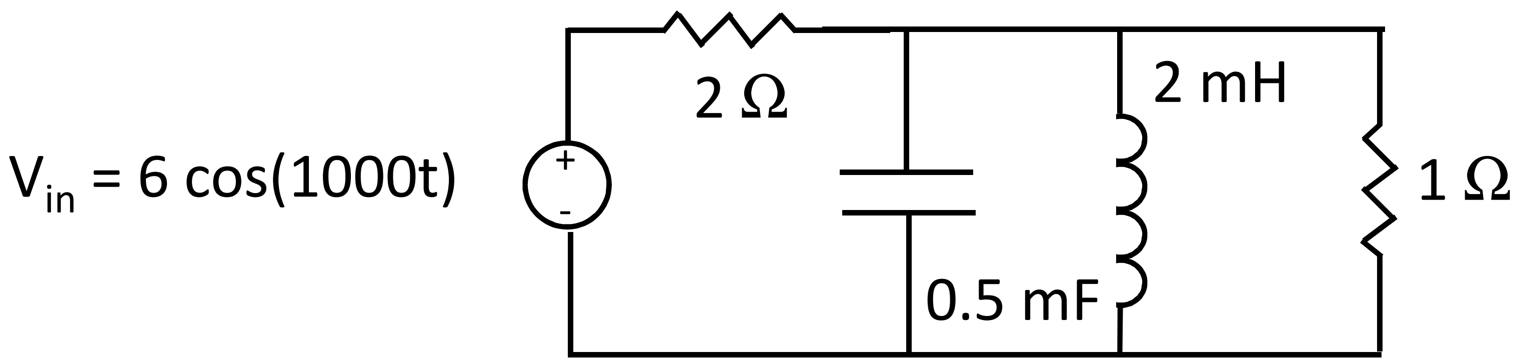
$$V_{out} = 784.5e^{j(4000t+41.82°)}$$

Example #5

- What total current flows through this circuit, and how much flows through the inductor?



Solution



- The impedance is given by:

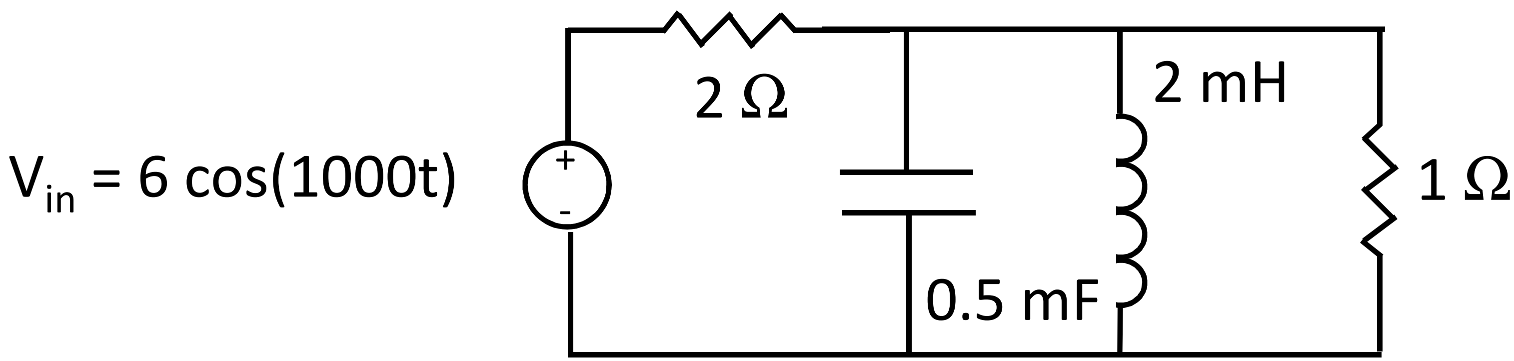
$$Z = 2 + \left[1 + j \cdot 10^3 \cdot 5 \cdot 10^{-4} + \frac{1}{j \cdot 10^3 \cdot 2 \cdot 10^{-3}}\right]^{-1}$$

$$Z = 3\ \Omega$$

- The total current is thus given by:

$$I_{tot} = \frac{V}{Z} = \frac{6e^{j1000t}}{3} = 2e^{j1000t}$$

Solution



- Applying the current division rule:

$$I_L = \frac{Y_L}{Y} I_{tot}$$

$$I_L = \frac{-0.5j}{1} 2e^{j1000t} = e^{j(1000t-90°)}$$

Homework

- HW #30 due today by 4:30 pm in EE 325B
- HW #31 due Fri.: DeCarlo & Lin, Chapter 10:
  - Problem 7
  - Problem 8
  - Problem 10 [Correction: In ANSWER, the phase angle of V_x is 71.56°]